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Statistical Distributions of the Lapped Transform Coefficients for Images

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Abstract—Discrete Cosine Transform (DCT) based transform coding is very popular in image, video and speech compression due to its good energy compaction and decorrelating properties. However, at low bit rates, the reconstructed images generally suffer from visually annoying blocking artifacts as a result of coarse quantization. Lapped transform was proposed as an alternative to the DCT with reduced blocking artifacts and increased coding gain. Lapped transforms are popular for their good performance, robustness against oversmoothing and availability of fast implementation algorithms. However, there is no proper study reported in the literature regarding the statistical distributions of block Lapped Orthogonal Transform (LOT) and Lapped Biorthogonal Transform (LBT) coefficients. This study performs two goodness-of-fit tests, the Kolmogorov-Smirnov (KS) test and the χ^2-test, to determine the distribution that best fits the LOT and LBT coefficients. The experimental results show that the distribution of a majority of the significant AC coefficients can be modeled by the Generalized Gaussian distribution. The knowledge of the statistical distribution of transform coefficients greatly helps in the design of optimal quantizers that may lead to minimum distortion and hence achieve optimal coding efficiency.

Keywords—Lapped orthogonal transform, Lapped biorthogonal transform, Image compression, KS test, χ^2-test

I. Introduction

The lapped transforms (LT) [9], [10], [11], [12] have been proposed as an alternative to the discrete cosine transform (DCT) with reduced blocking artifacts and better energy compaction. The two important properties of LTs which lead to significant reduction of blocking artifacts are

- The basis functions are longer than the block size
- The basis functions decay to zero smoothly at the boundaries

The basis function of LOT decay nearly to zero at the boundaries which leads to considerable reduction in blocking artifacts, though not totaly eliminated [11], [12]. Since the LBT synthesis basis functions decay to zero at the boundaries, the blocking artifacts are almost eliminated in LBT. The LOT and LBT have almost the same computational complexity and LBT has higher coding gain compared to LOT and DCT [11], [12]. Higher coding gain provides lower reconstruction error energies. Successful application of an transform in image compression requires knowledge of the statistical distribution of the transform coefficients. However, till now, no definitive study has been reported on the distribution of lapped transform coefficients of images.

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In contrast, there is a large body of studies in the literature dealing with the distributions of 2D DCT coefficients of natural images [15], [14], [4], [3]. Rieninger et al. [15] study the statistical distribution of the 2D DCT coefficients considering Gaussian, Laplacian, Gamma and Rayleigh distributions as probable models. Based on the Kolomogrov-Smirnov (KS) goodness of fit test they found that the statistics of the DC coefficients are best approximated by a Gaussian distribution while the non DC coefficients are best approximated by the Laplacian distribution. Through some mathematical analysis, Lam and Goodman [13] proved that 2D DCT coefficients are better modeled by the Laplacian distribution. Eggerton et al. [4] concluded that no particular density function can be used for each of the coefficients but Laplacian fits the majority of the coefficients. They also found that when all the coefficients are lumped into one density function, the Cauchy distribution provides the best fit. Muller [14], found that the Generalized Gaussian distribution best approximates the statistics of the 2D DCT coefficients. In [6], Joshi and Fischer compared the performance of Generalized Gaussian and Laplacian models when applied to image coding; the authors concluded that the more complex Generalized Gaussian model does not give significant advantage over the Laplacian model. In [3], Chang et al. concluded that the Generalized Gamma distribution best models the statistics of the 2D DCT coefficients. Smoot and Reeve [18] study the statistics of the DCT coefficients of the differential signal obtained after motion estimation. They observe that the statistics are best approximated by the Laplacian distribution. Bellifemine et al. [1] demonstrate that the DCT coefficients of the differential signal obtained after motion estimation are best approximated by the Laplacian distribution. Recently, Malavika et al. [2] studied the statistics of 3D DCT coefficients for video, considering Gaussian, Laplacian, Gamma and Rayleigh distributions as probable models and concluded that no single distribution can be used to model the distributions of all the coefficients for different video sequences; however Gamma distribution fits the majority of the significant AC coefficients while the DC coefficients can be well approximated by Gaussian distribution.

This paper studies the statistical distributions that best approximates the statistics of 2D LOT and LBT coefficients. We use KS and χ^2 goodness of fit test considering Gaussian, Laplacian, Gamma and Generalized Gaussian distributions as probable models as these distributions are commonly used for statistical modeling of DCT coefficients [15], [14], [4], [2]. The paper is organized as follows. Section II gives an brief overview of the Lapped transform. KS and χ^2 goodness of fit test are described in Section III. Section IV discuss the probability distributions used in the study. Experimental results are presented in Section V. Finally, Section VI concludes the paper.

