

Stability analysis of impulsive stochastic fuzzy cellular neural networks with time-varying delays and reaction-diffusion terms

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Abstract—In this paper, the problem of stability analysis for a class of impulsive stochastic fuzzy neural networks with time-varying delays and reaction-diffusion is considered. By utilizing suitable Lyapunov-Krasovskii functional, the inequality technique and stochastic analysis technique, some sufficient conditions ensuring global exponential stability of equilibrium point for impulsive stochastic fuzzy cellular neural networks with time-varying delays and diffusion are obtained. In particular, the estimate of the exponential convergence rate is also provided, which depends on system parameters, diffusion effect and impulsive disturbed intention. It is believed that these results are significant and useful for the design and applications of fuzzy neural networks. An example is given to show the effectiveness of the obtained results.

Keywords—Exponential stability; stochastic fuzzy cellular neural networks; time-varying delays; impulses; reaction-diffusion terms.

I. INTRODUCTION

IN mathematical modelling of real world problems, we encounter inconveniences, namely, the complexity and the uncertainty or vagueness. In order to take vagueness into consideration, fuzzy theory is considered as a suitable setting. Based on traditional CNN, Yang et al. proposed the fuzzy cellular neural networks (FCNN) [1], [2], which integrates fuzzy logic into the structure of the traditional CNN and maintains local connectedness among cells. Unlike previous CNN structures, FCNN has fuzzy logic between its template input and/or output besides the sum of product operation. FCNN is very useful paradigm for image processing problems (e.g., see, [10], [11]), which is a cornerstone in image processing and pattern recognition. In such applications, the stability of networks plays an important role, it is of significance and necessary to investigate the stability. It is well known, in both biological and artificial neural networks, the delays arise because of the processing of information. Time delays may lead to oscillation, divergence, or instability which may be harmful to a system. Therefore, study of neural dynamics with consideration of the delayed problem becomes extremely important to manufacture high quality neural networks. In recent years, there have been many analytical results for FCNNs with various axonal signal transmission delays, for example, see [3]-[11] and references therein. However, strictly speaking, diffusion effects cannot be avoided in the neural networks when electrons are moving in

asymmetric electromagnetic fields. So we must consider that the activations vary in space as well as in time. In order to take the reaction-diffusion phenomena into neural networks, many reaction-diffusion neural network models have been formulated and investigated in recent years, for example, see [12]-[24] and references therein.

On the other hand, in respect of the complexity, besides delay effect, impulsive effect likewise exists in a wide variety of evolutionary processes in which states are changed abruptly at certain moments of time, involving such fields as medicine and biology, economics, mechanics, electronics and telecommunications, etc. Many interesting results on impulsive effect have been gained, e.g., Refs. [25]-[32]. As artificial electronic systems, neural networks such as CNN, bidirectional neural networks and recurrent neural networks often are subject to impulsive perturbations which can affect dynamical behaviors of the systems just as time delays. Therefore, it is necessary to consider both impulsive effect and delay effect on the stability of neural networks.

In recent years, the dynamic behavior of stochastic neural networks, especially the stability of stochastic neural networks, has become a hot study topic. The main reason is that in practice, a real system is usually affected by external perturbations which, in many cases, are of great uncertainty and hence may be treated as random. As pointed out by Haykin [36], in real nervous systems, synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. Therefore, it is of significant importance to consider stochastic effects for the stability of neural networks. In Ref. [32], the authors studied the exponential stability of several delayed neural networks with impulsive and stochastic effect. In Ref. [33], a delay-independent sufficient condition for the exponential stability of stochastic Cohen-Grossberg neural networks with time-varying delays and reaction-diffusion terms. In Ref. [34], [35], the authors investigated the theory and application of stability for stochastic reaction diffusion systems.

Motivated by the above discussions, the objective of this paper is to formulate and study impulsive stochastic FCNNs with time-varying delays and reaction-diffusion terms. Under quite general conditions, by employing suitable Lyapunov-Krasovskii functional, the inequality technique and stochastic analysis technique, some sufficient conditions ensuring the existence, uniqueness and exponential stability of equilibrium point for impulsive stochastic FCNNs with time-varying delays and reaction-diffusion terms are obtained.

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The paper is organized as follows. In Section II, the new neural network model is formulated, and the necessary knowledge is provided. We give some sufficient conditions of exponential stability of impulsive stochastic FCNNs with time-varying delays and reaction-diffusion terms in Section III. In section IV, an example is given to show the effectiveness of the obtained results. Finally, we give the conclusion in section V.

