# Stable Tending Control of Complex Power Systems: An Example of Localized Design of Power System Stabilizers

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Abstract—The phase compensation method was proposed based on the concept of the damping torque analysis (DTA). It is a method for the design of a PSS (power system stabilizer) to suppress local-mode power oscillations in a single-machine infinite-bus power system. This paper presents the application of the phase compensation method for the design of a PSS in a multi-machine power system. The application is achieved by examining the direct damping contribution of the stabilizer to the power oscillations. By using linearized equal area criterion, a theoretical proof to the application for the PSS design is presented. Hence PSS design in the paper is an example of stable tending control by localized method.

**Keywords**—Phase compensation method, power system small-signal stability, power system stabilizer

#### I. INTRODUCTION

Power system oscillations threaten the safe operation of power systems. They were first observed in Northern American power network in Oct. 1964 during a trial interconnection of the Northwest Power Pool and the Southwest Power Pool [1]. Since then incidents of power system oscillations have been reported in power transmission networks in many countries. Over the last half century, many power system researchers and engineers have worked on and contributed to understanding and solution of the problem of power system oscillations. It is now well recognized that the cause of power system oscillations is lack of damping of the so-called "electromechanical oscillation modes" in a power system. To increase the damping of power system oscillations and improve system stability, the installation of supplementary excitation controller, power system stabilizer (PSS), is a simple and effective method. To date, most major electric power plants in many countries are equipped with PSS.

For the design of a PSS, the technique of damping torque analysis (DTA) was firstly introduced in [2] for a single-machine infinite-bus power system to investigate the effect of excitation control on power system small-disturbance rotor angle stability. It is based on the linearized Philips-Heffron model of the single-machine infinite-bus power system [3]. The well-known method of phase compensation (PC) to design the PSS was proposed and developed on the basis of the DTA [4]. The PC method is considered as a milestone contribution to the field and has been used in power

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industry for many decades to tune and set parameters of the PSS

Since 1980s, considerable effort has been spent on developing schemes to design the PSS installed in a complex multi-machine power system. Though attempt was made to extend the PC method to the case of the multi-machine power system [5-6], the modal analysis (MA) is a more popular method for the design of PSSs in the multi-machine power system. The normal procedure of the MA method is to establish the linearized model of the multi-machine power system firstly at a given operating condition of the power system. Then by use of the MA (computing the electromechanical oscillation modes of interests), various schemes of optimization can be developed to design PSSs. The objective of the design is to move the electromechanical oscillation modes of interests to the given positions in the complex plane such that the modes are of sufficient damping. Hence the basic idea to design the PSSs in fact is very similar to the pole assignment of control systems in control theory. Thus though a PSS is a decentralized controller as only local feedback signal is used to form the close-loop control system, its design has to be based on the model of whole power system in order to ensure global system stability. Hence the design is centralized, which contradicts the basic idea of decentralization of PSS application. The immediate problem coming from the contradiction is to obtain parameters of whole power system, which in practice may not always be readily available and are difficult to be validated when the system is large and complex.

It has been well accepted that to set a decentralized local PSS or several of them to guarantee global stability of a large complex power system may be possible. The price to pay for this is that the model of whole power system must be used. This paper proposes to investigate the option to change the regime of PSS design from ensuring global system stability to a looser condition of design to just achieve a "stable tending" control. If a PSS is designed to ensure making the power system more stable instead of globally stable, the design may not need to be based on the model of the whole power system. Hence the change of the regime could mean much simpler procedure of PSS design in practical applications.

In fact, the objective of installing a PSS in the power system is to provide extra damping to the electromechanical oscillation modes of interests. It does not really have to be set for achieving exact assignment of relevant electromechanical oscillation modes ti given positions in the complex plane. Hence the strategy of PSS design is changed to providing more

positive damping to the modes to achieve stable tending control, i.e., making the system more stable, fits the practical requirement of PSS installation as well.