II. Lapped Transforms

The lapped orthogonal transform (LOT) [9][10] has been proposed to overcome the blocking artifacts of the DCT and has increased coding gain. The lapped transforms has extended basis functions which overlaps across the block boundaries. In lapped transforms, the input signal length is two times its output signal length.

$$L = 2M \tag{1}$$

where *M* is the output signal length and *L* is the input signal length. The initial LOT matrix *P* which may not be necessarily optimal is given by

$$P = \frac{1}{2}\begin{pmatrix} D_e - D_o & D_e - D_o \\ J(D_e - D_o) & -J(D_e - D_o) \end{pmatrix} \tag{2}$$

where D_e and D_o are the *M x M/2* matrices containing the even and odd DCT functions respectively and *J* is the counter identity matrix. The optimal LOT matrix [9] is given by

$$P_0 = PZ \tag{3}$$

for an optimal *Z*. The covariance matrix of LOT coefficients is given by

$$R_0 = Z'P'R_{xx}PZ \tag{4}$$

where R_{xx} is the given signal covariance matrix [9].

$$R_{xx} = \begin{pmatrix} 1 & \rho & \rho^2 & .... & \rho^L \\ \rho & 1 & \rho & .... & \rho^{L-1} \\ . & & . & & . \\ . & & . & & . \\ \rho^{L-1} & .... & \rho & 1 & \rho \\ \rho^L & ... & \rho^2 & \rho & 1 \end{pmatrix} \tag{5}$$

We assume the signal model to be first order markov model with $\rho = 0.95$. From equation no. (4), when columns of *Z* are the eigen vectors of $P'R_{xx}P$ that is R_0 is diagonal, the transform coding gain [9] is maximized. The LOT matrix P_0 is optimal for such *Z*.

Although LOT can significantly reduce the blocking artifacts of images, some blocking artifacts are still visible as the LOT basis functions do not decay exactly to zero at the boundaries. In contrast the LBT [11], [12] bases decay to zero at the boundaries resulting in LBT exhibiting fewer blocking artifacts as compared to LOT. The LBT can be computed from the original LOT computation flowgraph with few modifications [11], [12]. The lapped transforms can be viewed as critically sampled multirate filter banks.

III. Goodness-of-fit tests

A. Kolomogrov - Smirnov (KS) Goodness of Fit Tests

Kolomogrov-Smirnov test [16] is one of the popular goodness of fit test used in [15], [4], [18], [2] for determination of distributions of DCT coefficients of an image or video. The KS goodness of fit test statistic is a distance measure between the empirical cumulative distribution function (CDF) for a given data set and the given model cumulative distribution function. For a given sample data set

$$X = \{x_1, x_2, ..., x_T\} \tag{6}$$

having the order statistics $x_{(s)}, s = 1, 2.............T$, the empirical cumulative distribution function is given as

$$\tilde{F}_X(x) = \begin{cases} 0, x < x_{(1)} \\ 1, x \geq x_{(T)} \\ \frac{s}{T}, x_{(s)} \leq x < x_{(s+1)}, s = 1, 2........, T-1 \end{cases} \tag{7}$$

The KS goodness of fit test statistic is given as

$$KS_{stat} = \max_{j=1,2,.........,T} \left| F_X(x_{(j)}) - \tilde{F}_X(x_{(j)}) \right| \tag{8}$$

When testing several distributions against the sample data, the one that gives the smallest KS statistic KS_{stat} is considered to be the best fit for the data.

B. χ^2 Goodness of Fit Tests

χ^2 test [16] is also one of the widely used goodness of fit test [1], [14], [3], [2] for determination of distributions of DCT coefficients of an image or video. The χ^2 goodness of fit test compares the model probability density functions with the empirical data and finds out the distortion by the following equation

$$\chi^2 = \sum_{i=1}^{k_d} \frac{(O_i - E_i)^2}{E_i} \tag{9}$$

where the range of data is partitioned into k_d disjoint and exhaustive bins B_i, *i*=1,2,3,......k_d. $E_i = n_c p_i$ is the expected frequency in bin B_i where $p_i = P(x \in B_i)$ and O_i is the observed frequency in bin B_i. n_c is the total number of data samples. The model probability density function which gives the minimum χ^2 value can be considered as the best fit.

IV. Probability Distributions

In the KS and the χ^2 goodness of fit tests, Gaussian, Laplacian, Gamma and Generalized Gaussian distributions were considered. The parameters of all the distributions were found using the maximum likelihood (ML) method [7].