II. MODEL DESCRIPTION AND PRELIMINARIES

In this section, we will consider the model of impulsive stochastic fuzzy neural networks with time-varying delays and diffusion, it is described by the following functional differential equation

$$\left\{ \begin{aligned} du_i(t, x) &= \left[\sum_{r=1}^m \frac{\partial}{\partial x_r} (D_{ir} \frac{\partial u_i(t, x)}{\partial x_r}) - a_i u_i(t, x) \right. \\ &+ \sum_{j=1}^n a_{ij} f_j(u_j(t, x)) \\ &+ \bigwedge_{j=1}^n \alpha_{ij} g_j(u_j(t - \tau_{ij}(t), x)) \\ &+ \bigvee_{j=1}^n \beta_{ij} g_j(u_j(t - \tau_{ij}(t), x)) + I_i \Big] dt \\ &+ \sum_{j=1}^n \sigma_{ij}(u_j(t, x), u_j(t - \tau_{ij}(t), x)) dw_j(t), \\ & \qquad \qquad \qquad t \geq 0, t \neq t_k, x \in \mathcal{G}, \\ u_i(t_k^+, x) &= u_i(t_k^-, x) + J_{ik}(u_i(t_k^-, x)), \\ & \qquad \qquad \qquad k \in N \triangleq \{1, 2, \dots\}, \\ u_i(s, x) &= \phi_i(s, x), \quad -\tau < s \leq 0, x \in \mathcal{G}, \\ u_i(t, x) &= 0, \quad t \geq 0, x \in \partial \mathcal{G} \end{aligned} \right. \quad (1)$$

for $i \in \mathcal{I} \triangleq \{1, 2, \dots, n\}$, where $x = (x_1, x_2, \dots, x_m)^T \in \mathcal{G} \subset R^m$, $|x_r| < l_r, r = 1, 2, \dots, m$ and $mes\mathcal{G} > 0$; $u = (u_1, u_2, \dots, u_n)^T \in R^n$; $u_i(t, x)$ is the state of the i th neuron at time t and in space x ; $D_{ir} \geq 0$ correspond to the transmission diffusion coefficient along the i th neuron; f_i and g_i denote the signal functions of the i th neuron at time t and in space x ; $I_i = \sum_{j=1}^n b_{ij} v_j + \tilde{I}_i + \bigwedge_{j=1}^n T_{ij} v_j + \bigvee_{j=1}^n H_{ij} v_j$, and v_i

and \tilde{I}_i denote input and bias of the i th neuron, respectively; $a_i > 0, a_{ij}, \alpha_{ij}, \beta_{ij}$ are constants, a_i represent the rate with which the i th unit will reset its potential to the resting state in isolation when disconnected from the networks and external inputs; a_{ij} and b_{ij} are elements of feedback template and feedforward template, respectively; α_{ij}, β_{ij} denote connection weights of the delays fuzzy feedback MIN template and the delays fuzzy feedback MAX template, respectively; T_{ij} and H_{ij} are elements of fuzzy feedforward MIN template and fuzzy feedforward MAX template, respectively; \bigwedge and \bigvee denote the fuzzy AND and fuzzy OR operation, respectively; $\tau_{ij}(t)$ ($0 \leq \tau_{ij}(t) \leq \tau_{ij} \leq \tau$, τ_{ij} are constants, $\tau = \max_{1 \leq i, j \leq n} \{\tau_{ij}\}$) correspond to the transmission delays at time t ; $\sigma_i(u, v) = (\sigma_{i1}(u, v), \sigma_{i2}(u, v), \dots, \sigma_{in}(u, v))^T$, where $\sigma_i(u, v)$ denote the weight function of random perturbation, $\omega(t) = (\omega_1(t), \omega_2(t), \dots, \omega_n(t))^T$ is an n -dimensional Brownian motion defined on complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P)$ with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the

usual conditions. t_k is called impulsive moment, and satisfies $0 < t_1 < t_2 < \dots, \lim_{k \rightarrow +\infty} t_k = +\infty$; $u_i(t_k^-, x)$ and $u_i(t_k^+, x)$ denote the left-hand and right-hand limits at t_k , respectively; J_{ik} show impulsive perturbation of the i th neuron at time t_k , respectively. We always assume $u_i(t_k^+, x) = u_i(t_k, x)$, $k \in N$.

If $J_{ik}(u_i(t, x)) = 0, i \in \mathcal{I}, k \in N$, then system (1) may reduce to the following model:

$$\left\{ \begin{aligned} du_i(t, x) &= \left[\sum_{r=1}^m \frac{\partial}{\partial x_r} (D_{ir} \frac{\partial u_i(t, x)}{\partial x_r}) - a_i u_i(t, x) \right. \\ &+ \sum_{j=1}^n a_{ij} f_j(u_j(t, x)) \\ &+ \bigwedge_{j=1}^n \alpha_{ij} g_j(u_j(t - \tau_{ij}(t), x)) \\ &+ \bigvee_{j=1}^n \beta_{ij} g_j(u_j(t - \tau_{ij}(t), x)) + I_i \Big] dt \\ &+ \sum_{j=1}^n \sigma_{ij}(u_j(t, x), u_j(t - \tau_{ij}(t), x)) dw_j(t), \\ & \qquad \qquad \qquad t > 0, x \in \mathcal{G}, \\ u_i(s, x) &= \phi_i(s, x), \quad -\tau < s \leq 0, x \in \mathcal{G}, \\ u_i(t, x) &= 0, \quad t \geq 0, x \in \partial \mathcal{G}. \end{aligned} \right. \quad (2)$$

System (2) is called the continuous system of model (1).

