International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:5, No:4, 2011

# Stability analysis of linear fractional order neutral system with multiple delays by algebraic approach

Lianglin Xiong, Yun Zhao, and Tao Jiang

*Abstract*—In this paper, we study the stability of $n$-dimensional linear fractional neutral differential equation with time delays. By using the Laplace transform, we introduce a characteristic equation for the above system with multiple time delays. We discover that if all roots of the characteristic equation have negative parts, then the equilibrium of the above linear system with fractional order is Lyapunov globally asymptotical stable if the equilibrium exist that is almost the same as that of classical differential equations. An example is provided to show the effectiveness of the approach presented in this paper.

*Keywords*—Fractional neutral differential equation, laplace transform, characteristic equation.

## I. INTRODUCTION

FRACTIONAL Fractional differential equations have gained considerable importance due to their application in various sciences, such as viscoelasticity, electroanalytical chemistry, electric conductance of biological systems, modeling of neurons, diffusion processes, damping laws, rheology, etc. Fractional order differential equation is represented in continuous-time domain by differential equations of non integer-order. Moreover, time delay is often present in real processes due to transportation of materials or energy. Therefore, most fractional systems may contain a delay term, such as fractional order neutral systems or some other fractional order delay systems.

Analysis of stability is fundamental to any control system. Recently, considerable attention has given to the stability problems arising from neutral systems. And various analysis techniques have been utilized to derive stability criteria for the systems by many researchers [1]-[8], and the references therein. On the other hand, although the problem of stability is a very essential and crucial issue for control systems including fractional order systems, due to the complexity of the relations, it has been discussed and investigated only in some recent literature [9]-[20], and the references therein. In the last five years, considerable attention has also been paid to obtain analytical robust stability conditions for fractional order linear time invariant (FO-LTI) systems, and the Cauchy initial value problem for various kinds of fractional order systems. The Cauchy initial value problem were discussed for various type fractional neutral functional differential equations and many criteria on existence and uniqueness were obtained. However, there is not much work on the subject of stability for fractional order neutral system, besides [21]-[29]. More recently, used the characteristic equation of the neutral system with single delay, stability criteria were derived in terms of the spectral radius of modulus matrices in [30]-[31], and the examples were showed that the conditions obtained in those paper are less conservative than some existing results. However, the stability for the fractional neutral functional differential equations hasn't been get attention by many researchers. All of those have motivated our research.

In this paper, we are interested in the stability of $n$-dimensional linear fractional neutral differential equation with multiple time delays. Similar to the approach of [9], making use of the Laplace transform, a characteristic equation for the above system with multiple time delays is introduced. We discover that if all roots of the characteristic equation have negative parts, then the equilibrium of the above linear system with fractional order is Lyapunov globally asymptotical stable if the equilibrium exist that is almost the same as that of classical differential equations. Finally, one special numerical example is given to illustrate the effectiveness of the obtained results.

## II. PROBLEM STATEMENT

This section start with recalling the essentials of the fractional calculus. The idea of fractional calculus has been known since the development of the regular calculus, with the first reference probably being associated with Leibniz and L'Hospital in 1695 where half-order derivative was mentioned.

The fractional calculus is a name for the theory of integrals and derivatives of arbitrary order, which unifies and generalizes the notions of integer-order differentiation and n-fold integration. There are three main uesed definitions of fractional integration and differentiation, such as Grunwald-Letnikov's definition, Riemann-Liouville's definition, Caputo's fractional derivative. The former two definitions are often used by pure mathematicians, while the last one is adopted by applied scientists, since it is more convenient in engineering applications. Here we only discuss Caputo derivative:

$$ {}_0^C D_t^{\alpha} x(t) = \frac{d^q x(t)}{dt^q} = J^{m-q} x^{(m)}(t), \quad \alpha > 0 $$

where $m = [q]$, i.e., $m$ is the first integer that is not less than $q$, $x^{(m)}$ is a conventional $mth$ order derivative, $J^{\beta}$ is the $\beta th$ order Riemann-Liouville integral operator, which is expressed as follows:

$$ J^{\beta} x(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-s)^{\beta-1} x(s) ds, \quad \beta > 0. $$

Lianglin Xiong is with School of Mathematics and Computer Science, Yunnan University of Nationalities, Kunming 650031, P.R. China,(Corresponding author's e-mail: lianglin_5318@126.com).

