Stabilization of the Lorenz Chaotic Equations by Fuzzy Controller

Behrooz Rezaie, Zahra Rahmani Cherati, Mohammad Reza Jahed Motlagh, and Mohammad Farrokhi

***Abstract*—**In this paper, a fuzzy controller is designed for stabilization of the Lorenz chaotic equations. A simple Mamdani inference method is used for this purpose. This method is very simple and applicable for complex chaotic systems and it can be implemented easily. The stability of close loop system is investigated by the Lyapunov stabilization criterion. A Lyapunov function is introduced and the global stability is proven. Finally, the effectiveness of this method is illustrated by simulation results and it is shown that the performance of the system is improved.

***Keywords*—**Chaotic system, Fuzzy control, Lorenz equation.

I. Introduction

In recent years, intelligent methods such as neural networks and fuzzy logic have been used to identify and control chaotic systems [1].. To achieve desired responses, intelligent systems based on robustness, accuracy and adaptation are useful and can be appropriate for chaos control [2-4]. Wang applied the fuzzy logic control based on adaptive control method for chaotic systems [5]. In 1997, Tanaka, Ikeda and Wang present a LMI-base fuzzy control for chaotic system [6]. Also they controlled the chaotic behaviors by the Sugeno-type fuzzy system [7]. Chen in 2000 proposed a fuzzy modeling method for control and prediction of uncertain chaotic system [8]. Li et. al. in 2001 introduced Takagi-Sugeno(TS) fuzzy system to make a chaotic system [9]. Park et. Al. in 2002 presented an adaptive fuzzy control scheme based on well-known Takagi–Sugeno (T–S) fuzzy models for the MIMO plants [2]. Feng and Chen in 2005 used T-S model of discrete-time for henon map [10]. In these references the methods are complicated and often difficult to implementation. Therefore, in this paper, we use a well-known and simple method that can be embedded easily in chaotic systems.

Determination of an accurate mathematical model for chaotic systems is often difficult [11]. So, model-free control methods are useful for these systems [11]. In this paper, a fuzzy method is used to control chaotic systems such as the Lorenz equation. For this purpose, the Lorenz equation is introduced first. Then a fuzzy controller based on the Mamdani inference approach is designed. This controller is based on IF-THEN rules obtained from experimental data. By applying this method to the Lorenz chaotic system in the next step, simulation results of the proposed method are compared with results of [7]. Simulation results show that the stability and performance of this method will be better than the Sugeno fuzzy control method. These important results will be achieved by a minimum control effort.

This paper is organized in seven sections. In section 2, the Lorenz equation will be introduced. In section 3, the Mamdani fuzzy control method will be studied. The Lyapunov stability criterion will be investigated in section 4. Simulation results will be shown and compared with the Sugeno method in fifth and sixth sections. Finally, conclusion remarks will be shown in the last section.

II. Lorenz Equation

In chaotic systems, any small variation of initial conditions can result nonlinear and non-deterministic performance of the system. Lorenz equation is one of the most popular chaotic systems [12].

The Lorenz equations are as following:

$$\begin{aligned}\dot{x}_1(t) &= -ax_1(t) + ax_2(t) + u(t)\\ \dot{x}_2(t) &= cx_1(t) - x_2(t) - x_1(t)x_3(t)\\ \dot{x}_3(t) &= x_1(t)x_2(t) - bx_3(t)\\ y(t) &= x_1(t)\end{aligned} \tag{1}$$

Where $x_1(t)$, $x_2(t)$ and $x_3(t)$ are state variables, y is output and a, b and c are parameters of the system.

Fig. 1 shows the performance of state variables of Lorenz equation with a=10, b=8/3 and c=28.

It can be seen that the Lorenz chaotic output signal is completely random and aperiodic, when the input is step signal.