If $\sigma_{ik}(u_i(t, x)) = 0, J_{ik}(u_i(t, x)) = 0, i \in \mathcal{I}, k \in N$, then system (1) may reduce to the following model:

$$\left\{ \begin{aligned} \frac{du_i(t, x)}{dt} &= \sum_{r=1}^m \frac{\partial}{\partial x_r} (D_{ir} \frac{\partial u_i(t, x)}{\partial x_r}) - a_i u_i(t, x) \\ &+ \sum_{j=1}^n a_{ij} f_j(u_j(t, x)) \\ &+ \bigwedge_{j=1}^n \alpha_{ij} g_j(u_j(t - \tau_{ij}(t), x)) \\ &+ \bigvee_{j=1}^n \beta_{ij} g_j(u_j(t - \tau_{ij}(t), x)) + I_i, \\ & \qquad \qquad \qquad t > 0, x \in \mathcal{G}, \\ u_i(s, x) &= \phi_i(s, x), \quad -\tau < s \leq 0, x \in \mathcal{G}, \\ u_i(t, x) &= 0, \quad t \geq 0, x \in \partial \mathcal{G}. \end{aligned} \right. \quad (3)$$

System (3) is called fuzzy cellular neural networks with time-delays and reaction-diffusion terms.

Throughout this paper, we make the following assumptions:

- (H1) The activation functions f_i and g_i are bounded and there exist two positive diagonal matrices $F = \text{diag}(F_1, F_2, \dots, F_n)$ and $G = \text{diag}(G_1, G_2, \dots, G_n)$, such that

$$|f_i(u) - f_i(v)| \leq F_i |u - v|, \quad |g_i(u) - g_i(v)| \leq G_i |u - v|$$

for all $u, v \in R, i \in \mathcal{I}$.

- (H2) Let $h_{ik}(u) = u + J_{ik}(u)$ be Lipschitz continuous in R , that is, there exist nonnegative constants $\gamma_{ik}, i \in \mathcal{I}, k \in N$, such that

$$|h_{ik}(u) - h_{ik}(v)| \leq \sqrt{\gamma_{ik}} |u - v|,$$

for all $u, v \in R$.

Remark 1. It should be noted that assumption (H1) guarantees the existence of an equilibrium point for system (3) by the well-known Brouwer fixed-point theorem. Let $u^* =$

$(u_1^*, \dots, u_n^*)^T$ be an equilibrium point of system (3). For the stability of equilibrium of system (1), we furthermore assume that

(H3) $\sigma(u, v)$ is linear adding and exist two non-negative matrices $S = (s_{ij})_{n \times n}$ and $W = (w_{ij})_{n \times n}$ such that

$$\begin{aligned} & \text{trace} \left[(\sigma_i(u, v) - \sigma_i(u^*, u^*))^T (\sigma_i(u, v) - \sigma_i(u^*, u^*)) \right] \\ & \leq \sum_{j=1}^n s_{ij} (u_j - u_j^*)^2 + \sum_{j=1}^n w_{ij} (v_j - u_j^*)^2 \end{aligned}$$

for all $u = (u_1, \dots, u_n)^T, v = (v_1, \dots, v_n)^T$.

(H4) $\sigma_{ij}(u_i^*, u_i^*) = 0, J_{ik}(u_i^*) = 0, i, j \in \mathcal{I}, k \in N$.

To begin with, we introduce some notation and recall some basic definitions.

$PC[J \times \mathcal{G}, R^n] \triangleq \left\{ u(t, x) : J \times \mathcal{G} \rightarrow R^n \mid u(t, x) \text{ is continuous at } t \neq t_k, u(t_k^+, x) = u(t_k, x) \text{ and } u(t_k^-, x) \text{ exists for } t, t_k \in J, k \in N \right\}$, where $J \subset R$ is an interval.

$PC[J, R^n] \triangleq \left\{ u(t) : J \rightarrow R^n \mid u(t) \text{ is continuous at } t \neq t_k, u(t_k^+) = u(t_k) \text{ and } u(t_k^-) \text{ exists for } t, t_k \in J, k \in N \right\}$, where $J \subset R$ is an interval.

$PC(\mathcal{G}) \triangleq \{ \varphi : (-\infty, 0] \times \mathcal{G} \rightarrow R^l \mid \varphi(s^+, x) = \phi(s, x) \text{ for } s \in (-\infty, 0), \varphi(s^-, x) \text{ exists for } s \in (-\infty, 0], \varphi(s^-, x) = \varphi(s, x) \text{ for all but at most a finite number of points } s \in (-\infty, 0] \}$.

$PC \triangleq \left\{ \phi : (-\infty, 0] \rightarrow R^n \mid \phi(s) \text{ is bounded on } (-\infty, 0], \text{ and } \phi(s^+) = \phi(s) \text{ for } s \in (-\infty, 0), \phi(s^-) \text{ exists for } s \in (-\infty, 0], \phi(s^-) = \phi(s) \text{ for all but at most a finite number of points } s \in (-\infty, 0] \right\}$.