In engineering, the fractional order $q$ often lies in $(0,1)$, so we always suppose that the 'order' $q$ is a positive number but less than 1 in this paper.

In the present article, we study the following $n-$dimensional linear fractional order neutral systems with multiple time delays:

$$\begin{cases} {}_0^C D_t^{\alpha_1} x_1(t) - \sum_{i=1}^{n} c_{1i}\, {}_0^C D_t^{\alpha_1} x_i(t-r_{1i}) = \\ \qquad \sum_{i=1}^{n} [a_{1i}x_i(t) + b_{1i}x(t-\tau_{1i})] \\ {}_0^C D_t^{\alpha_2} x_2(t) - \sum_{i=1}^{n} c_{2i}\, {}_0^C D_t^{\alpha_2} x_i(t-r_{2i}) = \\ \qquad \sum_{i=1}^{n} [a_{2i}x_i(t) + b_{2i}x(t-\tau_{2i})] \\ \qquad \vdots \\ {}_0^C D_t^{\alpha_n} x_n(t) - \sum_{i=1}^{n} c_{ni}\, {}_0^C D_t^{\alpha_n} x_i(t-r_{ni}) = \\ \qquad \sum_{i=1}^{n} [a_{ni}x_i(t) + b_{ni}x(t-\tau_{ni})] \end{cases} \tag{1}$$

where $\alpha_j$ is real and lies in $(0,1)$. the initial values $x_j(t) = \phi_j(t)$ are given for $-\max_{i,j}(\tau_{ji}, r_{ji}) = -\rho_{\max} \leq 0$ and $j = 1, \cdots, n$.

Next, we study the stability of system (1). Taking Laplace transform [32] on both sides of (1) gives

$$\begin{cases} s^{\alpha_1}X_1(s) - \sum_{i=1}^{n} c_{1i}[s^{\alpha_1}e^{-sr_{1i}}X_i(s) - e^{-sr_{1i}}s^{\alpha_1-1}\phi_i(0) \\ \quad + \int_{-r_{1i}}^{0} e^{-s(r_{1i}+t)}\, {}_0^C D_t^{\alpha_1}\phi_i(t)dt] \\ = s^{\alpha_1-1}\phi_1(0) + \sum_{i=1}^{n}[a_{1i}X_i(s) + b_{1i}e^{-s\tau_{1i}}(X_i(s) \\ \quad + \int_{-\tau_{1i}}^{0} e^{-st}\phi_i(t)dt)] \\ s^{\alpha_2}X_2(s) - \sum_{i=1}^{n} c_{2i}[s^{\alpha_2}e^{-sr_{2i}}X_i(s) - e^{-sr_{2i}}s^{\alpha_2-1}\phi_i(0) \\ \quad + \int_{-r_{2i}}^{0} e^{-s(r_{2i}+t)}\, {}_0^C D_t^{\alpha_2}\phi_i(t)dt] \\ = s^{\alpha_2-1}\phi_2(0) + \sum_{i=1}^{n}[a_{2i}X_i(s) + b_{2i}e^{-s\tau_{2i}}(X_i(s) \\ \quad + \int_{-\tau_{2i}}^{0} e^{-st}\phi_i(t)dt)] \\ \qquad \vdots \\ s^{\alpha_n}X_n(s) - \sum_{i=1}^{n} c_{ni}[s^{\alpha_n}e^{-sr_{ni}}X_i(s) - e^{-sr_{ni}}s^{\alpha_n-1}\phi_i(0) \\ \quad + \int_{-r_{ni}}^{0} e^{-s(r_{ni}+t)}\, {}_0^C D_t^{\alpha_n}\phi_i(t)dt] \\ = s^{\alpha_n-1}\phi_n(0) + \sum_{i=1}^{n}[a_{ni}X_i(s) + b_{ni}e^{-s\tau_{ni}}(X_i(s) \\ \quad + \int_{-\tau_{ni}}^{0} e^{-st}\phi_i(t)dt)] \end{cases}$$

where $X_i(s)$ is the Laplace transform of $x_i(t)$ with $X_i(s) = \mathcal{L}(x_i(t)), 1 \leq i \leq n$. We can rewrite (2) as follows:

$$\Delta(s) \bullet \begin{pmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_n(s) \end{pmatrix} = \begin{pmatrix} d_1(s) \\ d_2(s) \\ \vdots \\ d_n(s) \end{pmatrix}, \tag{2}$$

in which

$$\begin{aligned} d_j(s) = &\sum_{i=1}^{n} b_{ji}e^{-s\tau_{ji}} \int_{-\tau_{ji}}^{0} e^{-st}\phi_i(t)dt \\ &+ \sum_{i=1}^{n} c_{ji}e^{-sr_{ji}} \int_{-r_{ji}}^{0} e^{-st}\, {}_0^C D_t^{\alpha_j}\phi_i(t)dt \\ &+ \sum_{i=1}^{n} c_{ji}e^{-sr_{ji}}s^{\alpha_j-1}\phi_i(0) + s^{\alpha_j-1}\phi_j(0), \\ &j = 1, \cdots, n. \end{aligned} \tag{3}$$

$$\Delta(s) = \begin{pmatrix} \Delta_{11}(s) & \Delta_{12}(s) & \cdots & \Delta_{1n}(s) \\ 0 & \Delta_{22}(s) & \cdots & \Delta_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \Delta_{nn}(s) \end{pmatrix} \tag{4}$$

where

$$\Delta_{11}(s) = s^{\alpha_1} - c_{11}s^{\alpha_1}e^{-sr_{11}} - a_{11} - b_{11}e^{-s\tau_{11}},$$
$$\Delta_{22}(s) = s^{\alpha_2} - c_{22}s^{\alpha_2}e^{-sr_{22}} - a_{22} - b_{22}e^{-s\tau_{22}},$$
$$\Delta_{nn}(s) = s^{\alpha_n} - c_{nn}s^{\alpha_n}e^{-sr_{nn}} - a_{nn} - b_{nn}e^{-s\tau_{nn}},$$
$$\Delta_{ji}(s) = s^{\alpha_j} - c_{ji}s^{\alpha_j}e^{-sr_{ji}} - a_{ji} - b_{ji}e^{-s\tau_{ji}}.$$

We call $\Delta(s)$ a characteristic matrix of system (1) for simplicity and $\det(\Delta(s))$ a characteristic polynomial of (1). The distribution of $\det(\Delta(s))'s$ eigenvalues totally determines the stability of system (1). This can be seen from the following discussion.

## III. Main results

In this section, we establish several stability condition for fractional order neutral systems.

Obviously, if a linear fractional differential equation has a non-zero equilibrium, we can move this equilibrium to the origin by the translation transform. Throughout the paper, we always suppose that (1) has a zero solution and all complex computations are done in the branch of the principle value of argument.

*Theorem 1:* If all the roots of the characteristic equation $\det(\Delta(s)) = 0$ have negative real parts, then the zero solution of system (1) is Lyapunov globally asymptotically stable.

*Proof:* Multiplying $s$ on both sides of (2) gives

$$\Delta(s) \bullet \begin{pmatrix} sX1(s) \\ sX_2(s) \\ \vdots \\ sX_n(s) \end{pmatrix} = \begin{pmatrix} sd_1(s) \\ sd_2(s) \\ \vdots \\ sd_n(s) \end{pmatrix}. \tag{5}$$

If all roots of the transcendental equation $\det(\Delta(s)) = 0$ lie in open left half complex plane,i.e., $Re(s) < 0$, then we consider

(5) in $Re(s) \geq 0$. In this restricted area, (5) has a unique solution $\begin{pmatrix} sX_1(s) & sX_2(s) & \cdots & sX_n(s) \end{pmatrix}$ So, we have

$$\lim_{s\to 0, Re(s)\geq 0} sX_i(s) = 0, i = 1, 2, \cdots, n.$$

From the assumption of all roots of the characteristic equation $\det(\Delta(s)) = 0$ and the final-value theorem of Laplace transform [32], we get

$$\lim_{t\to+\infty} x_i(t) = \lim_{s\to 0, Re(s)\geq 0} sX_i(s) = 0, i = 1, 2, \cdots, n.$$

It implies that the fractional order neutral system is Lyapunov globally asymptotically stable. It completes the proof. ■

**Remark 1.** This result contains that of Theorem 1 in [9]. In fact, when $C = 0$, the neutral system is typical fractional order system with multiple time delay system, the result is obviously consist with that of [9]. Although this theorem is an extension of [9] in some sense, this results will be important to the stability analysis for fractional order neutral systems.