In order to demonstrate the possibility to develop theory of stable tending control for complex power system, this paper examines the scheme proposed by Gurrala and Sen in [7] as an example where a localized Phillips-Heffron model in a multi-machine power system is established for the design of the PSS. In [7] it is shown that when the secondary bus voltage of generators step-up transformer is used to derive the Phillips-Heffron model, only parameters locally available at the synchronous generator on which the PSS is installed are needed for the design of the PSS. Several numerical examples are presented in [7] which demonstrate the success of applying the PC method to design the PSS. Hence method proposed in [7] for the PSS design does not need to establish the model of the whole power system. Instead, it is based on a localized model. Only locally available information at the place where the PSS is installed is needed to derive the localized model. However, [7] does not provide any theoretical explanation on why the localized model can be used for the successful design of PSS.

In this paper, by using linearized equal area criterion, a theoretical proof to the localized method of PSS design proposed in [7] is presented. The paper demonstrates that localized design in [7] can ensure provision of extra positive damping by the PSS to the low-frequency power oscillation associated with the machine where the PSS is installed. Hence in fact it is an example of stable tending control.

## II. THE EXAMPLE OF LOCALIZED DESIGN OF PSS

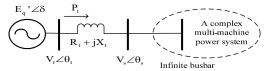


Fig. 1 Single machine in a complex multi-machine power system

Fig. 1 shows the configuration of a single-machine connected to a complex multi-machine power system, where  $R_t + jX_t$  denotes the impedance of the transformer,  $V_t \angle \theta_t$ and  $V_s \angle \theta_s$  the voltage at the primary and secondary side of the transformer respectively. For the design of PSS installed on the single machine to damp the low-frequency power oscillation along the transmission line connecting the machine to the system, there are two types of linearized models to be used. The first one is the linearized model of the whole multi-machine power system, based on which a MA method can be used to set the parameters of the PSS. This involves the procedure to obtain and validate parameters and information of the whole complex multi-machine power system, obviously complicated if not impossible. The second is the linearized model of a single-machine infinite-bus power system where the PC method can be used to design the PSS. This has to find a suitable "infinite busbar" inside the complex multi-machine power system as the reference, which also needs information of external multi-machine power system and is not a straightforward work.

It is proposed in [7] that by simply taking the secondary side of the transformer as the reference busbar to replace the "infinite busbar" to be used and  $V_s \angle \theta_s$  as the reference voltage, a similar Phillips-Heffron model to that for a single-machine infinite-bus power system can be established as shown in Fig. 2. In Fig. 2, prefix  $\Delta$  denotes the small increment of a variable,  $PSS_{\omega}(s)$  and  $V_{pss}$  the transfer function and stabilizing control signal of the PSS respectively. Because the reference voltage  $V_s \angle \theta_s$  is not a constant, there are three extra blocks,  $G_{v1}$ ,  $G_{v2}$  and  $G_{v3}$ , as compared to the conventional Phillips-Heffron model of the single-machine infinite-bus power system.

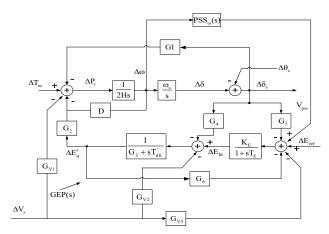


Fig. 2 Phillips-Heffron model proposed in [7]

It is proposed in [7] that simply to derive the transfer function of the forward path of the stabilizing control signal of the PSS GEP(s) as shown by Fig. 3, the conventional PC method is used to design the PSS. As the establishment of the linearized model in Fig. 2 does not need any external information of the complex multi-machine power system, the method proposed in [7] is indeed localized and very simple. However, as shown in Fig. 3, the variable involved in the electromechanical oscillation loop in the proposed Phillips-Heffron is  $\delta_s = \delta - \theta_s$ , no longer only the angular position of the machine. It cannot be seen clearly if positive damping torque contribution by the PSS to the machine can suppress the low-frequency power oscillation because  $\theta_s$  is variable. In [7] three numerical examples are presented to demonstrate that the PC method can be used to design the PSS to damp the low-frequency power oscillation, though no theoretical explanation is provided in [7] about why the PSS can be successful designed as proposed.

