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Stability Analysis of Neural Networks with Leakage,Discrete and Distributed Delays

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Abstract—This paper deals with the problem of stability of neural networks with leakage, discrete and distributed delays. A new Lyapunov functional which contains some new double integral terms are introduced. By using appropriate model transformation that shifts the considered systems into the neutral-type time-delay system, and by making use of some inequality techniques, delay-dependent criteria are developed to guarantee the stability of the considered system. Finally, numerical examples are provided to illustrate the usefulness of the proposed main results.

Keywords—Neural networks, Stability, Time-varying delays, Linear matrix inequality.

I. Introduction

STABILITY analysis for neural network is an issue of both theoretical and practical importance due to the fact that its wide application in various areas such as image processing,automatic control,pattern recognition,and so on. Therefore, the study of stability analysis for neural networks has been attracting the interest of many researchers in the past years, and many important and interesting results have been proposed in terms of all sorts of methods in the literature [1-10] and the references therein.

Now, many sufficient conditions ensuring global asymptotic stability and global exponential stability for delayed neural networks have been derived [3-25], however, only little attention has been paid towards the stability analysis of neural networks and dynamic systems involving time-delay in the leakage term [20-30]. To the bast of authors'knowledge,up to now there are no result on the problem of stability analysis for neural networks with time-delays in the leakage term.

Motivated by this mentioned above, in this paper, two new delay-dependent stability criteria for neural networks with interval time-varying delay will be proposed by dividing the delay interval, constructing new Lyapunov-Krasovskii functional and shifting the considered systems into the neutral-type time-delay system.The obtained criterion are less conservative because free-weighting matrices method and a convex optimization approach are considered. Finally, numerical examples are given to illustrate the the usefulness and feasibility of the proposed main results.

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II. Problem statement

Consider the following neural networks with mixed time-varying delays:

$$\begin{aligned}\dot{x}(t) = -Ax(t-\sigma) + Bg(x(t)) + Cg(x(t-\varsigma(t))) \\ +D\int_{t-r(t)}^{t} g(x(s))ds + I_0\end{aligned} \tag{1}$$

$$x(t) = \varphi(t),\ \ t \in [-h, 0]$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$ is the neuron state vector, $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T$ denotes the neuron activation function, and $I_0 = [I_1, I_2, \ldots, I_n]^T \in R^n$ is a constant input vector,$A = diag\{a_i\} \in R^n$ is a positive diagonal matrix , $B = (b_{ij})_{n\times n} \in R^n$ is the connection weight matrix,$C = (c_{ij})_{n\times n} \in R^n$,and $D = (d_{ij})_{n\times n} \in R^n$ are the delayed connection weight matrices,and σ is the leakage delay satisfying $\sigma \geq 0$.The initial vector $\varphi(t)$ is bounded and continuous on $[-h, 0]$.
The following assumptions are adopted throughout the paper.
Assumption 1: The delays $\varsigma(t), r(t)$ are time-varying continuous function and satisfy:

$$0 \leq \varsigma(t) \leq \varsigma, 0 \leq r(t) \leq r, \dot{\varsigma}(t) \leq \mu \tag{2}$$

where ς, r, μ are constant.
Assumption 2: Each neuron activation function $g_i(\cdot), i = 1, 2, \ldots, n$,in (1) satisfies the following condition:

$$\phi_i^- \leq \frac{g_i(\alpha) - g_i(\beta)}{\alpha - \beta} \leq \phi_i^+, \forall \alpha, \beta \in R, \alpha \neq \beta \tag{3}$$

where $\phi_i^-, \phi_i^+, i = 1, 2, \ldots, n$ are constants,and matrices $\Phi_1 = diag\{\phi_1^+, \phi_2^+, \ldots, \phi_n^+\}, \Phi_2 = diag\{\phi_1^-, \phi_2^-, \ldots, \phi_n^-\}$.
Based on Assumption 1-2, it can be easily proven that there exists one equilibrium point for (1) by Brouwer's fixed-point theorem. Assuming that $x^* = [x_1^*, x_2^*, \ldots, x_n^*]^T$ is the equilibrium point of (1) and using the transformation $y(\cdot) = x(\cdot) - x^*$,system (1) can be converted to the following system :

$$\begin{aligned}\dot{y}(t) = Ay(t-\sigma) + Bf(y(t)) + Cf(y(t-\varsigma(t))) \\ +D\int_{t-r(t)}^{t} f(y(s))ds\end{aligned} \tag{4}$$