**Remark 2.** If $\tau_{ji} = \tau > 0, r_{ji} = r > 0$ for $i, j = 1, 2, \cdots, n$ and $\alpha_1 = \alpha_2 = \cdots = \alpha_n = 1$, then the characteristic matrix and characteristic equation of (1) are reduced to $sI - se^{-sr}C - A - e^{-s\tau}B$ and $det(sI - se^{-sr}C - A - e^{-s\tau}B) = 0$ respectively. They coincide with the usual definitions of the characteristic matrix and characteristic equation of neutral delayed equations. Especially, if $B = C = 0$, then the characteristic matrix and characteristic equation of (1) are respectively reduced to $sI - A$ and $det(sI - A) = 0$, which agree with the typical definitions for typical differential equations.

Further more, If $\tau_{ji} = r_{ji} = \tau = r > 0$ for $i, j = 1, 2, \cdots, n$ and $\alpha_1 = \alpha_2 = \cdots = \alpha_n = \alpha$, then systems (1) become as

$$ {}_0^C D_t^\alpha x_j(t) - \sum_{i=1}^{n} c_{ji}\, {}_0^C D_t^\alpha x_i(t-\tau) = \sum_{i=1}^{n} [a_{ji}x_i(t) + b_{ji}x(t-\tau)] \tag{6}$$

In short, equation (6) can be written as

$$ {}_0^C D_t^\alpha x(t) - C\, {}_0^C D_t^\alpha x(t-\tau) = Ax(t) + Bx(t-\tau) \tag{7}$$

where $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, $C = (c_{ij})_{n\times n}$, $x^T(t) = \begin{pmatrix} x_1(t) & x_2(t) & \cdots & x_n(t) \end{pmatrix}^T$, and the characteristic matrix of (1) is reduced to

$$s^\alpha I - s^\alpha e^{-s\tau}C - A - e^{-s\tau}B$$

and the characteristic equation is reduced to

$$det(s^\alpha I - s^\alpha e^{-s\tau}C - A - e^{-s\tau}B) = 0 \tag{8}$$

Moreover, If $\tau_{ji} = \tau, r_{ji} = r > 0$ for $i, j = 1, 2, \cdots, n$ and $\alpha_1 = \alpha_2 = \cdots = \alpha_n = \alpha$, then systems (1) return to

$$ {}_0^C D_t^\alpha x_j(t) - \sum_{i=1}^{n} c_{ji}\, {}_0^C D_t^\alpha x_i(t-r) = \sum_{i=1}^{n} [a_{ji}x_i(t) + b_{ji}x(t-\tau)] \tag{9}$$

Similarly, equation (9) can be given as

$$ {}_0^C D_t^\alpha x(t) - C\, {}_0^C D_t^\alpha x(t-r) = Ax(t) + Bx(t-\tau) \tag{10}$$

and the characteristic matrix of (1) is reduced to

$$s^\alpha I - s^\alpha e^{-sr}C - A - e^{-s\tau}B$$

and the characteristic equation is reduced to

$$det(s^\alpha I - s^\alpha e^{-sr}C - A - e^{-s\tau}B) = 0 \tag{11}$$

Therefore, based on these characteristic equations (8) and (11), one can obtain the stability analysis for the fractional order neutral systems in different case, similar to some existing results for neutral systems.

## IV. Example

This section will list one example to show the effectiveness of our new criteria for asymptotic stability of fractional order neutral systems.

**Example 1.** Consider system (7) with

$$A = \begin{pmatrix} 4 & 1 \\ -2 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1.4 & -0.2 \\ -0.5 & 2.1 \end{pmatrix},$$

$$C = \begin{pmatrix} -0.71 & 0.44 \\ -0.64 & 0.32 \end{pmatrix}, \quad \tau = 0.9874, \quad \alpha = 0.18$$

Clearly, the characteristic equation of this systems is

$$det(s^{0.18}I - s^{0.18}e^{-0.9874s}C - A - e^{-0.9874s}B) = 0 \tag{12}$$

With a simple calculation in the Matlab toolbox, all the roots of the characteristic equation have negative real parts. According to Theorem 1, the system is asymptotically stable.