In the following section, a theoretical explanation by use of linearized equal area criterion is presented that proves indeed the localized PCM proposed [7] can always provide extra positive damping to the low-frequency power oscillation associated with the machine. Hence, though it is still unknown

if the localized method proposed in [7] can guarantee the global stability of the whole power system or not, the scheme proposed in [7] is a truly example of stable tending control.

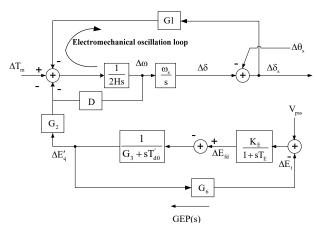


Fig. 3 Forward path of the PSS proposed in [7]

#### III. THEORETICAL EXPLANATION FOR THE EXAMPLE

The linearized model of Fig. 2 can be written in the form of state-space representation as ( $\Delta T_m=0,\ \Delta E_{ref}=0$ ).

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{b}_{E}\Delta\mathbf{Z}_{E} + \mathbf{b}_{pss}\Delta\mathbf{V}_{pss}$$

$$\Delta\mathbf{Z}_{I} = \mathbf{C}\mathbf{X} + \mathbf{d}_{E}\Delta\mathbf{Z}_{E} + \mathbf{d}_{pss}\Delta\mathbf{V}_{pss}$$
(1)

where

$$\begin{split} \mathbf{X} &= \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E_{q} \\ \Delta E_{fd} \end{bmatrix}, & \Delta \mathbf{Z}_{E} = \begin{bmatrix} \Delta V_{s} \\ \Delta \theta_{s} \end{bmatrix}, & \Delta \mathbf{Z}_{I} = \begin{bmatrix} \Delta V_{t} \\ \Delta \theta_{t} \end{bmatrix} \\ \mathbf{A} &= \begin{bmatrix} 0 & \omega_{s} & 0 & 0 \\ -\frac{G_{1}}{2H} & -\frac{D}{2H} & -\frac{G_{2}}{2H} & 0 \\ -\frac{G_{4}}{T_{d0}}, & 0 & -\frac{G_{3}}{T_{d0}}, & \frac{1}{T_{d0}}, \\ -\frac{K_{A}G_{5}}{T_{A}} & 0 & -\frac{K_{A}G_{6}}{T_{A}} & -\frac{1}{T_{A}} \end{bmatrix}, \\ \mathbf{b}_{pss} &= \begin{bmatrix} 0 & 0 & 0 & \frac{K_{A}}{T_{A}} \end{bmatrix}^{T}, \\ \mathbf{b}_{E} &= \begin{bmatrix} 0 & 0 & 0 & \frac{G_{1}}{2H} & \frac{G_{1}}{2H} \\ -\frac{G_{V2}}{T_{d0}}, & \frac{G_{4}}{T_{d0}}, \\ -\frac{K_{A}G_{V3}}{T_{A}} & -\frac{K_{A}G_{5}}{T_{A}} \end{bmatrix}, \end{split}$$

 $\Delta V_t$  is the input signal to AVR. Hence, from Fig. 2 it can have:

$$\Delta V_{t} = G_{5}(\Delta \delta - \Delta \theta_{s}) + G_{6}\Delta E_{q}' + G_{v3}\Delta V_{s}$$
 (2)

In the common coordinate system, terminal voltage of the generator can be expressed to be ((19) in [7]):

$$\begin{aligned} &V_{q} = R_{t}i_{q} - X_{t}i_{d} + V_{s}\cos\delta_{s} \\ &V_{d} = R_{t}i_{d} + X_{t}i_{q} - V_{s}\sin\delta_{s} \end{aligned} \tag{3}$$