where $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^T$ is the state vector of the transformed system, and the neuron activation function in system (4) $f(y(t)) = [f_1(y_1(t)), f_2(y_2(t)), \ldots, f_n(y_n(t))]^T$, satisfy that $f_i(y_i(\cdot)) = g_i(x_i(\cdot)+x_i^*)-g_i(x_i^*)$, $i = 1, 2, \ldots, n$. From $Eq.(3)$,$f_i(\cdot)$ satisfies the following condition:

$$\phi_i^- \le \frac{f_i(\alpha)}{\alpha} \le \phi_i^+, \forall \alpha \neq 0, i = 1, 2, \ldots, n. \tag{5}$$

Moreover, the above system has an equivalent form as follows:

$$\frac{d}{dt}\left[y(t) - A\int_{t-\sigma}^{t} y(s)ds\right] = -Ay(t) + Bf(y(t)) + Cf(y(t-\varsigma(t))) + D\int_{t-r(t)}^{t} f(y(s))ds \tag{6}$$

Lemma 1 [9].For any constant matrices $G, S \in R^{n\times n}, S = S^T, G = G^T > 0$,the following inequality holds:

$$-\varsigma\int_{t-\varsigma}^{t} x^T(s)Gx(s)ds \le -\begin{bmatrix}\int_{t-\varsigma(t)}^{t} x(s)ds \\ \int_{t-\varsigma}^{t-\varsigma(t)} x(s)ds\end{bmatrix}^T \begin{bmatrix} G & S \\ * & G\end{bmatrix}\begin{bmatrix}\int_{t-\varsigma(t)}^{t} x(s)ds \\ \int_{t-\varsigma}^{t-\varsigma(t)} x(s)ds\end{bmatrix} \tag{7}$$

III. Main results

In this section,a new Lyapunov functional is constructed and a less conservative delay-dependent stability criterion is obtained.

Theorem 1 Given that the Assumption 1-2 hold, the system (6) is globally asymptotic stability if there exist symmetric positive definite matrices $P, Q_i, i = 1, 2, 3, 4, R_j, j = 1, \ldots,$ 6, positive diagonal matrices $L = diag\{l_1, l_2, \ldots, l_n\}, W_1,$ $W_2, K = diag\{k_1, k_2, \ldots, k_n\}$, and symmetric matrix T_1, T_2, such that the following LMIs hold:

$$\Pi < 0 \tag{8}$$

$$\begin{bmatrix} Q_2 & T_1 \\ * & Q_3\end{bmatrix} < 0 \tag{9}$$

where

$$\Pi = [\pi_{ij}]_{11\times 11}$$

$$\pi_{11} = -2PA + R_1 + R_2 + R_3 + R_4 + R_5 + \varsigma Q_2 + \sigma^2 R_6 + T_1 - 2\Phi_1 W_1 \Phi_2$$

$$\pi_{12} = \Phi_2 LA - \Phi_1 KA$$

$$\pi_{17} = PB - \Phi_2 LB + \Phi_1 KB + W_1(\Phi_1 + \Phi_2)$$

$$\pi_{18} = PC - \Phi_2 LC + \Phi_1 KC$$

$$\pi_{19} = PD - \Phi_2 LD + \Phi_1 KD, \pi_{1,11} = A^T PA$$

$$\pi_{22} = -R_5 + \varsigma A^T Q_3 A$$

$$\pi_{27} = -AL + AK - \varsigma A^T Q_3 B, \pi_{28} = -\varsigma A^T Q_3 C$$

$$\pi_{29} = -\varsigma A^T Q_3 D, \pi_{33} = -R_2 - T_1, \pi_{44} = -R_3$$

$$\pi_{55} = -R_4, \pi_{66} = -(1-\mu)R_1 - 2\Phi_1 W_2 \Phi_2$$

$$\pi_{68} = W_2(\Phi_1 + \Phi_2)$$

$$\pi_{77} = 2LB - 2KB + \varsigma B^T Q_3 B + Q_1 + r^2 Q_4 - 2W_1$$

$$\pi_{78} = LC - KC + \varsigma B^T Q_3 C$$

$$\pi_{79} = LD - KD + \varsigma B^T Q_3 D, \pi_{7,11} = -B^T PA$$

$$\pi_{88} = -(1-\mu)Q_1 - 2W_2 + \varsigma C^T Q_3 C$$

$$\pi_{89} = \varsigma C^T Q_3 D, \pi_{8,11} = -C^T PA$$

$$\pi_{99} = \varsigma D^T Q_3 D - Q_4, \pi_{9,10} = -T_2$$

$$\pi_{9,11} = -D^T PA, \pi_{10,10} = -Q_4, \pi_{11,11} = -R_6$$

Proof: Construct a new class of Lyapunov functional candidate as follow:

