Stabilization and Observation of Attitude Control Systems for Micro Satellites

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Abstract—In this paper, we are interested in attitude control of a satellite, which using wheels of reaction, by state feedback. First, we develop a method allowing us to put the control and its integral in the state-feedback form. Then, by using the theorem of Gronwall-Bellman, we put the sufficient conditions so that the nonlinear system modeling the satellite is stabilisable and observed by state feedback.

Keywords—Satellite, attitude control, state feedback, attitude stabilization.

I. INTRODUCTION

THE attitude control, in the aeronautic area, occupies a particular place in the automatic control researches. In more of the severe criteria imposed by the schedules of conditions, in terms of precision and control robustness, the interest related to such systems also rests on structural considerations.

Indeed, these nonlinear systems being, multivariable and strongly sensitive to the disturbances, constitute benchmarks for the checking of the control laws having to be synthesized for less constrained industrial systems.

In addition, several works were established on the attitude control, where various approaches were used, and in particular PID regulators [7], quadratic regulation (LQR) [5], robust control [8,10]. Many works considers simplified linear models, which neglect a part of the system dynamics, or are based on an adaptive identification being able to weigh down calculations of the control laws.
Our results are based on a nonlinear model while avoiding any simplifications, as it is the case in many related works.

The dynamic model of a satellite with earth pointing, using a SCA 3axes maneuvered by the reaction wheels and the magneto couplers (MC), and evolving under the effect of the disturbances, results from the equations of the dynamics and the movement cinematic of the satellite[4,7,9].

The mathematical model of the system is that given by the following state space representation:

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$$\dot{x}(t) = Ax(t) + Bu(t) + G\int_0^t u(s)ds + \sum_{i=1}^{3}\left(\int_0^t u_i(s)ds\right)C_i x(t) \qquad (1)$$

Where:

- $x(0) = x_0$;

- $u(t) = -h_w^{\cdot}(t)$: is the control where h(t) is the angular momentum of the wheel cluster;

- $x(t) = (\phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi})$: the state representing respectively the Eulerian angles and their derivatives.

- A, B, G, C_i : matrixes with respectively (n, n) ;(n, p) ;(n, p) ;(n, n) dimensions.

In this work, we present a control law computed by double poles placement; the studied model is nonlinear. In the second section, we present a control design method applied to the linear part of the system, which we generalize thereafter with the nonlinear model. In the next section, we treat a feedback control based on an observer that is generally used when a part or all variables of state are not measurable. The proof of the controlled system stability is established using the Bellman-Gronwall lemma [2, 3].

II. STATE-FEEDBACK STABILIZATION

In this section, we consider the system of attitude control for the satellite described by the nonlinear model above. We will carry out the synthesis, by steps, of the stabilizing control.

A. Linear System Design

Neglecting the quasi-bilinear term in the system of equation (1), we obtains the following linear system:

$$\dot{x}(t) = Ax(t) + B u(t) + G\int_0^t u(s)ds \text{ Where x }(0) = x_0 \qquad (2)$$

Proposition:

Consider the system described by the simplified model (2), and let the pair (A_1, B_1) be controllable, with:

$$A_1 = \begin{pmatrix} A & G \\ 0 & 0 \end{pmatrix} \text{ is } (n+p, n+p) \text{ and } B_1 = \begin{pmatrix} B \\ I \end{pmatrix} \text{ is } (n+p, p)$$

There exist a state feedback of the form u (t) = - K x (t) that stabilise the system.

Proof:

$$\text{Let}: \ z = \int_0^t u(s)ds , \ \text{hence}: \ \dot{z} = u(t) \tag{3}$$

From (2) and (3), the new system becomes:

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A & G \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} + \begin{pmatrix} B \\ I \end{pmatrix} u \tag{4}$$

Where I and 0 are respectively identity and null matrix.

Let : $X = \begin{pmatrix} x \\ z \end{pmatrix}$; it follows:

$$\dot{X} = A_1 X + B_1 u \tag{5}$$

Where:

$A_1 = \begin{pmatrix} A & G \\ 0 & 0 \end{pmatrix}$ is $(n+p, n+p)$ and $B_1 = \begin{pmatrix} B \\ I \end{pmatrix}$ is $(n+p, p)$

If the pair (A_1, B_1) is controllable, then there is a linear state feedback allowing the system (5) to be stable. To design this control, we will develop two successive poles placements; the first pole placement allows having a relation between

u (t) and its integral

$$u(t) = -K_1 x(t) - K_2 \int_0^t u(s)ds \tag{6}$$

The second pole placement is then:

$$u(t) = -K_0 x(t) \tag{7}$$

So, the two successive poles placements, enabled us to put

u(t) and its integral in the state – feedback form.