For $u(t, x) = (u_1(t, x), u_2(t, x), \dots, u_n(t, x))^T \in R^m$, we define $\|u_i(t, x)\|_2 = \left[\int_{\mathcal{G}} |u_i(t, x)|^2 dx \right]^{\frac{1}{2}}, \|u_i(t, x)\|_2^2 = \int_{\mathcal{G}} |u_i(t, x)|^2 dx, i \in \mathcal{I}$, and for any $\phi(s, x) = (\phi_1(s, x), \phi_2(s, x), \dots, \phi_n(s, x))^T \in PC(\mathcal{G})$, the norm on $PC(\mathcal{G})$ is defined by

$$\|\phi\|_2^2 = \sup_{-\tau \leq s \leq 0} \sum_{i=1}^n \|\phi_i(s, x)\|_2^2,$$

then it can be proved that $PC(\mathcal{G})$ is a Banach space.

For an $m \times n$ matrix A , $|A|$ denotes the absolute value matrix given by $|A| = (|a_{ij}|)_{m \times n}$. For $A = (a_{ij})_{m \times n}, B = (b_{ij})_{m \times n} \in R^{m \times n}, A \geq B (A > B)$ means that each pair of corresponding elements of A and B such that the inequality $a_{ij} \geq b_{ij} (a_{ij} > b_{ij})$.

Definition 1: The equilibrium point u^* of system (1) is said to be globally exponentially stable, if there exist constants $\lambda > 0$ and $M \geq 1$ such that

$$\sum_{i=1}^n \left(E(\|u_i(t, x) - u_i^*\|_2^2) \right) \leq ME \left(\|\phi - u^*\|_2^2 \right) e^{-\lambda(t-t_0)}$$

for all $t \geq t_0$, where $u = (u_1(t, x), u_2(t, x), \dots, u_n(t, x))^T$ is any solution of system (1) with the initial condition $\phi = (\phi_1, \phi_2, \dots, \phi_n)^T \in PC(\mathcal{G})$.

Lemma 1: [12] For any positive integer n , let $g_j : R \rightarrow R$ be a function ($j \in \mathcal{J}$), then we have

$$\left| \bigwedge_{j=1}^n \alpha_j g_j(u_j) - \bigwedge_{j=1}^n \alpha_j g_j(v_j) \right| \leq \sum_{j=1}^n |\alpha_j| \cdot |g_j(u_j) - g_j(v_j)|,$$

$$\left| \bigvee_{j=1}^n \alpha_j g_j(u_j) - \bigvee_{j=1}^n \alpha_j g_j(v_j) \right| \leq \sum_{j=1}^n |\alpha_j| \cdot |g_j(u_j) - g_j(v_j)|$$

for all $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)^T, u = (u_1, u_2, \dots, u_n)^T, v = (v_1, v_2, \dots, v_n)^T \in R^n$.

Lemma 2: [16] Let \mathcal{G} be a cube $|x_r| < l_r (r = 1, 2, \dots, l)$ and let $h(x)$ be a real-valued function belonging to $C^1(\mathcal{G})$ which vanish on the boundary $\partial\mathcal{G}$ of \mathcal{G} , i.e., $h(x)|_{\partial\mathcal{G}} = 0$. Then

$$\int_{\mathcal{G}} h^2(x) dx \leq l_r^2 \int_{\mathcal{G}} \left| \frac{\partial h}{\partial x_r} \right|^2 dx.$$

Lemma 3: [12] Let $a < b \leq +\infty$, and let $y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T \in PC[[a, b], R^n]$ satisfy the following delay differential inequality with the initial condition $\tilde{u}(a+s) \in PC$:

$$D^+ y_i(t) \leq -r_i y_i(t) + \sum_{j=1}^n p_{ij} y_j(t) + \sum_{j=1}^n q_{ij} y_j(t - \tau_{ij}(t)) \quad (4)$$

for all $i \in \mathcal{I}$, where $r_i > 0, p_{ij} > 0, q_{ij} > 0, i \in \mathcal{I}$. If the initial conditions satisfies

$$y_i(s) \leq \kappa \xi_i e^{-\lambda(s-a)}, \quad s \in [-\tau, a], \quad i \in \mathcal{I}, \quad (5)$$

where $\lambda > 0, \xi = (\xi_1, \xi_2, \dots, \xi_n)^T > 0$ and satisfy

$$(\lambda - r_i) \xi_i + \sum_{j=1}^m (p_{ij} + e^{\lambda \tau_{ij}} q_{ij}) \xi_j < 0, \quad i \in \mathcal{I}. \quad (6)$$

Then

$$y_i(t) \leq \kappa \xi_i e^{-\lambda(t-a)}, \quad t \in [-a, b], \quad i \in \mathcal{I}. \quad (7)$$

III. MAIN RESULTS

In this section, we will discuss global exponential stability of impulsive stochastic FCNNs with time-varying delays and reaction-diffusion terms, and give their proofs.