$$V(y_t) = \sum_{i=1}^{4} V_i(y_t)$$

with

$$V_1(y_t) = [y(t) - A\int_{t-\sigma}^{t} y(s)ds]^T P[y(t) - A\int_{t-\sigma}^{t} y(s)ds]$$

$$V_2(y_t) = 2\sum_{i=1}^{n}\int_0^{y_i(t)} [l_i(f_i(s) - \phi_i^- s) + k_i(\phi_i^+ s - f_i(s))]ds$$

$$\begin{aligned} V_3(y_t) = &\int_{t-\varsigma(t)}^{t} y^T(s)R_1 y(s)ds + \int_{t-\varsigma}^{t} y^T(s)R_2 y(s)ds \\ &+ \int_{t-\frac{\varsigma}{3}} y^T(s)R_3 y(s)ds + \int_{t-\frac{2\varsigma}{3}} y^T(s)R_4 y(s)ds \\ &+ \int_{t-\sigma}^{t} y^T(s)R_5 y(s)ds + \int_{t-\varsigma(t)}^{t} f^T(y(s))Q_1 f(y(s))ds \end{aligned}$$

$$\begin{aligned} V_4(y_t) = &\int_{-\varsigma}^{0}\int_{t+\theta}^{t} [y^T(s)Q_2 y(s) + \dot{y}^T(s)Q_3\dot{y}(s)]dsd\theta \\ &+ \sigma\int_{-\sigma}^{0}\int_{t+\theta}^{t} y^T(s)R_6 y(s)dsd\theta \\ &+ r\int_{-r}^{0}\int_{t+\theta}^{t} f^T(y(s))Q_4 f(y(s))dsd\theta \end{aligned}$$

Then, taking the time derivative of V(t) with respect to t along the system (6) yield

$$\begin{aligned} \dot{V}_1(y_t) = &2[y(t) - A\int_{t-\sigma}^{t} y(s)ds]^T P[-Ay(t) + Bf(y(t)) \\ &+ Cf(y(t-\varsigma(t))) + D\int_{t-r(t)}^{t} f(y(s))ds] \end{aligned} \tag{10}$$

$$\begin{aligned} \dot{V}_2(y_t) = &2(f^T(y(t)) - y^T(t)\Phi_2)L\dot{y}(t) \\ &+ 2(y^T(t)\Phi_1 - f^T(y(t)))K\dot{y}(t) \end{aligned} \tag{11}$$

$$\begin{aligned}\dot{V}_3(y_t) \le\ & y^T(t)(R_1+R_2+R_3+R_4+R_5)y(t)\\ & -(1-\mu)y^T(t-\varsigma(t))R_1y(t-\varsigma(t))+f^T(y(t))Q_1f(y(t))\\ & -y^T(t-\varsigma)R_2y(t-\varsigma)-y^T(t-\frac{\varsigma}{3})R_3y(t-\frac{\varsigma}{3})\\ & -y^T(t-\frac{2\varsigma}{3})R_4y(t-\frac{2\varsigma}{3})-y^T(t-\sigma)R_5y(t-\sigma)\\ & -(1-\mu)f^T(y(t-\varsigma(t)))Q_1f(y(t-\varsigma(t)))\end{aligned} \tag{12}$$

$$\begin{aligned}\dot{V}_4(y_t) =\ & \varsigma[y^T(t)Q_2y(t)+\dot{y}^T(t)Q_3\dot{y}(t)]\\ & -\int_{t-\varsigma}^{t}[y^T(s)Q_2y(s)+\dot{y}^T(s)Q_3\dot{y}(s)]\\ & +\sigma^2y^T(t)R_6y(t)-\sigma\int_{t-\sigma}^{t}y^T(s)R_6y(s)ds\\ & +r^2f^T(y(s))Q_4f(y(s))-r\int_{t-r}^{t}f^T(y(s))Q_4f(y(s))ds\end{aligned} \tag{13}$$