A. Gaussian probability density function

The Gaussian probability density function is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{10}$$

where μ is the mean and σ^2 is the variance. The ML estimates of μ and σ^2 are given by

$$\widehat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{11}$$

$$\widehat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i - \widehat{\mu})^2 \tag{12}$$

B. Laplacian probability density function

The Laplacian probability density function is given by

$$f_X(x) = \frac{1}{2b} \exp\left(\frac{-|x-\mu|}{b}\right) \quad (13)$$

where μ is the mean and b is the scale parameter. The variance is given by $2b^2$. The ML estimate of the parameter b is given by

$$\widehat{b} = \frac{1}{N} \sum_{i=1}^{N} |x_i - \widehat{\mu}| \quad (14)$$

where μ is estimated using (11).

C. Gamma probability density function

The Gamma probability density function [5] is given by

$$f_X(x) = \frac{\sqrt[4]{3}}{\sqrt{8\pi\sigma\,|x-\mu|}} \exp\left(\frac{-\sqrt{3}\,|x-\mu|}{2\sigma}\right) \quad (15)$$

The parameters μ and σ are estimated using (11) and (12) respectively.

D. Generalized Gaussian probability density function

The Generalized Gaussian probability density function [8] is given by

$$f_X(x) = \frac{\beta}{2\alpha\Gamma\left(\frac{1}{\beta}\right)} \exp - \left(\frac{|x-\mu|}{\alpha}\right)^{\beta} \quad (16)$$

where $\Gamma(.)$ is the Gamma function given by

$$\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt, z > 0 \quad (17)$$

and α, β respectively are known as the scale parameter and the shape parameter. For the special cases $\beta = 2$ or $\beta = 1$, the generalized Gaussian pdf becomes a Gaussian or a Laplacian pdf respectively. The ML estimation of the parameters α and β can be obtained as follows: The shape parameter $\widehat{\beta}$ is the solution of the equation

$$1 + \frac{\psi\left(\frac{1}{\beta}\right)}{\beta} - \frac{\sum_{i=1}^{N} |x_i|^{\beta} \log |x_i|}{\sum |x_i|^{\beta}} + \frac{\log\left(\frac{\beta}{N} \sum_{i=1}^{N} |x_i|^{\beta}\right)}{\beta} = 0 \quad (18)$$

where $\psi(.)$ is the digamma function given by

$$\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)} \quad (19)$$

The ML estimate of α (for known ML estimate $\widehat{\beta}$) is given by

$$\widehat{\alpha} = \left(\frac{\widehat{\beta}}{N} \sum_{i=1}^{N} |x_i|^{\widehat{\beta}}\right)^{\frac{1}{\widehat{\beta}}} \quad (20)$$

where N is number of observations. The $\widehat{\beta}$ is determined using the Newton Raphson iterative procedure [7] with the initial guess from the moment based method [17]. Experimental results indicates that only 3-4 iteration steps are required to compute the solutions within an accuracy of 10^{-6}.

In Table I, the estimated shape parameter $\widehat{\beta}$ of M=8 LOT coefficients for different test images are shown. Table II, shows $\widehat{\beta}$ values of M=8 LBT coefficients for the same images. In Table I and II, we can see that ML estimates of β are considerably different from 1 in most of the cases. For the Mandrill and the Bridge images, the statistics for a few coefficients is likely to be Laplacian (β=1). Further it may be observed that within a given image the estimated shape parameter values of different coefficients do not vary much.

TABLE I
ESTIMATED SHAPE PARAMETER $\widehat{\beta}$ OF LOT (M=8) COEFFICIENTS FOR DIFFERENT TEST IMAGES

	Lena	Barbara	Mandrill	Boat	Bridge	Couple
C_{10}	0.4583	0.5160	0.8329	0.5192	0.8495	0.4170
C_{11}	0.4148	0.4972	1.0039	0.5129	0.9812	0.4711
C_{01}	0.4574	0.5032	1.0553	0.4585	0.8654	0.5157
C_{20}	0.5678	0.5758	0.6934	0.5377	0.8856	0.5912
C_{02}	0.4927	0.5725	0.9316	0.4632	0.9025	0.4911
C_{12}	0.4363	0.4905	0.9139	0.5081	0.9345	0.4889
C_{21}	0.4680	0.5637	0.7568	0.5895	1.0562	0.5235
C_{03}	0.5746	0.5888	0.8918	0.4835	0.8572	0.4763
C_{30}	0.6747	0.7108	0.6918	0.6030	0.9673	0.5378

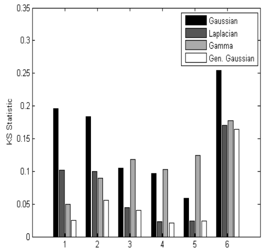
TABLE II
ESTIMATED SHAPE PARAMETER $\widehat{\beta}$ OF LBT (M=8) COEFFICIENTS FOR DIFFERENT IMAGES