## V. Conclusion

Some new stability conditions for of a class of fractional order neutral systems are achieved in this paper. By using the Laplace's transformation the characteristic equation is introduced for the fractional order neutral systems. All the roots of the characteristic equation have negative real parts implies the asymptotically stable for the corresponding systems. An Illustrative example is given to demonstrate the effectiveness of the main results presented in this paper.

## Acknowledgment

This work was supported by the science study fund of Yunnan Province Office of Education(2010Y430).

The author would like to thank the associate editor and the anonymous reviewers for their constructive comments and suggestions to improve the quality of the paper.

## References

[1] M.-W. Spong, A theorem on neutral delay systems Original Research Article Systems & Control Letters, 6(1985): 291-294.
[2] Y. He, M. Wu, J.-H. She, G.-P. Liu, Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays, Systems & Control Letters, 51(2004): 57-65.
[3] C.-H. Lien, K.-W. Yu, Y.-J. Chung, Y.-F. Lin, L.-Y. Chung, J.-D. Chen, Exponential stability analysis for uncertain switched neutral systems with interval-time-varying state delay, Nonlinear Analysis: Hybrid Systems, 3(2009):334-342.
[4] L.-L. Xiong, S.-M. Zhong, M. Ye, S.-L. Wu, New stability and stabilization for switched neutral control systems Original Research Article Chaos, Solitons & Fractals, 42(2009):1800-1811.
[5] X.-G. Liu, M. Wu, Ralph Martin, M.-L. Tang, Stability analysis for neutral systems with mixed delays Original Research Article Journal of Computational and Applied Mathematics, 202(2007):478-497.