Consider the following zero equality with any symmetric matrix T_1 :

$$y^T(t)T_1y(t)-y^T(t-\varsigma)T_1y(t-\varsigma)-2\int_{t-\varsigma}^{t}y^T(s)T_1\dot{y}(s)ds=0 \tag{14}$$

Using Jensen's inequality,we can obtain that:

$$-\sigma\int_{t-\sigma}^{t}y^T(s)R_6y(s)\le-(\int_{t-\sigma}^{t}y(s)ds)^TR_6(\int_{t-\sigma}^{t}y(s)ds) \tag{15}$$

Using Lemma 1,we can obtain that:

$$\begin{aligned}& -r\int_{t-r}^{t}f^T(y(s))Q_4f(y(s))ds\\ & \le-\begin{bmatrix}\int_{t-r(t)}^{t}f(y(s))ds\\ \int_{t-r}^{t-r(t)}f(y(s))ds\end{bmatrix}^T\begin{bmatrix}Q_4 & T_2\\ * & Q_4\end{bmatrix}\begin{bmatrix}\int_{t-r(t)}^{t}f(y(s))ds\\ \int_{t-r}^{t-r(t)}f(y(s))ds\end{bmatrix}\end{aligned} \tag{16}$$

From (5), we can obtain that:

$$\begin{aligned}-2f^T(y(t))W_1f(y(t))+2y^T(t)W_1(\Phi_1+\Phi_2)f(y(t))\\ -2y^T(t)\Phi_1W_1\Phi_2y(t)\ge0\end{aligned} \tag{17}$$

$$\begin{aligned}& -2f^T(y(t-\varsigma(t)))W_2f(y(t-\varsigma(t)))\\ & +2y^T(t-\varsigma(t))W_2(\Phi_1+\Phi_2)f(y(t-\varsigma(t)))\\ & -2y^T(t-\varsigma(t))\Phi_1W_2\Phi_2y(t-\varsigma(t))\ge0\end{aligned} \tag{18}$$

Basing on (10)-(18),one can obtain that:

$$\dot{V}(y_t)\le\xi^T(t)\Pi\xi(t)-\int_{t-\varsigma}^{t}\begin{bmatrix}y(s)\\ \dot{y}(s)\end{bmatrix}^T\begin{bmatrix}Q_2 & T_1\\ * & Q_3\end{bmatrix}\begin{bmatrix}y(s)\\ \dot{y}(s)\end{bmatrix}$$

where

$$\begin{aligned}\xi^T(t)=\ & [y^T(t),y^T(t-\sigma),y^T(t-\varsigma),y^T(t-\frac{\varsigma}{3}),y^T(t-\frac{2\varsigma}{3}),\\ & y^T(t-\varsigma(t)),f^T(y(t)),f^T(y(t-\varsigma(t))),\int_{t-r(t)}^{t}f^T(y(s))ds,\\ & \int_{t-r}^{t-r(t)}f^T(y(s))ds,\int_{t-\sigma}^{t}y^T(s)ds]\end{aligned}$$

Hence,we can obtain that:

$$\dot{V}(y_t)\le-\xi^T(t)\Xi\xi(t)$$

where $\Xi=-\Pi$. Thus,it can be deduced that

$$V(y_t)+\int_0^t\xi^T(s)\Xi\xi(s)ds\le V(y_0),\ t>0$$

where

$$\begin{aligned}V(y_0)\le\ & \Big\{2\lambda_{max}(P)(1+\sigma^2\max_{i\in A}a_i^2)+2\lambda_{max}(\Phi_1-\Phi_2)(\lambda_{max}(L)\\ & +\lambda_{max}(K))+\varsigma(\lambda_{max}(R_1)+\lambda_{max}(R_2)+\frac{1}{3}\lambda_{max}(R_3)\\ & +\frac{2}{3}\lambda_{max}(R_4))+\sigma\lambda_{max}(R_5)+\varsigma\phi^2\lambda_{max}(Q_1)\\ & +2\varsigma^2[\lambda_{max}(A^TA)+\phi^2\lambda_{max}(B^TB)+\phi^2\lambda_{max}(C^TC)\\ & +r^2\phi^2\lambda_{max}(D^TD)]\Big\}\sup_{-h\le s\le0}\|y(s)\|^2<\infty\end{aligned}$$