	Lena	Barbara	Mandrill	Boat	Bridge	Couple
C_{10}	0.4682	0.5466	0.8695	0.5165	0.8172	0.3889
C_{11}	0.4232	0.5341	1.0496	0.5014	1.0032	0.4596
C_{01}	0.4732	0.5450	1.0381	0.4734	0.8739	0.5374
C_{20}	0.6329	0.6040	0.6603	0.5585	0.8559	0.5892
C_{02}	0.5189	0.6038	0.9353	0.4750	0.9091	0.5043
C_{12}	0.4296	0.4983	0.9286	0.5007	0.9455	0.4918
C_{21}	0.4794	0.5922	0.7590	0.5791	1.0770	0.5229
C_{03}	0.5882	0.6962	0.9673	0.5130	0.8847	0.5163
C_{30}	0.5784	0.6874	0.7127	0.6213	0.9974	0.5287

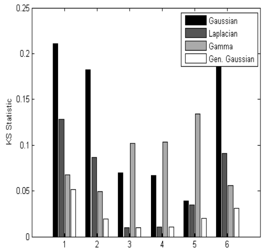
V. EXPERIMENTAL RESULTS

We use Lena, Barbara, Mandrill, Bridge, Aerial and Couple test images for experiments because these images contains variety of image details and textural informations. The KS and Chi-square goodness of fit performance was evaluated using 2D block LOT and 2D block LBT coefficients with M=8 against the model distributions. The model distribution which provides the minimum KS statistic or Chi-square statistic is considered to be the best fit under the KS or χ^2 criterion. The nine AC coefficients C_{10}, C_{11}, C_{01}, C_{20}, C_{02}, C_{12}, C_{21}, C_{03} and C_{30} used in the experiments were chosen because they usually have the most effect on the image quality.

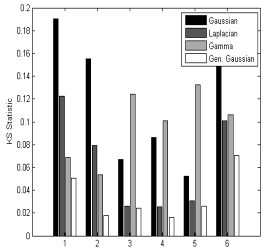
If N_1xN_2 is the size of the image after computation of 2D LOT or LBT, then each frequency coefficient will have (N_1/M)x(N_2/M) values for the image to be used in KS and χ^2 test. The KS statistic results for modeling of block LOT and LBT coefficients are shown in Fig.1 and Fig.2 in form of graphs where the X-axis is composed of six discrete points representing the six test images with bargraphs showing the KS statistic for the Gaussian, Laplacian, Gamma and Generalized



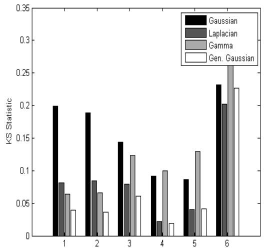
(a) For coefficient C_{10}



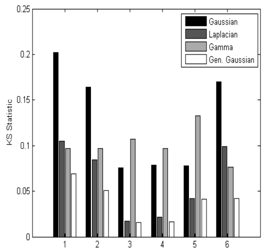
(b) For coefficient C_{11}



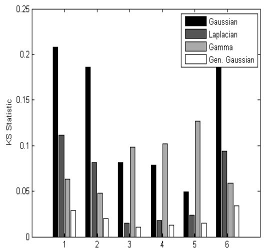
(c) For coefficient C_{01}



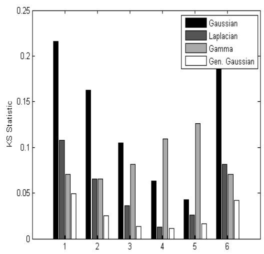
(d) For coefficient C_{20}



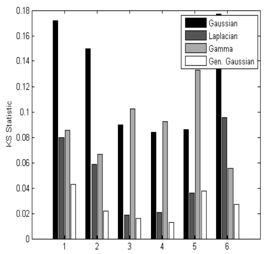
(e) For coefficient C_{02}



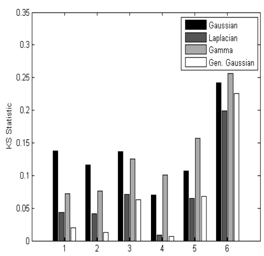
(f) For coefficient C_{12}



(g) For coefficient C_{21}

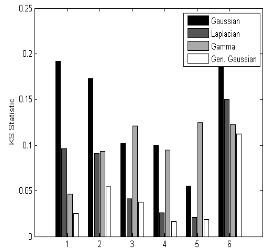


(h) For coefficient C_{03}

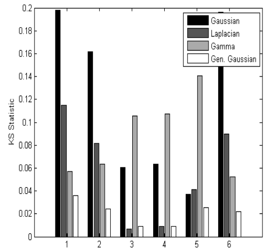


(i) For coefficient C_{30}

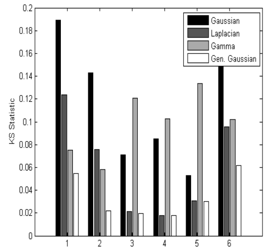
Fig. 1. KS test statistic for LOT (M=8) coefficients C_{10}, C_{11}, C_{01}, C_{20}, C_{02}, C_{12}, C_{21}, C_{03} and C_{30} (1=Lena, 2=Barbara, 3=Mandrill, 4=Bridge, 5=Aerial, 6=Couple)