1ˢᵗ poles placement

Let us seek a gain matrix K such as $(A_1 - B_1 K)$ is asymptotically stable, and then stabilizes the system (5). Hence:

$$u = -KX = -(K_1 \quad K_2)\begin{pmatrix} x \\ z \end{pmatrix} = -K_1 x - K_2 z = -K_1 x - K_2 \int_0^t u(s)ds$$

Remark: K_2 is chosen to be invertible matrix.

So we have:

$$K_2 \int_0^t u(s)ds = -K_1 x - u$$

$$\int_0^t u(s)ds = (K_2^{-1})[-K_1 x - u] = -K_2^{-1} K_1 x - K_2^{-1} u.$$

Let us replace this integral in the linear system (2), we obtains:

$$\begin{aligned} x(t) &= Ax(t) + Bu(t) + G[-K_2^{-1}K_1 x - K_2^{-1}u] \\ &= (A - GK_2^{-1}K_1)x + (B - GK_2^{-1})u \\ &= A_2 x + B_2 u \end{aligned} \tag{8}$$

Such as:

$$A_2 = A - G.K_2^{-1}.K_1 \ \text{et} \ B_2 = B - G.K_2^{-1}$$

2ⁿᵈ poles placement

Let us seek a gain matrix K_0, such that the matrix $(A_2 - B_2 K_0)$ is asymptotically stable and then stabilizes the system (8).

Consider the control:

$$u(t) = -K_0 x(t)$$

The closed loop system is:

$$\dot{x}(t) = (A_2 - B_2 K_0)x = F_0 x(t) \tag{9}$$

The eigenvalues of the matrix F_0, λ_i ($\lambda_i \neq \lambda_j$ pour $i \neq j$) are such as Re $(\lambda_i) < 0$. Hence the system that models the satellite attitude movement is stable.

The following result enables us to give the stability conditions of the nonlinear system.

B. Application to the Non Linear System Stabilization

Consider the non linear system (1) that models the satellite:

$$\dot{x}(t) = Ax(t) + Bu(t) + G\int_0^t u(s)ds + \sum_{i=1}^3 (\int_0^t u_i(s)ds)(C_i x(t))$$

Where the choice of the control law can be derived from the two poles placement, it's easy to see that the non linear system can be written in the following form:

$$\dot{x}(t) = F_0 x(t) + \sum_{i=1}^3 (\int_0^t u_i(s)ds)C_i x(t). \tag{10}$$

Where, according to the previous calculation, F_0 is asymptotically stable.

The matrix F_0 is asymptotically stable, then from the Hille-Yoshida theorem, there exists M>0 and ω<0 such as:

$$\forall t \geq 0 : \|e^{F_0 t}\| \leq M \ e^{wt} \tag{11}$$

Theorem:

Let: $R = \dfrac{-\omega}{M^2 \sum_{i=1}^p \|\delta_i\|\|c_i\|}$ and $M_0 = \dfrac{M}{1 + \dfrac{M^2}{\omega}\sum_{i=1}^p \|\delta_i\|\|C_i\|\|x_0\|}$

For $\|x_0\| \leq R$, the system (10) controlled by the linear state feed back: $u(t) = -K_0 x(t)$

And then is asymptotically stable.

Where: δ_i is a matrix that depends on the control choice.

Proof:

From (1), (2) et (9), we can write:

$$\dot{x}(t) = F_0 x(t) + \sum_{i=1}^3 (\int_0^t u_i(s)ds)C_i x(t).$$

And: $\int_0^t u(s)ds = -K_2^{-1}K_1 x - K_2^{-1}u.$

It follows: $\int_0^t u_i(s)ds = (-K_2^{-1}K_1 x)_i - (K_2^{-1}u)_i$

And from the second poles placement $u(t) = -K_0 x(t)$

$$\int_0^t u_i(s)ds = (-K_2^{-1}K_1 x)_i + (K_2^{-1}K_0 x)_i = ((-K_2^{-1}K_1 + K_2^{-1}K_0)x)_i$$

$$\text{Hence}: \int_0^t u_i(s)ds = (\delta \ x(t))_i \tag{12}$$

With: $(\delta)_i = (-K_2^{-1}K_1 + K_2^{-1}K_0)_i$

The studied system is:

$$\begin{cases} \dot{x} = F_0 x + \sum_{i=1}^{p} (\delta\ x(t))_i\ C_i x(t) \\ x(t_0) = x_0 \end{cases} \quad (13)$$

The solution of the system (13) is:

$$x(t) = e^{F_0 t} x_0 + \int_0^t e^{F_o(t-s)} (\sum_{i=1}^{p} (\delta\ x(t))_i\ C_i x(t)) ds$$