By using (3) above and linearizing  $\theta_t = \arctan^{-1} \frac{V_d}{V_n}$  it can have:

$$\Delta\theta_{t} = G_{7}(\Delta\delta - \Delta\theta_{s}) + G_{8}\Delta E_{q}' + G_{9}\Delta V_{s}$$
(4)

where  $G_7, G_8, G_9$  are constant. Hence, from (3) and (4) the following is obtained for (1):

$$\mathbf{C} = \begin{bmatrix} G_5 & 0 & G_6 & 0 \\ G_7 & 0 & G_8 & 0 \end{bmatrix}, \mathbf{d}_{\mathbf{E}} = \begin{bmatrix} G_{v3} & -G_5 \\ G_9 & -G_7 \end{bmatrix}, \mathbf{d}_{pss} = 0$$
 (5)

Power oscillation along the transmission line connecting the machine and system as shown in Fig. 1 can be expressed as:

$$P_{t} = \frac{V_{t}V_{s}}{X_{t}}\sin(\theta_{t} - \theta_{s}) = \frac{V_{t}V_{s}}{X_{t}}\sin\theta_{ts}$$

Linearization of above equation gives:

$$\Delta P_{t} = C_{t} \Delta V_{t} + C_{s} \Delta V_{s} + C_{ts} (\Delta \theta_{t} - \Delta \theta_{s})$$

$$= \begin{bmatrix} C_{s} & -C_{ts} \end{bmatrix} \mathbf{Z}_{E} + \begin{bmatrix} C_{t} & C_{ts} \end{bmatrix} \mathbf{Z}_{I}$$

$$= C_{E} \mathbf{Z}_{E} + C_{I} \mathbf{Z}_{I}$$
(6)

where

$$\begin{split} &C_t = \frac{V_{s0}}{X_t}\sin\theta_{ts0}, C_s = \frac{V_{t0}}{X_t}\sin\theta_{ts0}, \\ &C_{ts} = \frac{V_{t0}V_{s0}}{X_t}\cos\theta_{ts0} \end{split} \label{eq:cts}$$

From (1) it can be obtained that:

$$\Delta \mathbf{Z}_{\mathbf{I}} = [\mathbf{C}(\mathbf{s}\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}_{\mathbf{E}} + \mathbf{d}_{\mathbf{E}}]\Delta \mathbf{Z}_{\mathbf{E}} + [\mathbf{C}(\mathbf{s}\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}_{pss} + \mathbf{d}_{pss}]\Delta \mathbf{V}_{pss} = \mathbf{F}_{\mathbf{E}}(\mathbf{s})\Delta \mathbf{Z}_{\mathbf{E}} + \mathbf{F}_{\mathbf{PSS}}(\mathbf{s})\Delta \mathbf{V}_{pss}$$
(7)

Substituting (7) into (6) gives:

$$\Delta P_{t} = [C_{I}F_{E}(s) + C_{E}]\Delta Z_{E} + C_{I}F_{PSS}(s)\Delta V_{pss}$$
(8)

Denote:

$$\Delta P_{t}(\Delta \mathbf{Z}_{E}) = [\mathbf{C}_{I} \mathbf{F}_{E}(\mathbf{s}) + \mathbf{C}_{E}] \Delta \mathbf{Z}_{E} = \mathbf{F}(\mathbf{s}) \Delta \mathbf{Z}_{E}$$

$$\Delta P_{t}(\Delta \mathbf{V}_{DSS}) = \mathbf{C}_{I} \mathbf{F}_{PSS}(\mathbf{s}) \Delta \mathbf{u}_{DSS} = \mathbf{f}_{DSS}(\mathbf{s}) \Delta \mathbf{V}_{DSS}$$
(9)

and decompose  $\Delta P_t(\Delta \mathbf{Z}_E)$  under  $\Delta \theta_{ts} - \Delta \dot{\theta}_{ts}$  coordinate as ( $\Delta \dot{\theta}_{ts}$  is the time derivative of  $\Delta \theta_{ts}$ ):

$$\Delta P_{t}(\Delta \mathbf{Z}_{E}) = C_{e} \Delta \theta_{ts} + D_{e} \Delta \dot{\theta}_{ts}$$
(10)

where  $C_e$  and  $D_e$  are constant. Thus by use of (9) and (10), (8) can be written as:

$$\Delta P_{t} = C_{e} \Delta \theta_{ts} + D_{e} \Delta \dot{\theta}_{ts} + f_{pss}(s) \Delta V_{pss}$$
 (11)