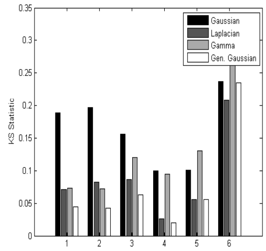
(a) For coefficient C_{10}



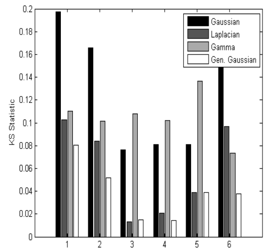
(b) For coefficient C_{11}



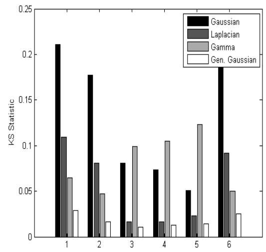
(c) For coefficient C_{01}



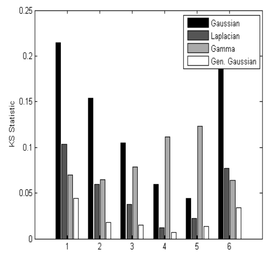
(d) For coefficient C_{20}



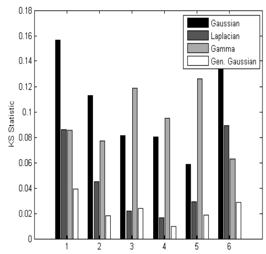
(e) For coefficient C_{02}



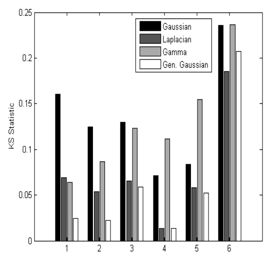
(f) For coefficient C_{12}



(g) For coefficient C_{21}



(h) For coefficient C_{03}



(i) For coefficient C_{30}

Fig. 2. KS test statistic for LBT (M=8) coefficients C_{10}, C_{11}, C_{01}, C_{20}, C_{02}, C_{12}, C_{21}, C_{03} and C_{30} (1=Lena, 2=Barbara, 3=Mandrill, 4=Bridge, 5=Aerial, 6=Couple)

TABLE III
χ^2 STATISTICS FOR A FEW LOT (M=8) COEFFICIENTS OF LENA IMAGE.

	Gaussian	Laplacian	Gamma	Gen. Gaussian
C_{10}	1.77x10^7	5876	54	**15**
C_{11}	446713	2307	66	**15**
C_{01}	1856825	1578	25	**22**
C_{20}	3.53x10^{13}	107612	117	**44**
C_{02}	4.60x10^7	4518	**49**	53
C_{12}	4.42x10^7	8664	**11**	12
C_{21}	1.88x10^7	15356	66	**62**
C_{03}	2.63x10^6	1008	29	**8**
C_{30}	1.59x10^{12}	5617	**36**	44

TABLE IV
χ^2 STATISTICS FOR A FEW LOT (M=8) COEFFICIENTS OF BARBARA IMAGE.

	Gaussian	Laplacian	Gamma	Gen. Gaussian
C_{10}	2.62x10^5	1366	25	**7**
C_{11}	25526	1164	26	**4**
C_{01}	1645	365	10	**9**
C_{20}	2.98x10^7	5208	**34**	43
C_{02}	7798	343	10498	**20**
C_{12}	2.72x10^5	2740	65	**14**
C_{21}	4.78x10^6	1226	23	**4**
C_{03}	6.89x10^9	2742	20	**7**
C_{30}	1194	107	109	**5**

Gaussian distributions. The results for χ^2 test are provided in Tables III-XIV. We consider the Gaussian and Gamma distributions with mean and variance equal to sample mean and sample variance. The Laplacian and Generalized Gaussian model parameters were calculated using maximum likelihood method.