Theorem 1: Under assumptions (H1)-(H4), if the following conditions hold:

(C1) there exist constant $\lambda > 0$ and vectors $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T > 0$, such that

$$\begin{aligned} 0 & > \left[\lambda - 2a_i - 2 \sum_{r=1}^m \frac{D_{ir}}{l_r^2} + \sum_{j=1}^n (F_j |a_{ij}| + G_j |\alpha_{ij}|) \right. \\ & \left. + G_j |\beta_{ij}| \right] \xi_i + \sum_{j=1}^n \left(F_j |a_{ij}| + s_{ij} \right) \xi_j \\ & + \sum_{j=1}^n \left(G_j |\alpha_{ij}| + G_j |\beta_{ij}| + w_{ij} \right) e^{\lambda \tau_{ij}} \xi_j; \end{aligned}$$

(C2) $\mu = \sup_{k \in N} \left\{ \frac{\ln \mu_k}{t_k - t_{k-1}} \right\} < \lambda$, where $\mu_k = \max_{1 \leq i \leq n} \{1, \gamma_{ik}\}, k \in N$;

then system (1) has exactly one globally exponentially stable equilibrium point, and its exponential convergence rate equals $\lambda - \mu$.

Proof. Let $u(t, x) = (u_1, u_2, \dots, u_n)^T$ be an arbitrary solution of system (1) with the initial condition $\phi = (\phi_1, \phi_2, \dots, \phi_n)^T \in PC(\mathcal{G})$, set $v_i(t, x) = u_i(t, x) - u_i^*$ for $i \in \mathcal{I}$. It is easy to see that system (1) can be transformed into the following system

$$\left\{ \begin{aligned} dv_i(t, x) &= \left[\sum_{r=1}^m \frac{\partial}{\partial x_r} (D_{ir} \frac{\partial v_i(t, x)}{\partial x_r}) - a_i v_i(t, x) \right. \\ &\quad + \sum_{j=1}^n a_{ij} (f_j(v_j + u_j^*) - f_j(u_j^*)) \\ &\quad + \bigwedge_{j=1}^n \alpha_{ij} g_j(v_j(t - \tau_{ij}(t), x) + u_j^*) \\ &\quad - \bigwedge_{j=1}^n \alpha_{ij} g_j(u_j^*) \\ &\quad + \bigvee_{j=1}^n \beta_{ij} g_j(v_j(t - \tau_{ij}(t), x) + u_j^*) \\ &\quad \left. - \bigvee_{j=1}^n \beta_{ij} g_j(u_j^*) \right] dt \\ &\quad + \sum_{j=1}^n \sigma_{ij} (v_j + u_j^*, u_j(t - \tau_{ij}(t), x) + u_j^*) dw_j(t), \\ &\quad t \geq 0, \quad t \neq t_k, \quad x \in \mathcal{G}, \\ v_i(t_k^+, x) &= v_i(t_k^-, x) + J_{ik}(v_i(t_k^-, x) + u_i^*), \quad k \in N, \\ v_i(s, x) &= \phi_i(s, x) - u_i^*, \quad -\tau < s \leq 0, \quad x \in \mathcal{G}, \\ v_i(t, x) &= 0, \quad t \geq 0, \quad x \in \partial \mathcal{G} \end{aligned} \right. \quad (8)$$

for all $i \in \mathcal{I}$.

For the system (8), construct the following Lyapunov functional:

$$V_i(t) = \int_{\mathcal{G}} v_i^2(t, x) dx$$

By the Itô differential formula, we get

$$\begin{aligned} LV_i(t) &= \int_{\mathcal{G}} 2v_i \left[\sum_{l=1}^m \frac{\partial}{\partial x_l} (D_{il} \frac{\partial v_i}{\partial x_l}) - a_i v_i \right. \\ &\quad + \sum_{j=1}^n a_{ij} (f_j(v_j + u_j^*) - f_j(u_j^*)) \\ &\quad + \bigwedge_{j=1}^n \alpha_{ij} g_j(v_j(t - \tau_{ij}(t), x) + u_j^*) \\ &\quad - \bigwedge_{j=1}^n \alpha_{ij} g_j(u_j^*) \\ &\quad + \bigvee_{j=1}^n \beta_{ij} g_j(v_j(t - \tau_{ij}(t), x) + u_j^*) \\ &\quad \left. - \bigvee_{j=1}^n \beta_{ij} g_j(u_j^*) \right] dx \\ &\quad + 2 \sum_{j=1}^n \int_{\mathcal{G}} (u_i - u_i^*) \\ &\quad \times \sigma_{ij} (v_j + u_j^*, v_j(t - \tau_{ij}(t), x) + u_j^*) dw_j(t) dx \\ &\quad + \int_{\mathcal{G}} \text{trace}(\tilde{\sigma}_i^T \tilde{\sigma}_i) dx, \end{aligned} \quad (9)$$

where $\tilde{\sigma}_i = \sigma_i(v_j + u_j^*, v_j(t - \tau_{ij}(t), x) + u_j^*)$.