[6] L.-L. Xiong, S.-M. Zhong, J.-K. Tian, Novel robust stability criteria of uncertain neutral systems with discrete and distributed delays, Chaos, Solitons & Fractals, 40(2009):771-777.
[7] W.-J. Xiong , J.-L. Liang. Novel stability criteria for neutral systems with multiple time delays. Chaos,solitions & Fractals, 32(2007): 1735-1741.
[8] H. Li, H.-B. Li, S.-M. Zhong. Some new simple stability criteria of linear neutral systems with a single delay. Journal of Computational and Applied Mathematics, 200(2007):441-447.
[9] W.-H. Deng, C.-P. Li, J.-H. Lv, Stability analysis of linear fractional differential system with multiple time delays, Nonlinear Dynamics, 48(2007):409-416.
[10] Y.-Q. Chen, H.S. Ahn, I. Podlubny, Robust stability check of fractional order linear time invariant systems with interval uncertainties. Signal Process, 86(2006):2611-2618.
[11] H.-S. Ahn, Y.-Q. Chen, I. Podlubny, Robust stability test of a class of linear time-invariant interval fractional-order system using Lyapunov inequality. Appl.Math Comput, 187(2007):27-34.
[12] Moze M, Sabatier J. LMI tools for stability analysis of fractional systems. In: Proceedings ASME 2005 international design engineering technical conferences & computers and information in engineering conference, Paper DETC2005-85182, Long Beach, CA, USA, September 24-28, 2005.
[13] Petras I, Chen Y, Vinagre BM. Robust stability test for interval fractional order linear systems. In: Blondel VD, Megretski A, editors. Unsolved problems in the mathematics of systems and control, vols. 208210. Princeton, NJ: Princeton University Press; 2004 [Chapter 6.5].
[14] Petras I, Chen Y, Vinagre BM, Podlubny I. Stability of linear time invariant systems with interval fractional orders and interval coefficients. In: Proceedings of the international conference on computation cybernetics (ICCC04), Vienna Technical University, Vienna, Austria, 8/30-9/1; 2005. p. 14.
[15] R. Hotzel, Some stability conditions for fractional delay systems. J Math Syst Estimat Control, 8(1998):1-9.
[16] C. Hwang, Y.C. Cheng, A numerical algorithm for stability testing of fractional delay systems, Automatica, 42 (2006): 825-831.
[17] D. Matignon, Stability properties for generalized fractional differential systems, ESAIM: Proc. 5 (1998): 145-158.
[18] P. Ostalczyk, Nyquist characteristics of a fractional order integrator, Journal of Fractional Calculus, 19(2001):67-78.
[19] Mohammad Saleh Tavazoei, Mohammad Haeri, Sadegh Bolouki, Milad Siami, Stability preservation analysis for frequency-based methods in numerical simulation of fractional order systems, Siam Journal of Numerical Analysis, 47(2008): 321-338.
[20] J.-G. Lu, Y.-Q. Chen, Robust Stability and Stabilization of Fractional-Order Interval Systems with the Fractional Order $\alpha$: The $0 < \alpha < 1$ case, IEEE transactions on automatic control, 55(2010): 152-158.
[21] Z.-X. Tai, X.-C. Wang, Controllability of fractional-order impulsive neutral functional infinite delay integrodifferential systems in Banach spaces, Applied Mathematics Letters, 22(2009):1760-1765.
[22] Y. Zhou, F. Jiao, J. Li, Existence and uniqueness for fractional neutral differential equations with infinite delay, Nonlinear Analysis: Theory, Methods & Applications, 71(2009): 3249-3256.
[23] Y. Zhou, F. Jiao, J. Li, Existence and uniqueness for $p-$ type fractional neutral differential equations, Nonlinear Analysis: Theory, Methods & Applications, 71(2009):2724-2733.
[24] R.P. Agarwal, Y. Zhou, Y.-Y. He, Existence of fractional neutral functional differential equations,
[25] Y. Zhou, F. Jiao, Computers and Mathematics with Applications, 59(2010):1095-1100.
[26] Y. Zhou, F. Jiao, Existence of mild solutions for fractional neutral evolution equations, Computers and Mathematics with Applications, 59(2010):1063-1077.
[27] G.M. Mophou, G. M. N'Guerekata, Existence of mild solutions of some semilinear neutral fractional functional evolution equations with infinite delay, Applied Mathematics and Computation, 216(2010):61-69.
[28] Kamran Akbari Moornani, Mohammad Haeri, On robust stability of LTI fractional-order delay systems of retarded and neutral type, Automatica 46 (2010) 362-368.
[29] A.-H. Lin, Y. Ren, N.-M. Xia, On neutral impulsive stochastic integro-differential equations with infinite delays via fractional operators, Mathematical and Computer Modelling, 51(2010):413-424.
[30] G.-D. Hu, Some new simple stability criteria of neutral delay-differential systems, Applied mathematics and computaiton, 80(1996):257-271.
[31] D.-Q. Cao, P. He, Stability criteria of linear neutral systems with a single delay, Applied Mathematics and Computation, 148(2004):135-143.
[32] E.J. Muth, Transform Methods with Applications to Engineering and Operations Research. Prentice-Hall, New Jersey, 1977.
[33] P. Lancaster, M. Tismenetsky, The Theory of Matrices, Academic Press, Orlando, FL, 1985.

**Lianglin Xiong** was born in Sichuan Province, China, in 1981. He received the B.S. degree from Neijiang teacher university, Sichuan, Neijiang, China, in 2004, obtained the M.S. and Ph.D degree from the University of Electronic Science and Technology of China (UESTC), Sichuan, in 2007 and 2009, respectively. He is a associate professor with the School of Mathematics and Computer Science, Yunnan University of Nationalities. His research interests include neural systems, neutral systems, hybrid systems, fractional-order systems and so on.

**Yun Zhao** was born in Yunnan Province, China, in 1979. She received the B.S. and M.S. degree from Yunnan University, Kunming, Yunnan, China, in 2001 and 2004, respectively. She is a teacher with the School of Mathematics and Computer Science, Yunnan University of Nationalities. Her research interests include intelligent information processing.

**Tao Jiang** was born in Yunnan Province, China. He received the B.S. degree from Nanjing University, Nanjing, Jiangsu, China, in 1996, obtained the M.S. and PhD degree from School of Software, Yunnan University, Yunnan, in 2003 and 2010 respectively. He is a teacher with the School of Mathematics and Computer Science, Yunnan University of Nationalities. His research interests include formal method of software development, Domain-Specific Modeling, formalization and validation of models and so on.