Furthermore,one can obtain that:

$$\begin{aligned}\|y(t)\| &= \|y(t)-y(t+\theta)+y(t+\theta)\|,\ \ (t\in[0,1])\\ &\le\|\int_t^{t+\theta}\dot{y}(s)ds\|+\|\int_t^{t+1}y(s)ds\|\\ &\le\int_t^{t+\theta}\|\dot{y}(s)\|ds+\int_t^{t+1}\|y(s)\|ds\\ &\le\int_t^{t+1}(\|\dot{y}(s)\|+\|y(s)\|)ds\\ &\le\frac{2}{\sqrt{\lambda_{min}(\Xi)}}\int_t^{t+1}\xi^T(s)\Xi\xi(s)ds\\ &\le\frac{2}{\sqrt{\lambda_{min}(\Xi)}}\int_t^{\infty}\xi^T(s)\Xi\xi(s)ds\to0,\ (t\to\infty)\end{aligned} \tag{19}$$

Therefore, combined with (8)-(9) and (19), we conclude that model (6) has a unique equilibrium point which is globally asymptotically stable. ■

Remark 1 In this paper, time-varying delay is assumed to be differentiable. But in many cases μ is unknown, considering this situation, we can set $Q_1=R_1=0$ in Lyapunov-Krasovskii functional $V(y_t)$ of Theorem 1, then the time-varying delay becomes non-differentiable.

Remark 2 In system (6), if we consider the $\sigma=0$,we can get the following system:

$$\begin{aligned}\dot{y}(t)=Ay(t)+Bf(y(t))+Cf(y(t-\varsigma(t)))\\ +D\int_{t-r(t)}^{t}f(y(s))ds\end{aligned} \tag{20}$$

For system (20), we have the following result.

Theorem 2 Given that the Assumption 1-2 hold, the system (20) is globally asymptotic stability if there exist symmetric positive definite matrices $P,Q_i,i=2,3,4,R_j,j=2,\dots,6,$ positive diagonal matrices $L=diag\{l_1,l_2,\dots,l_n\},W_1,W_2,$ $K=diag\{k_1,k_2,\dots,k_n\}$, and symmetric matrix T_1,T_2, such that the following LMIs hold:

$$\begin{bmatrix}Q_2 & T_1\\ * & Q_3\end{bmatrix}<0 \tag{21}$$

$$E=\begin{bmatrix} E_{11} & 0 & 0 & 0 & 0 & E_{16} & E_{17} & E_{18} & 0 \\ * & E_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -R_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -R_4 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & E_{55} & 0 & E_{57} & 0 & 0 \\ * & * & * & * & * & E_{66} & E_{67} & E_{68} & 0 \\ * & * & * & * & * & * & E_{77} & E_{78} & 0 \\ * & * & * & * & * & * & * & E_{88} & -T_3 \\ * & * & * & * & * & * & * & * & -Q_4 \end{bmatrix}<0 \tag{22}$$

where

$$E_{11} = -2PA + R_1 + R_2 + R_3 + R_4 + R_5 + \varsigma Q_2 + \sigma^2 R_6 + T_1 - 2\Phi_1 W_1 \Phi_2 + \Phi_2 LA - \Phi_1 KA$$

$$E_{16} = PB - \Phi_2 LB + \Phi_1 KB + W_1(\Phi_1 + \Phi_2)$$

$$E_{17} = PC - \Phi_2 LC + \Phi_1 KC$$

$$E_{18} = PD - \Phi_2 LD + \Phi_1 KD$$

$$E_{22} = -R_2 - T_1$$

$$E_{55} = -(1-\mu)R_1 - 2\Phi_1 W_2 \Phi_2$$

$$E_{57} = W_2(\Phi_1 + \Phi_2)$$

$$E_{66} = 2LB - 2KB + \varsigma B^T Q_3 B + Q_1 + r^2 Q_4 - 2W_1$$

$$E_{67} = LC - KC + \varsigma B^T Q_3 C$$

$$E_{68} = LD - KD + \varsigma B^T Q_3 D$$

$$E_{77} = -(1-\mu)Q_1 - 2W_2 + \varsigma C^T Q_3 C$$

$$E_{78} = \varsigma C^T Q_3 D,\ E_{88} = \varsigma D^T Q_3 D - Q_4$$

Proof: Following the similar procedure in Theorem 1,we have

$$\dot{V}(y_t) \le \xi_1^T(t) E \xi_1(t) - \int_{t-\varsigma}^{t} \begin{bmatrix} y(s) \\ \dot{y}(s) \end{bmatrix}^T \begin{bmatrix} Q_2 & T_1 \\ * & Q_3 \end{bmatrix} \begin{bmatrix} y(s) \\ \dot{y}(s) \end{bmatrix}$$