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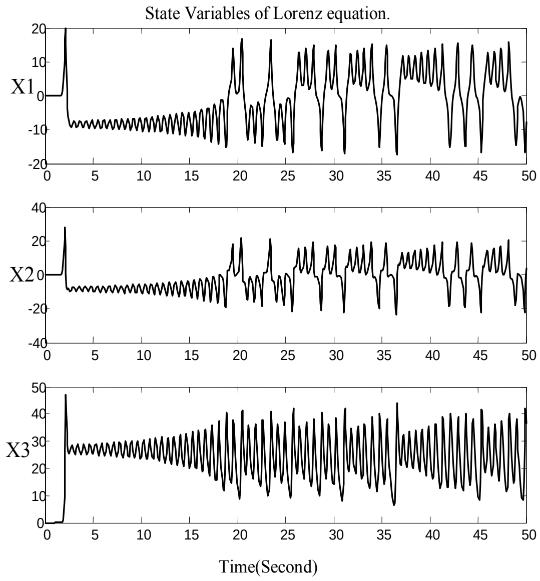

Fig. 1 the state variables of the Lorenz chaotic equations with a=10, b=8/3, c=28

III. Stabilization of the Lorenz Equations by Fuzzy Logic Approach

A. The Mamdani Fuzzy Control Method

The Mamdani inference method is used to stabilize the Lorenz chaotic equation. This technique is based on observation of experimental data and IF-THEN rules. It is completed by selection of triangular fuzzifier; centers mean defuzzifier and the Mamdani product as the inference engine.

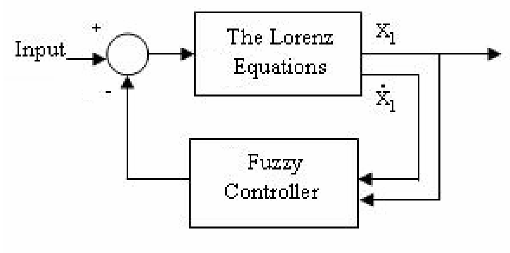

Fig. 2 The fuzzy control of the Lorenz equation

Fig. 2 shows the block diagram of close loop system. By the step response of closed loop system and analyzing experimental observations, required rules for input and output of the fuzzy system are obtained, which is shown as Table I.

The variation ranges of x_1 and $\dot{x}_1$ are [–30, +30] and [-3000, +3000], respectively. These ranges are resulted from the variation range of x_1 in Fig. 1. Input and output membership functions are shown in Fig. 3 and 4.

TABLE I
THE MAMDANI FUZZY RULES

$\dot{x}_1$ \ x_1	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NM	NM	NM	NS	NS	ZE
NM	NM	NM	NS	NS	ZE	PS	PS
NS	NM	NM	NS	NS	ZE	PS	PS
ZE	NM	NS	NS	ZE	PS	PS	PM
PS	NS	NS	ZE	PS	PS	PM	PM
PM	NS	ZE	PS	PS	PM	PM	PB
PS	ZE	PS	PS	PM	PM	PB	PB

NB = Negative Big,
NM = Negative Medium,
NS = Negative Small,
ZE = Zero Equal,
PS = Positive Small,
PM = Positive Medium,
PB = Positive Bog