A. *KS test results*

- **For LOT (M=8)**: In Fig.1, coefficients C_{10}, C_{01}, C_{02}, C_{12} and C_{21} show the smallest KS statistic for the Generalized Gaussian distribution in all the tested images. Coefficient C_{11} also shows the smallest Generalized Gaussian KS statistic for most of the test images except for Mandrill and Bridge where Laplacian KS statistic is the smallest. For coefficient C_{20}, Laplacian KS statistic is the smallest only for Aerial and Couple images and for the rest Generalized Gaussian KS statistic is the smallest. For C_{03}, except for Aerial image where Laplacian KS statistic is the smallest, for all others Generalized Gaussian KS statistic is the smallest. However for the coefficient C_{30}, except for Aerial and Couple, Generalized Gaussian KS statistic is the smallest.
- **For LBT (M=8)**: Fig.2 shows that Coefficients C_{10}, C_{01}, C_{12} and C_{21} have the smallest KS statistic for Generalized Gaussian distribution for all the test images. Coefficient C_{11} for the Mandrill image shows smallest KS statistic in the case of the Laplacian distribution and coefficient C_{20} for the Aerial and Couple images shows the minimum KS statistic for the Laplacian distribution and for the rest of the images the KS statistic is smallest for the Generalized Gaussian distribution. Coefficients C_{02} and C_{03} exhibit the smallest KS statistic for Laplacian distribution in case of Mandrill image and for the rest, KS statistic for Generalized Gaussian is the smallest. Coefficient C_{30} shows the smallest KS statistic in case of Laplacian for Bridge and Couple images and for all other images KS statistic for Generalized Gaussian is the minimum.

TABLE V
χ^2 STATISTICS FOR A FEW LOT (M=8) COEFFICIENTS OF MANDRILL IMAGE.

	Gaussian	Laplacian	Gamma	Gen. Gaussian
C_{10}	15429	186	359	**79**
C_{11}	1470	6	529	**6**
C_{01}	2907	9	395	**16**
C_{20}	9.93x10^5	1205	**106**	121
C_{02}	53312	20	306	**10**
C_{12}	13243	21	231	**6**
C_{21}	2.90x10^5	55	152	**8**
C_{03}	1.85x10^7	64	100	**20**
C_{30}	87999	527	**38**	90

TABLE VI
χ^2 STATISTICS FOR A FEW LOT (M=8) COEFFICIENTS OF BRIDGE IMAGE.

	Gaussian	Laplacian	Gamma	Gen. Gaussian
C_{10}	23879	140	237	**57**
C_{11}	10896	6	393	**5**
C_{01}	2.19x10^5	54	167	**12**
C_{20}	51332	88	241	**36**
C_{02}	1698	25	351	**20**
C_{12}	28458	17	325	**12**
C_{21}	2359	4	433	**3**
C_{03}	69320	29	201	**6**
C_{30}	1124	8	131	**7**

TABLE VII
χ^2 STATISTICS FOR A FEW LOT (M=8) COEFFICIENTS OF AERIAL IMAGE.

	Gaussian	Laplacian	Gamma	Gen. Gaussian
C_{10}	86	14	217	**8**
C_{11}	24	37	264	**9**
C_{01}	91	13	183	**11**
C_{20}	3.73x10^5	42	142	**34**
C_{02}	274	**11**	165	12
C_{12}	68	13	186	**5**
C_{21}	41	18	269	**3**
C_{03}	5838	20	124	**18**
C_{30}	2.21x10^6	194	201	**130**

TABLE VIII
χ^2 STATISTICS FOR A FEW LOT (M=8) COEFFICIENTS OF COUPLE IMAGE.

	Gaussian	Laplacian	Gamma	Gen. Gaussian
C_{10}	2.08x10^7	4.54x10^5	579	**375**
C_{11}	3.42x10^{13}	4.16x10^5	167	**30**
C_{01}	1.29x10^7	603	**6**	**6**
C_{20}	2.09x10^{11}	9.57x10^5	**680**	1065
C_{02}	7.46x10^6	409	**2**	24
C_{12}	9.31x10^{1}4	3.22x10^5	131	**20**
C_{21}	4750	1393	**2**	16
C_{03}	1.40x10^9	543	**10**	15
C_{30}	7.07x10^7	6667	**203**	221

TABLE IX
χ^2 STATISTICS FOR A FEW LBT (M=8) COEFFICIENTS OF LENA IMAGE.

	Gaussian	Laplacian	Gamma	Gen. Gaussian
C_{10}	1.65x10^9	9903	117	**47**
C_{11}	1.07x10^7	3252	78	**57**
C_{01}	5.34x10^7	1154	82	**70**
C_{20}	1.03x10^{13}	4.33x10^4	87	**61**
C_{02}	9.97x10^9	1.02x10^4	254	**158**
C_{12}	3.43x10^9	1.12x10^4	92	**42**
C_{21}	5.27x10^9	2.39x10^4	233	**100**
C_{03}	1.08x10^{11}	368	**28**	43
C_{30}	1.67x10^6	845	**48**	58

TABLE X
χ^2 STATISTICS FOR A FEW LBT (M=8) COEFFICIENTS OF BARBARA IMAGE.