From Green's formula and the initial condition, we have

$$\begin{aligned} &\int_{\mathcal{G}} v_i \sum_{r=1}^m \frac{\partial}{\partial x_r} (D_{ir} \frac{\partial v_i}{\partial x_r}) dx \\ &= - \sum_{r=1}^m \int_{\mathcal{G}} D_{ir} \left(\frac{\partial v_i}{\partial x_r} \right)^2 dx. \end{aligned} \quad (10)$$

By Lemma 2, we can obtain

$$\begin{aligned} &\int_{\mathcal{G}} v_i \sum_{r=1}^m \frac{\partial}{\partial x_r} (D_{ir} \frac{\partial v_i}{\partial x_r}) dx \\ &\leq - \sum_{r=1}^m \frac{D_{ir}}{l_r^2} \|v_i(t, x)\|_2^2. \end{aligned} \quad (11)$$

From assumption (H1) and Höder inequality, we have

$$\begin{aligned} &\int_{\mathcal{G}} v_i \sum_{j=1}^n a_{ij} (f_j(v_j + u_j^*) - f_j(u_j^*)) dx \\ &\leq \sum_{j=1}^n |a_{ij}| \int_{\mathcal{G}} |v_j(t, x)| |f_j(v_j + u_j^*) - f_j(u_j^*)| dx \\ &\leq \sum_{j=1}^n |a_{ij}| \int_{\mathcal{G}} |v_i(t, x)| |v_j(t, x)| F_j dx \\ &\leq \sum_{j=1}^n F_j |a_{ij}| \|v_i(t, x)\|_2 \|v_j(t, x)\|_2. \end{aligned} \quad (12)$$

By Lemma 1, assumption (H1) and Höder inequality, we have

$$\begin{aligned} &\int_{\mathcal{G}} v_i \times \left[\bigwedge_{j=1}^n \alpha_{ij} g_j(v_j(t - \tau_{ij}(t), x) + u_j^*) \right. \\ &\quad \left. - \bigwedge_{j=1}^n \alpha_{ij} g_j(u_j^*) \right] dx \\ &\leq \int_{\mathcal{G}} |v_i| \\ &\quad \times \left| \bigwedge_{j=1}^n \alpha_{ij} g_j(v_j(t - \tau_{ij}(t), x) + u_j^*) - \bigwedge_{j=1}^n \alpha_{ij} g_j(u_j^*) \right| dx \\ &\leq \int_{\mathcal{G}} |v_i| \sum_{j=1}^n |\alpha_{ij}| |g_j(v_j(t - \tau_{ij}(t), x) + u_j^*) - g_j(u_j^*)| dx \\ &= \sum_{j=1}^n |\alpha_{ij}| \int_{\mathcal{G}} |v_i| |g_j(v_j(t - \tau_{ij}(t), x) + u_j^*) - g_j(u_j^*)| dx \\ &\leq \sum_{j=1}^n |\alpha_{ij}| G_j \int_{\mathcal{G}} |v_i(t, x)| |v_j(t - \tau_{ij}(t), x)| dx \\ &\leq \sum_{j=1}^n G_j |\alpha_{ij}| \|v_i(t, x)\|_2 \|v_j(t - \tau_{ij}(t), x)\|_2. \end{aligned} \quad (13)$$

By the same reason, we have

$$\begin{aligned} &\int_{\mathcal{G}} v_i \times \left[\bigvee_{j=1}^n \beta_{ij} g_j(v_j(t - \tau_{ij}(t), x) + u_j^*) \right. \\ &\quad \left. - \bigvee_{j=1}^n \beta_{ij} g_j(u_j^*) \right] dx \end{aligned}$$

$$\leq \sum_{j=1}^n G_j |\beta_{ij}| \|v_i(t, x)\|_2 \|v_j^*(t - \tau_{ij}(t), x)\|_2. \quad (14)$$

Taking the mathematical expectation of both sides of (16), we have

According to assumption (H3), we have

$$\begin{aligned} & \int_{\mathcal{G}} \tilde{\sigma}_i^T \tilde{\sigma}_i dx \\ &= \int_{\mathcal{G}} [\tilde{\sigma}_i - \sigma_i(u^*, u^*)]^T [\tilde{\sigma}_i - \sigma_i(u^*, u^*)] dx \\ &\leq \sum_{j=1}^n s_{ij} \int_{\mathcal{G}} v_j^2(t, x) dx \\ &\quad + \sum_{j=1}^n w_{ij} \int_{\mathcal{G}} v_j^2(t - \tau_{ij}(t), x) dx \\ &\leq \sum_{j=1}^n s_{ij} \|v_j(t, x)\|_2^2 + \sum_{j=1}^n w_{ij} \|v_j(t - \tau_{ij}(t), x)\|_2^2. \end{aligned} \quad (15)$$

i.e.

$$\begin{aligned} D^+ E(V_i(t)) &\leq \left[-2a_i - 2 \sum_{r=1}^m \frac{D_{ir}}{l_r^2} \right. \\ &\quad \left. + \sum_{j=1}^n (F_j |a_{ij}| + G_j |\alpha_{ij}| + G_j |\beta_{ij}|) \right] \\ &\quad \times E(\|v_i(t, x)\|_2^2) \\ &\quad + \sum_{j=1}^n (F_j |a_{ij}| + s_{ij}) E(\|v_j(t, x)\|_2^2) \\ &\quad + \sum_{j=1}^n (G_j |\alpha_{ij}| + G_j |\beta_{ij}| + w_{ij}) \\ &\quad \times E(\|v_j(t - \tau_{ij}(t), x)\|_2^2) \end{aligned}$$