(11) shows that low-frequency power oscillation  $\Delta P_t$  is only affected by the PSS through the part,  $f_{pss}(s)\Delta V_{pss}$  in  $\Delta P_t$ . This part is the direct contribution from the PSS to the power oscillation (variation). The following is a main conclusion about the damping of low-frequency power oscillation as affected by this part of PSS stabilizing control.

Main Conclusion: If the PSS is designed to ensure  $f_{pss}(s)\Delta V_{pss}$  to be proportional to the time derivative of  $\Delta P_t$ , that is:

$$\Delta P_{t}(\Delta V_{pss}) = f_{pss}(s)\Delta V_{pss} = D_{pss}\Delta \dot{P}_{t}(D_{pss} > 0) \eqno(12)$$

The PSS will supply positive damping to suppress the power oscillation  $\Delta P_t$  . In (12)  $\Delta \dot{P}_t$  denotes the time derivative of  $\Delta P_t$  .

The above main conclusion can be proved by using the graphical explanation based on the linearized  $P-\delta$  curve and equal-area criterion as follows.

Fig. 4 shows the linearized  $P_t - \theta_{ts}$  curve where  $\theta_{ts0}$ ,  $P_{t0}$  is the steady-state operating point of the power system. It is assumed that the small-signal oscillation of  $\Delta P_t$  starts from point 'a' in Fig. 4 with the operating point moving down. Without affecting the result of discussion, it is assumed that  $D_e > 0$  in (11). When there is no PSS installed,  $f_{pss}(s)\Delta V_{pss} = 0$  and (11) becomes:

$$\Delta P_{t} = C_{e} \Delta \theta_{ts} + D_{e} \Delta \dot{\theta}_{ts}$$
(13)

 $\Delta P_t = C_e \Delta \theta_{ts}$  is a line shown in Fig. 4. When the operating point moves down,  $\Delta \dot{\theta}_{ts} < 0$ ,  $D_e \Delta \dot{\theta}_{ts} < 0$  is added on the line, Hence, (13) is expressed as the dashed curve in Fig. 4. When the operating point arrives at point 'f' and stops moving,  $D_e \Delta \dot{\theta}_{ts} = 0$ . Hence the operating point should be on the line at

point 'f',  $\Delta P_t = C_e \Delta \theta_{ts}$  as shown in Fig. 4. According to the equal area criterion, area 'ade' is equal to that of 'dgf'.

Consider the case that the PSS is installed and the stabilizing control is added. If the PSS is set to ensure (12) standing,  $D_{nss}\Delta\dot{P}_{t} < 0 \ (D_{nss} > 0)$  is added on the dashed curve  $\Delta P_t = C_e \Delta \theta_{ts} + D_e \Delta \theta_{ts}$  when the operating point moves down. Hence the operating point should move below the dashed curve along the highlighted trajectory as shown in Fig. 4. When the operating point stops moving, it should arrive on the line,  $\Delta P_t = C_e \Delta \theta_{ts}$ . According to the equal area criterion, area  $A_1$ must be equal to area A2 at point 'c'. It is apparent that addition of the part from the PSS stabilizing control with (12) standing, area  $A_1$  is reduced which results in a smaller area  $A_2$ . Obviously it can have  $\theta_{ts1} - \theta_{ts0} > \theta_{ts0} - \theta_{ts2}$ , which indicates extra positive damping provision from the stabilizing control to the power oscillation. A similar analysis can be carried out to examine the case when the operating point moves up from point 'c'.