	Gaussian	Laplacian	Gamma	Gen. Gaussian
C_{10}	8.29x10^9	2133	**56**	66
C_{11}	1.30x10^6	726	26	**24**
C_{01}	1.21x10^4	464	**19**	37
C_{20}	5.76x10^{13}	1.87x10^5	**112**	129
C_{02}	1.19x10^9	861	276	**121**
C_{12}	2.31x10^{10}	7571	101	**26**
C_{21}	2.06x10^{12}	3938	131	**31**
C_{03}	2.41x10^{10}	382	86	**16**
C_{30}	1.15x10^4	174	55	**25**

B. χ^2 test results

- **For LOT(M=8)**: Coefficients C_{10}, C_{11}, C_{01}, C_{20}, C_{21} and C_{03} for Lena image shows smallest χ^2 statistic for the Generalized Gaussian distribution among all the model distributions and coefficients C_{02}, C_{12} and C_{30} shows smallest χ^2 statistic for the Gamma distribution. For the Barbara image 8 of 9 distributions shows smallest χ^2 statistic for the Generalized Gaussian distribution and only coefficient C_{20} shows smallest χ^2 statistic for the Gamma distribution. For Mandrill image also 7 out of 9 coefficients show smallest χ^2 statistic for the Generalized Gaussian distribution and only coefficients C_{20} and C_{30} shows smallest χ^2 statistic for Gamma case. For Bridge image all the 9 tested coefficients show smallest χ^2 statistic for the Generalized Gaussian distribution. Aerial image shows smallest χ^2 statistic in case of the Generalized Gaussian distribution for 8 of 9 tested coefficients. Only coefficient C_{02} shows smallest χ^2 statistic for the Laplacian distribution among all the model distributions.

TABLE XI
χ^2 STATISTICS FOR A FEW LBT (M=8) COEFFICIENTS OF MANDRILL IMAGE.

	Gaussian	Laplacian	Gamma	Gen. Gaussian
C_{10}	1.24x10^4	189	266	**104**
C_{11}	623	9	550	**7**
C_{01}	7919	**13**	404	17
C_{20}	1.27x10^7	3014	228	**163**
C_{02}	40131	29	275	**14**
C_{12}	7012	17	239	**5**
C_{21}	9.27x10^4	57	176	**10**
C_{03}	6.91x10^6	37	203	**27**
C_{30}	5.27x10^4	456	**52**	78

TABLE XII
χ^2 STATISTICS FOR A FEW LBT (M=8) COEFFICIENTS OF BRIDGE IMAGE.

	Gaussian	Laplacian	Gamma	Gen. Gaussian
C_{10}	1.93x10^4	184	244	**53**
C_{11}	2.10x10^4	**9**	353	**9**
C_{01}	3.15x10^5	63	177	**14**
C_{20}	1.60x10^5	164	218	**63**
C_{02}	2287	25	372	**20**
C_{12}	9310	24	294	**19**
C_{21}	3392	6	474	**4**
C_{03}	6099	16	270	**3**
C_{30}	6312	**24**	507	**24**

TABLE XIII
χ^2 STATISTICS FOR A FEW LBT (M=8) COEFFICIENTS OF AERIAL IMAGE.

	Gaussian	Laplacian	Gamma	Gen. Gaussian
C_{10}	69	22	249	**15**
C_{11}	20	48	306	**12**
C_{01}	57	18	259	**8**
C_{20}	1.84x10^6	88	131	**52**
C_{02}	586	**19**	180	20
C_{12}	100	10	201	**2**
C_{21}	42	20	228	**8**
C_{03}	3228	**12**	91	13
C_{30}	8.33x10^4	**62**	248	77