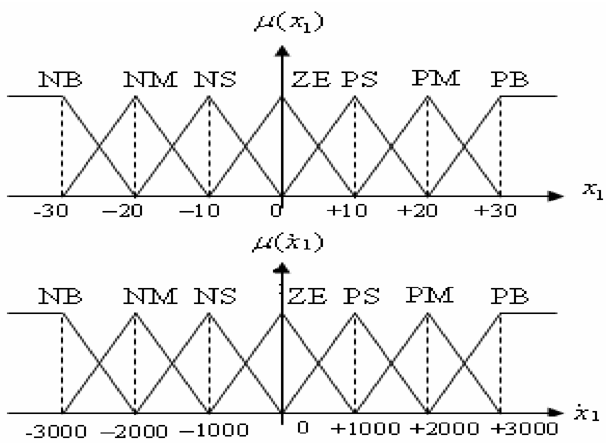

Fig. 3 Input membership functions of the Mamdani fuzzy controller

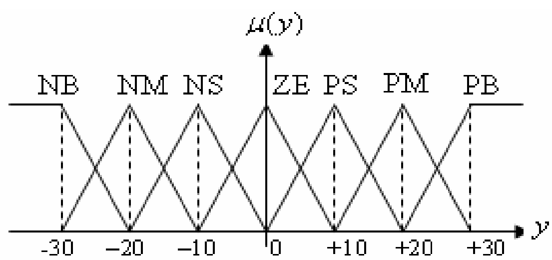

Fig. 4 Output membership functions of the Mamdani fuzzy controller

IV. Stability Analysis

Now, it is proven that the Lorenz system with fuzzy controller is globally asymptotic stable. For this purpose, the coordination of the system is changed as the following:

$$\begin{aligned}\tilde{x}_1(t) &= x_1(t) - \phi_1(x)\\ \tilde{x}_2(t) &= x_2(t) - \phi_2(x)\\ \tilde{x}_3(t) &= x_3(t) - \phi_3(x)\end{aligned} \tag{2}$$

Where $\phi_1(x), \phi_2$ and $\phi_3(x)$ are nonlinear and smooth functions.

Now the following functions are defined:

$$\begin{aligned}\gamma_1(t) &= \dot{\phi}_1(x) = u(x) - ax_1 + ax_2 - \tilde{x}_1\\ \gamma_2(t) &= \dot{\phi}_2(x) = cx_1 - x_2 - x_1x_3\\ \gamma_3(t) &= \dot{\phi}_3(x) = x_1x_2 - bx_3\end{aligned} \tag{3}$$

Then the Lyapunov function is defined as

$$V = \frac{1}{2}(\tilde{x}_1^2(t) + \tilde{x}_2^2(t) + \tilde{x}_3^2(t)) \tag{4}$$

By substitution equation (3) in derivation of the Lyapunov function:

$$\dot{V} = -\frac{1}{2}\tilde{x}_1^2(t) \tag{5}$$

$\dot{V} \le 0$ and $\dot{V} = 0$ if and only if $\tilde{x}_1(t) = 0$.

So, the system is globally asymptotic stable, if and only if $\gamma_1(x), \gamma_2(x)$ and $\gamma_3(x)$ defined in equation (3), are smooth functions.

Now, we claim that $\gamma_1(x), \gamma_2(x)$ and $\gamma_3(x)$ are smooth. The stability analysis is completed analysis by proving our claim.

$\gamma_2(x)$ and $\gamma_3(x)$ are smooth, because the states of the system are smooth. So, this is sufficient to show that u(x) is a smooth function. u(x) should be defined according to the Mamdani method as follows:

$$u(x) = \frac{\sum_{l_7=1}^{N_7} \cdots \sum_{l_1=1}^{N_1} \bar{y}^{l_7 \ldots l_1} \left(\prod_{i=1}^{7} \mu_{A_i^{l_i}}(x_i)\right)}{\sum_{l_7=1}^{N_7} \cdots \sum_{l_1=1}^{N_1} \left(\prod_{i=1}^{7} \mu_{A_i^{l_i}}(x_i)\right)} \tag{6}$$

Where $\mu_{Ai}(x)$ is membership function.

According to simulation result u(x) has the maximum change when $y = x_1 \in [-100\,100]$ and $x = \int x_1 dx \in [-10\,10]$, so u(x) can be approximated as the following:

$$u(x) = \begin{cases} (\frac{1}{10}y - 10)(\frac{1}{100}x - 100) & -10 < y \le 0 \quad and \quad -10 < x \le 0\\ (\frac{1}{10}y - 10)(\frac{-1}{100}x + 100) & -10 < y \le 0 \quad and \quad 0 \le x < 10\\ (\frac{-1}{10}y + 10)(\frac{1}{100}x - 100) & 0 \le y < 10 \quad and \quad -10 < x \le 0\\ (\frac{-1}{10}y + 10)(\frac{-1}{100}x + 100) & 0 \le y \le 10 \quad and \quad 0 \le x < 10 \end{cases} \tag{7}$$

Where $y = x_1$ and $x = \int x_1 dx$ that x_1, $\int x_1 dx$, x_2 and x_3 are smooth. So, $\gamma_1(x)$ is also smooth

V. Simulation Results

Figs. 5 and 6 show the simulation results of the Lorenz equations by Mamdani fuzzy controller.