Application of Hille-Yoshida theorem leads to:

The matrix F_0 is asymptotically stable, if there exists M>0 and ω<0 such as:

$$\forall\ t \geq 0 : \left\| e^{F_0 t} \right\| \leq M\ e^{wt}$$

Hence:

$$\|x(t)\| \leq M\ e^{wt} \|x_0\| + \int_0^t (M\ e^{w(t-s)}) \sum_{i=1}^{p} \|\delta_i\| \|C_i\| \|x(s)\|^2) ds$$

$$\|x(t)\ e^{-wt}\| \leq M \|x_0\| + \int_0^t (M\ e^{ws}) \sum_{i=1}^{p} \|\delta_i\| \|C_i\| \|e^{-ws} x(s)\|^2) ds$$

Let:

$$M\ \|x_0\| = k\ ;\ g(s) = M \sum_{i=1}^{3} \|\delta_i\| \|C_i\| \|e^{ws}\|\ ;$$

$$m(t) = x(t) e^{-\omega t}\ ;\ n = 2$$

Hence:

$$m(t) \leq k + \int_0^t g(s).[m(s)]^2 ds = k + \int_0^t [g(s).m(s)] m(s) ds$$

From the Bellman – Gronwall lemma [2]:

$$m(t) \leq k \exp\int_0^t g(s) ds \Rightarrow -m(t) \exp\int_0^t - g(s) m(s) ds \geq -k$$

$$\Rightarrow \frac{d}{dt} \exp\int_0^t - g(s) m(s) ds \geq -k g(s)$$

Hence

$$k(m(t))^{-1} \geq 1 - k \int_0^t g(s) ds$$

Under the following hypotheses: $1 - k \int_0^t g(s) ds \succ 0$

We obtain: $m(t) \leq \dfrac{k}{1 - k \int_0^t g(s) ds}$

III. OBSERVATION AND STABILIZATION

We consider the non linear system representing the satellite (10); we will investigate the case when states measurements are not all available, i.e. it is not possible to design a control of the form: $u(t) = g(x(t))$ .then a linear feedback on the estimate state of x (t) is considered:

$$g(z(t)) = -K.z(t) \quad (14)$$

Where z (t) is the estimation of x(t).

The system considered is written in the form:

$$x^{\bullet}(t) = F_0 x(t) + \sum_{i=1}^{p} (\int_0^t u_i(s) ds) C_i x(t).$$

Without loss generality, let p =1. The system considered will be described by:

$$\dot{x}(t) = F_0.x(t) + (\int_0^t u(s) ds) C_1 x(t). \qquad x(0) = x_0 \quad (15)$$

$$y(t) = D.x(t)$$

We design an observer modeled by the equation:

$$\dot{z}(t) = F_0 z(t) + (\int_0^t u(s) ds) C_1.z(t) + L(y(t) - \hat{y}(t))$$

$$\dot{z}(t) = (F_0 - L.D) z(t) + (\int_0^t u(s) ds) C_1.z(t) + L.y(t). \quad (16)$$

Thus:

$$\dot{z}(t) = F.z(t) - K.z(t).C.z(t) + L.y(t)$$

Where: $F = F_0 - L.D$ and $C_1 = C$

From (12), we have: $\int_0^t u(s) ds = -\delta.z(t)$

Let's put: $\delta = K$;

We are therefore: $\int_0^t u(s) ds = -K.z(t) \quad (17)$

Our objective is to show that the dynamic system with the augmented state vector $w(t) = \begin{bmatrix} x & \varepsilon \end{bmatrix}^T$ is asymptotically stable.

Main hypotheses

1-The pair (F_0, D) is detectable, i.e.: there exist a matrix L such that: $F = F_0 - LD$ is asymptotically stable, which implies that there exist $M_2 \succ 0$ and $w_2 \prec 0, \forall t \geq 0$:

$$\left\| e^{Ft} \right\| \leq M_2 e^{w_2 t}$$

2- The matrix F_0 is asymptotically stable, i.e.: there exist $M_1 \succ 0$ and $w_1 \prec 0, \forall t \geq 0$:

$$\left\| e^{F_0 t} \right\| \leq M_1 e^{w_1 t}$$

Theorem:

Let: $R = \dfrac{-w}{2.\tilde{M}^2 \|K\| \|C\|}$

And: $M_0 = \dfrac{\tilde{M}}{1 + 2.\tilde{M}^2 \|w_0\| \|K\| \dfrac{\|C\|}{w}}$

For $\|w_0\| \leq R$, and under the previous main hypotheses, the system (15) controlled by the linear state observer feedback (14) satisfies for all $t \geq 0$:

$$\|w(t)\| \leq M_0 \|w_0\| e^{wt} \quad (18)$$

Hence the system (15) is asymptotically stable and (16) asymptotically observes (15).