By applying (11)-(15) to (9), it follows from $a^2 + b^2 \geq 2ab$ that

$$\begin{aligned} LV_i(t) &\leq -\left(2a_i + 2 \sum_{r=1}^m \frac{D_{ir}}{l_r^2}\right) \|v_i(t, x)\|_2^2 \\ &\quad + 2 \sum_{j=1}^n F_j |a_{ij}| \|v_i(t, x)\|_2 \|v_j(t, x)\|_2 \\ &\quad + 2 \sum_{j=1}^n G_j (|\alpha_{ij}| + |\beta_{ij}|) \\ &\quad \times \|v_i(t, x)\|_2 \|v_j(t - \tau_{ij}(t), x)\|_2 \\ &\quad + \sum_{j=1}^n s_{ij} \|v_j(t, x)\|_2^2 \\ &\quad + \sum_{j=1}^n w_{ij} \|v_j(t - \tau_{ij}(t), x)\|_2^2 \\ &\quad + 2 \sum_{j=1}^n \int_{\mathcal{G}} v_i \\ &\quad \times \sigma_{ij}(v_j + u_j^*, v_j(t - \tau_{ij}(t), x) + u_j^*) dw_j(t) dx \\ &\leq \left[-2a_i + 2 \sum_{r=1}^m \frac{D_{ir}}{l_r^2} \right. \\ &\quad \left. + \sum_{j=1}^n (F_j |a_{ij}| + G_j |\alpha_{ij}| + G_j |\beta_{ij}|) \right] \\ &\quad \times \|v_i(t, x)\|_2^2 \\ &\quad + \sum_{j=1}^n (F_j |a_{ij}| + s_{ij}) \|v_j(t, x)\|_2^2 \\ &\quad + \sum_{j=1}^n (G_j |\alpha_{ij}| + G_j |\beta_{ij}| + w_{ij}) \\ &\quad \times \|v_j(t - \tau_{ij}(t), x)\|_2^2 \\ &\quad + 2 \sum_{j=1}^n \int_{\Omega} v_i \\ &\quad \times \sigma_{ij}(v_j + u_j^*, v_j(t - \tau_{ij}(t), x) + u_j^*) dw_j(t) dx. \end{aligned} \quad (16)$$

$$\begin{aligned} D^+ E(\|u_i - u_i^*\|_2^2) &\leq \left[-2a_i - 2 \sum_{r=1}^m \frac{D_{ir}}{l_r^2} \right. \\ &\quad \left. + \sum_{j=1}^n (F_j |a_{ij}| + G_j |\alpha_{ij}| + G_j |\beta_{ij}|) \right] \\ &\quad \times E(\|u_i - u_i^*\|_2^2) \\ &\quad + \sum_{j=1}^n (F_j |a_{ij}| + s_{ij}) E(\|u_j - u_j^*\|_2^2) \\ &\quad + \sum_{j=1}^n (G_j |\alpha_{ij}| + G_j |\beta_{ij}| + w_{ij}) \\ &\quad \times E(\|u_i(t - \tau_{ij}(t), x) - u_j^*\|_2^2) \end{aligned} \quad (17)$$

for all $i \in \mathcal{I}, t_{k-1} < t < t_k, k \in N$.

Let $y_i(t) = E(\|u_i - u_i^*\|_2^2)$, $i \in \mathcal{I}$, and $r_i = 2a_i + 2 \sum_{r=1}^m \frac{D_{ir}}{l_r^2} - \sum_{j=1}^n (F_j |a_{ij}| + G_j |\alpha_{ij}| + G_j |\beta_{ij}|)$, $p_{ij} = F_j |a_{ij}| + s_{ij}$, $q_{ij} = G_j |\alpha_{ij}| + G_j |\beta_{ij}| + w_{ij}$ for $i \in \mathcal{I}$, from (18), we have

$$D^+ y_i(t) \leq -r_i y_i(t) + \sum_{j=1}^n p_{ij} y_j(t) + \sum_{j=1}^n q_{ij} y_j(t - \tau_{ij}(t)) \quad (18)$$

for all $i \in \mathcal{I}$. From condition (C1), there exist constant $\lambda > 0$ and vectors $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T > 0$ such that

$$\begin{aligned} 0 &> \left[\lambda - 2a_i - 2 \sum_{r=1}^m \frac{D_{ir}}{l_r^2} \right. \\ &\quad \left. + \sum_{j=1}^n (F_j |a_{ij}| + G_j |\alpha_{ij}| + G_j |\beta_{ij}|) \right] \xi_i \\ &\quad + \sum_{j=1}^n (F_j |a_{ij}| + s_{ij}) \xi_j \\ &\quad + \sum_{j=1}^n (G_j |\alpha_{ij}| + G_j |\beta_{ij}| + w_{ij}) e^{\lambda \tau_{ij}} \xi_j. \end{aligned} \quad (9)$$