where

$$\xi_1^T(t) = [y^T(t), y^T(t-\varsigma), y^T(t-\frac{\varsigma}{3}), y^T(t-\frac{2\varsigma}{3}), y^T(t-\varsigma(t)), f^T(y(t)), f^T(y(t-\varsigma(t))), \int_{t-r(t)}^{t} f^T(y(s))ds, \int_{t-r}^{t-r(t)} f^T(y(s))ds]$$

Hence, we can obtain that system (20) is globally asymptotically stable. This completes the proof. ■

TABLE I
ALLOWABLE UPPER BOUND OF ς FOR EXAMPLE 1.

Method	Theorem 2
$\mu = 0$	11.382
$\mu = 0.3$	10.879
$\mu = 0.6$	10.075
$\mu = 0.9$	9.621

TABLE II
ALLOWABLE UPPER BOUND OF ς FOR EXAMPLE 1.

Method	Theorem 1
$\mu = 0, \sigma = 0.2$	11.051
$\mu = 0.3, \sigma = 0.4$	10.256
$\mu = 0.6, \sigma = 0.6$	9.832
$\mu = 0.9, \sigma = 0.8$	8.879

IV. NUMERICAL EXAMPLES

In this section,we provide two numerical examples to demonstrate the effectiveness and less conservatism of our delay-dependent stability criteria.

Example 1 Consider the system (4) with the following parameters:

$$A = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}, B = \begin{bmatrix} 1.2 & -0.8 & 0.6 \\ 0.5 & -1.5 & 0.7 \\ -0.8 & -1.2 & -1.4 \end{bmatrix},$$

$$C = \begin{bmatrix} -1.4 & 0.9 & 0.5 \\ -0.6 & 1.2 & 0.8 \\ 0.5 & -0.7 & 1.1 \end{bmatrix}, D = \begin{bmatrix} 1.8 & 0.7 & -0.8 \\ 0.6 & 0.4 & 1.0 \\ -0.4 & -0.6 & 1.2 \end{bmatrix}$$

Let $\Phi_2 = diag\{-1.2, 0, -2.4\}, \Phi_1 = diag\{0, 1.4, 0\}$.
For various μ,the maximum of ς calculated by Theorem 2.According to Table I,when $\sigma = 0$,this example shows that the stability criterion in this paper can lead to less conservative results.In Table II,we can obtain that the maximum of ς with various σ, μ.

Example 2 Consider a delayed recurrent neural networks with the following parameters:

$$\dot{y}(t) = -Ay(t-\sigma) + Bf(y(t)) + Cf(y(t-\varsigma(t)))$$

where

$$A=\begin{bmatrix} 1.5 & 0 \\ 0 & 0.7 \end{bmatrix}, B=\begin{bmatrix} 0.0503 & 0.0454 \\ 0.0987 & 0.2075 \end{bmatrix}, C = \begin{bmatrix} 0.2381 & 0.9320 \\ 0.0388 & 0.5062 \end{bmatrix}$$

The neuron activation functions are assumed to satisfy Assumption 2 with $\Phi_2 = diag\{0, 0\}, \Phi_2 = diag\{0.3, 0.8\}$.
,For various ς and μ,the maximum of σ calculated by Theorem 1.According to Table III,we can see that the stability criterion in this paper can lead to less conservative results.

TABLE III
ALLOWABLE UPPER BOUND OF σ FOR EXAMPLE 2.

Method	Theorem 1
$\mu = 0, \varsigma = 0.2$	1.976
$\mu = 0.3, \varsigma = 0.4$	1.549
$\mu = 0.6, \varsigma = 0.6$	1.025
$\mu = 0.9, \varsigma = 0.8$	0.946

V. Conclusion

In this present paper, we have deal with the problem of stability for neural networks with leakage,discrete and distributed delays. Two new delay-dependent stability criteria for neural networks with time-varying delay will be proposed. The obtained criteria are less conservative because free-weighting matrices method and a convex optimization approach are considered. Finally,two examples have been given to illustrate the effectiveness of the proposed method.

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