Proof:

The system and the estimation error $\varepsilon = z - x$ are described by equations:

$$\dot{x}(t) = F_0 x(t) - Kz(t).C.x(t) \quad (19)$$

$$\dot{\varepsilon}(t) = F.\varepsilon(t) - K.z(t).C.\varepsilon(t) \quad (20)$$

Where: F is chosen to verify: $F = F_0 - L.D$.

Hence, such dynamical system with a state vector $w(t)$ is written as:

$$w(t) = R.w(t) + S(t). \qquad (21)$$

Where:

$$R = \begin{pmatrix} F_0 & 0 \\ 0 & F \end{pmatrix} \quad \text{and} \quad S(t) = \begin{pmatrix} -Kz(t)Cx(t) \\ -Kz(t)C\varepsilon(t) \end{pmatrix}$$

The solution of the system (21) is written as:

$$w(t) = e^{Rt}w_0 + \int_0^t e^{R(t-s)} S(s)ds \qquad (22)$$

The theorem of Schumacher [6] asserts that:

$$\exists \; \widetilde{M} \succ 0 \quad \text{Such that:} \; \left\| e^{Rt} \right\| \leq \widetilde{M}.e^{wt}$$

Where $w = \sup(w_1, w_2)$.

Therefore: $\left\| w(t) \right\| e^{-wt} \leq \widetilde{M} \left\| w_0 \right\| + \int_0^t \widetilde{M} e^{-ws} \left\| S(s) \right\| ds$

It's easy to see that: $\left\| S(s) \right\| \leq 4. \left\| K \right\| \left\| C \right\| \left\| w(s) \right\|^2$

Hence:

$$\left\| w(t) \right\| e^{-wt} \leq \widetilde{M} \left\| w_0 \right\| + \int_0^t 4.\widetilde{M} \left\| K \right\| \left\| C \right\| e^{ws} \left(\left\| w(s) \right\| e^{-ws} \right)^2 ds \quad (23)$$

Application of the Bellman- Gronwall's lemma leads to the result in theorem; under the main hypotheses, we have:

$$\lim_{t\to\infty} \left\| w(t) \right\| = 0 \;.$$

Hence the system (15) is asymptotically stable and asymptotically observed by (1 6).

IV. CONCLUSION

In this paper, we proposed a general stabilization method, with linear state – feedback laws, for a micro satellite that uses the wheels of reaction and whose model is nonlinear. In the first part, we simplified the system by treating only its linear part; then we proposed a control law by successive poles placement.

By generalizing this control to the nonlinear model of the satellite, we studied the problem of state feedback stabilization and observation, while giving the conditions so conditions that the system considered is stabilisable.

The synthesis of the control suggested is based on the solution of the state space equation and on the Bellman-Gronwall lemma.

REFERENCES

[1] Desoer C. A., Vidyasagar M., Feedback systems: Input – Output Properties, Academic Press New York 1975.

[2] N.Elalami, « A generalisation of Gronwall's lemma »,2ème conférence internationale sur les équations différentielles, faculté des sciences Marrakech, Juin 1995

[3] N.Elalami, A.Echchatbi, A.Bouaziz; « Observation et Stabilisation Robust des systèmes Bilinéaires Incertains » CIFA 2002 ,Conférence Internationale Francophone d'Automatique, Nantes, 8-10 juillet 2002.

[4] P.C. Hughes (1986). Spacecraft Attitude Dynamics. John Wiley & sons, USA, New York.

[5] K.L. Musser and W .L . Elbert (1986). Autonomous spacecraft attitude control using magnetic torquing only. Proceeding of flight Mechanics Estimation Theory Symposium, pp. 23-38. NASA.

[6] Schumacher J. M., A direct approach to compensator. Design for distributed parameters systems, Siam J. Control and optimization, vol. 21, N.6,1983.

[7] M. J. Sidi (1977). Spacecraft Dynamics and control. Cambridge University Press, Cambridge, UK.

[8] A. Skullestad and J. Gilbert (2000). H_∞ control of gravity gradient stabilised Satellite. Control Engineering Practice, 8, 975-983.

[9] P. Tsiotras, H. Shen and C. Hall. Satellite attitude control and power tracking with momentum wheels, AAS, 99-317.

[10] C. Valentin- Charbonnel, G. Duc and S. Le Ballois (1999). Low-order robust attitude control of an earth observation satellile. Control Engineering Practice, 7, 493-506.