From (1), (5), (7)-(9) it is easy to prove  $f_{pss}(s)$  defined by (9) is GEP(s) in Figs. 2 and 3. In fact, according to the principle of superimposition of linear systems, from Fig. 2 it can obtain directly:

$$\begin{split} &\Delta P_{t} = f_{1}(s)\Delta V_{s} + f_{2}(s)\Delta \theta_{s} + GEP(s)\Delta V_{pss} \\ &= \begin{bmatrix} f_{1}(s) & f_{2}(s) \end{bmatrix} \Delta \mathbf{Z}_{E} + GEP(s)\Delta V_{pss} \end{split} \tag{14}$$

Comparing (14) with (8) and (9) it can have  $f_{pss}(s) = GEP(s)$ . Hence if the PCM is used to ensure a positive damping torque is provided, that is:

$$GEP(s)\Delta V_{pss} = D_{d}\Delta\omega, \ D_{d} > 0 \eqno(15)$$

It can have:

$$\begin{split} & GEP(s)\Delta V_{pss} = f_{pss}(s)\Delta V_{pss} = D_d\Delta\omega \\ & = D_d(2Hs+D)\Delta P_t = 2HD_d\Delta\dot{P}_t + DD_d\Delta P_t \end{split} \tag{16}$$

A positive damping part,  $2HD_d\Delta\dot{P}_t$ ,  $2HD_d>0$ , is provided by the PSS to help the suppression of low-frequency power oscillation. This explains why the localized design method proposed in [7] for the design of PSS can supply positive damping to low-frequency power oscillation.

### IV. CONCLUSIONS AND FURTHER COMMENTS

Installation of PSS is an effective way to suppress low-frequency oscillations in a power system to enhance system stability. So far, majority of schemes proposed to design PSS is based on the linearized model of the whole power system, as those schemes are developed to ensure

system global stability. This is very much the idea of decentralized stabilization of closed-loop control systems in control theory. However, validation of parameters and information of whole power system may be difficult in practice when the system is large and complex. Furthermore, objective of installing a PSS often is to supply extra damping to low-frequency power oscillation. It neither has to meet the strict condition of guaranteeing system global stability nor to assign the electromechanical oscillation modes of interests to given positions in a complex plane.

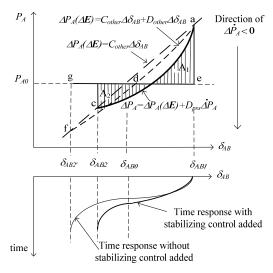


Fig. 4 Illustration of analysis based on the linearized  $P_t - \theta_{ts}$  curve

This paper proposes the investigation of PSS design to meet a looser condition of making the system more stable, named as "stable tending control" in the paper. With the stable tending control, PSS design may not need to be based on the model of whole power system. Thus the difficulty to validate the parameters and information of a complex multi-machine power system may be avoided. To demonstrate the feasibility of the proposed idea of stable tending control of PSS, this paper presents an example scheme of localized design of PSS proposed in [7] where only a localized model of a multi-machine power system is needed to design a PSS. The establishment of the localized model only needs information locally available at the machine on which the PSS is installed without having to validate the parameters of the whole power system. This paper has demonstrated that the scheme proposed in [7] is a type of stable tending control, as the PSS designed can ensure provision of extra damping to the low-frequency oscillation associated with the machine where the PSS is installed. Hence the stable tending control is possible and indeed much simpler as global issues of complex power system may be avoided for the design of a local decentralized PSS.

With the localized design of stable tending control of PSS, enhancement of robustness of PSS to variations of power system operating conditions could also become a simple issue. By on-line gathering of local information to update the localized model, PSS parameters can be set in real time to

always provide positive damping to the low-frequency oscillation of interests. Based on the same principle, plug-in PSS can also be designed. Localization could make intelligent stability control of power systems simpler and hence feasible.

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