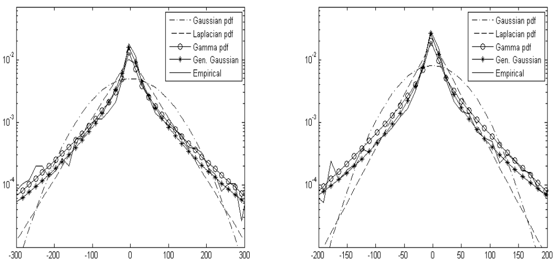
However for the Couple image, 5 of 9 coefficients show smallest χ^2 statistic for the Gamma distribution and coefficient C_{01} shows smallest and same χ^2 statistic for the Generalized Gaussian and the Gamma distributions. Coefficients C_{10}, C_{11} and C_{12} show smallest χ^2 statistic for the Generalized Gaussian distribution in case of Couple image.
- **For LBT (M=8)**: For Lena image, 7 of 9 coefficients show smallest χ^2 statistic for the Generalized Gaussian distribution among all the model distributions. Only coefficients C_{03} and C_{30} show smallest χ^2 statistic in case of the Gamma distribution among all the model distributions. For Barbara image, 6 of 9 tested coefficients show smallest χ^2 statistic for the Generalized Gaussian distribution among all the model distributions. Only coefficients C_{10}, C_{01} and C_{20} show smallest χ^2 statistic for the Gamma distributions among all the model distributions. Coefficients C_{10}, C_{11}, C_{20}, C_{02}, C_{12}, C_{21} and C_{03} for Mandrill image show smallest χ^2 statistic for the Generalized Gaussian distribution. Only coefficient C_{01} shows the smallest χ^2 statistic for the Laplacian case and the coefficient C_{30} shows the smallest χ^2 statistic for the Gamma distribution among all the model distributions. For Bridge image, 7 of 9 coefficients show smallest χ^2 statistic for the Generalized Gaussian case among all the model distributions. However coefficients C_{11} and C_{30} shows same χ^2 statistic for the Laplacian and Generalized Gaussian distributions which is minimum among all the model distributions. For Aerial image, 6 of 9 coefficients shows smallest χ^2 statistic for the Generalized Gaussian distribution among all the model distributions. Only coefficients C_{02}, C_{03} and C_{30} show smallest χ^2 statistic for the Laplacian case. For Couple image, 4 of 9 coefficients show smallest χ^2 statistic for the Generalized Gaussian case among all the model distributions and coefficient

TABLE XIV
χ^2 STATISTICS FOR A FEW LBT (M=8) COEFFICIENTS OF COUPLE IMAGE.

	Gaussian	Laplacian	Gamma	Gen. Gaussian
C_{10}	1.85x10^6	1.36x10^5	484	**205**
C_{11}	3.72x10^{13}	3.69x10^5	152	**20**
C_{01}	1.34x10^6	497	**4**	5
C_{20}	3.90x10^9	1.26x10^5	**782**	1261
C_{02}	2.52x10^5	306	**7**	36
C_{12}	3.64x10^{12}	5.39x10^4	103	**15**
C_{21}	1.19x10^{13}	5.48x10^5	**56**	88
C_{03}	1.10x10^4	376	**19**	**19**
C_{30}	3.66x10^8	1.75x10^4	1159	**1025**

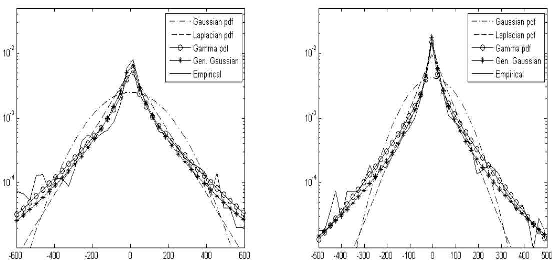
C_{03} shows the same χ^2 statistic for the Gamma and Generalized Gaussian case which is the smallest among all the model distributions. However coefficients C_{01}, C_{20}, C_{02} and C_{21} shows smallest χ^2 statistic for the Gamma distribution for Couple image.

The experimental results show that no single distribution provides the smallest KS and χ^2 statistic for all the tested AC coefficients of block LOT and LBT of different test images. The Generalized Gaussian distribution however provides the smallest KS and χ^2 statistic for the majority of the significant AC coefficients of the tested images. Fig.3 and Fig.4 shows the empirical pdf of the block LOT and LBT coefficients along with the fitted Gaussian, Laplacian, Gamma and Generalized Gaussian pdfs. From Fig.3 and Fig.4 as well as from the values of the KS and Chi-square statistics, it is clear that the Generalized Gaussian distribution provides a better fit to the empirical distribution as compared to the Gaussian, Laplacian and Gamma distribution.



(a) C_{01} (b) C_{10}

Fig. 3. Logarithmic histograms of the block LOT (M=8) coefficients for Lena image and the best Gaussian, Laplacian, Gamma, Generalized Gaussian pdfs fitted to this histogram in log domain.



(a) C_{01} (b) C_{10}

Fig. 4. Logarithmic histograms of the block LBT (M=8) coefficients for Lena image and the best Gaussian, Laplacian, Gamma, Generalized Gaussian pdfs fitted to this histogram in log domain.

VI. CONCLUSION

In this paper, we perform the KS and χ^2 goodness of fit tests to determine a suitable statistical distribution that best approximates the block LOT and LBT coefficients of natural images. The experimental results indicate that no single distribution can be used to model the distributions of all AC coefficients for all natural images. However the distribution of a majority of the significant AC coefficients can be modeled by the Generalized Gaussian distribution. The knowledge of the statistical distribution of transform coefficients is very important in the design of optimal quantizers that may lead to minimum distortion and hence achieve optimal coding efficiency.

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