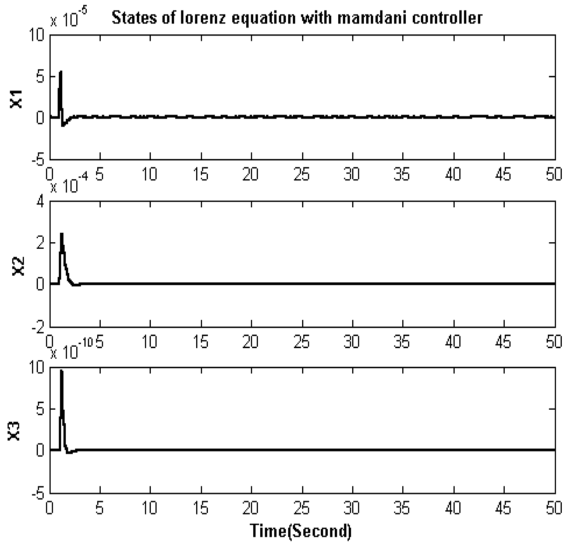

Fig. 5 the state variables of system with Mamdani fuzzy controller

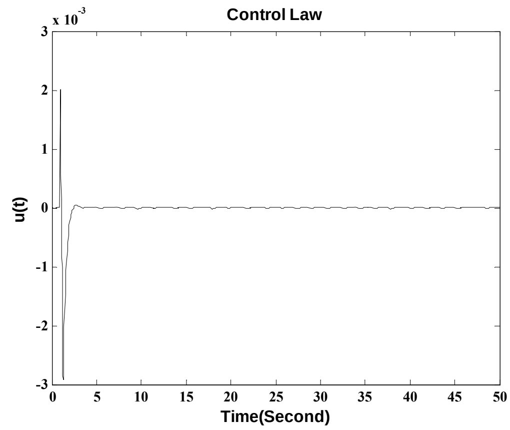

Fig. 6 The control signal

VI. Comparing Mamdani Fuzzy Controller with Sugeno Fuzzy Controller

Table II compares the Mamdani fuzzy controller that proposed in this paper with the Sugeno fuzzy controller in [7]

TABLE II
COMPARING MAMDANI AND SUGENO CONTROL METHODS

	MAMDANI FUZZY CONTROLLER	SUGENO FUZZY CONTROLLER [7]
$\int_0^\infty \lvert u(t)\rvert dt$ (CONTROL SIGNAL)	1.5x10^{-1}	1x10^{-1}
MAX. OF X_3	1x10^{-9}	3x10^{-8}
MAX. OF X_2	2.3x10^{-4}	7.2x10^{-3}
MAX. OF X_1	5.5x10^{-5}	5.1x10^{-5}
$\int_0^\infty \lvert x_3(t)\rvert dt$	1.765x10^{-10}	3.678x10^{-9}
$\int_0^\infty \lvert x_2(t)\rvert dt$	1.1686x10^{-4}	2.146x10^{-4}
$\int_0^\infty \lvert x_1(t)\rvert dt$	1.558x10^{-5}	7.678x10^{-5}

VII. Conclusion

In this paper, the Lorenz equations were stabilized by the fuzzy controller. A simple and effective Mamdani fuzzy controller is designed and applied to the Lorenz equations. The stability of the proposed method was proven and it was shown that the Lorenz equation is globally stable by this controller. Simulation results were shown that by applying this controller, performance of the system has been improved and the resulted system is stable.

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