Taking $\kappa = \frac{E(\|\phi - u^*\|_2^2)}{\min_{1 \leq i \leq n} \{\xi_i\}}$, it is easy to prove that

$$y_i(s) \leq \kappa \xi_i e^{-\lambda s}, \quad s \in [-\tau, 0], \quad i \in \mathcal{I}. \quad (20)$$

From Lemma 3, we obtain

$$y_i(t) \leq \kappa \xi_i e^{-\lambda t}, \quad 0 = t_0 \leq t < t_1. \quad (21)$$

Suppose that, for $l \leq k$, the following inequalities hold:

$$y_i(t) \leq \kappa \mu_0 \mu_1 \cdots \mu_{l-1} \xi_i e^{-\lambda t}, \quad t_{l-1} \leq t < t_l, \quad i \in \mathcal{I}, \quad (22)$$

where $\mu_0 = 1$. When $l = k + 1$, we note that

$$\begin{aligned} y_i(t_k) &= E\left(\|u_i(t_k, x) - u_i^*\|_2^2\right) \\ &= E\left(\|u_i(t_k^+, x) - u_i^*\|_2^2\right) \\ &= E\left(\|u_i(t_k^-, x) + J_{ik}(u_i(t_k^-, x) - u_i^*)\|_2^2\right) \\ &= E\left(\|h_{ik}(u_i(t_k^-, x) - u_i^*)\|_2^2\right) \\ &\leq \gamma_{ik} y_i(t_k^-) \\ &\leq \kappa \mu_0 \mu_1 \cdots \mu_{k-1} \mu_k \xi_i \lim_{t \rightarrow t_k^-} e^{-\lambda t} \\ &\leq \kappa \mu_0 \mu_1 \cdots \mu_{k-1} \mu_k \xi_i e^{-\lambda t_k}. \end{aligned} \quad (23)$$

From (22), (23) and $\mu_k \geq 1$, we have

$$y_i(t) \leq \kappa \mu_0 \mu_1 \cdots \mu_{k-1} \mu_k \xi_i e^{-\lambda t}, \quad -\tau \leq t \leq t_k. \quad (24)$$

Combining (19),(24) and Lemma 3, we obtain that

$$y_i(t) \leq \kappa \mu_0 \mu_1 \cdots \mu_k \xi_i e^{-\lambda t}, \quad t_k \leq t < t_{k+1}. \quad (25)$$

Applying the mathematical induction, we can obtain the following inequalities

$$y_i(t) \leq \kappa \mu_0 \mu_1 \cdots \mu_k \xi_i e^{-\lambda t}, \quad t \in [t_k, t_{k+1}), \quad k \in N. \quad (26)$$

According to (C2), we have $\mu_k \leq e^{\mu(t_k - t_{k-1})} < e^{\lambda(t_k - t_{k-1})}$, so we get

$$\begin{aligned} y_i(t) &\leq \kappa e^{\mu t_1} e^{\mu(t_2 - t_1)} \dots e^{\mu(t_{k-1} - t_{k-2})} \xi_i e^{-\lambda t} \\ &= \kappa \xi_i e^{\mu t_{k-1}} e^{-\lambda t} \\ &\leq \kappa \xi_i e^{-(\lambda - \mu)t}, \quad t \in [t_{k-1}, t_k), \quad k \in N. \end{aligned}$$

That is

$$y_i(t) \leq \kappa \xi_i e^{-(\lambda - \mu)t}, \quad t \in [-\tau, t_k), \quad k \in N. \quad (27)$$

It follows that

$$\begin{aligned} &\sum_{i=1}^n \left(E(\|u_i(t, x) - u_i^*\|_2^2) \right) \\ &= \sum_{i=1}^n y_i(t) \\ &\leq \sum_{i=1}^n \kappa \xi_i e^{-(\lambda - \mu)t} \\ &= \frac{\sum_{i=1}^n \xi_i}{\min_{1 \leq i \leq n} \{\xi_i\}} E(\|\phi - u^*\|_2^2) e^{-(\lambda - \mu)t}. \end{aligned}$$

Let $M = \frac{\sum_{i=1}^n \xi_i}{\min_{1 \leq i \leq n} \{\xi_i\}} > 1$, then we have

$$\sum_{i=1}^n \left(E(\|u_i(t, x) - u_i^*\|_2^2) \right) \leq M E(\|\phi - u^*\|_2^2) e^{-(\lambda - \mu)t}.$$

The proof is completed.

Remark 2. when $D_{il} = 0$, system (1) may reduce to the following model:

$$\left\{ \begin{aligned} du_i(t) &= \left[-a_i u_i(t) + \sum_{j=1}^n a_{ij} f_j(u_j(t)) \right. \\ &\quad \left. + \sum_{j=1}^n \alpha_{ij} g_j(u_j(t - \tau_{ij}(t))) \right. \\ &\quad \left. + \sum_{j=1}^n \beta_{ij} g_j(u_j(t - \tau_{ij}(t))) + I_i \right] dt \\ &\quad + \sum_{j=1}^n \sigma_{ij}(u_j(t), u_j(t - \tau_{ij}(t))) dw_j(t), \\ &\quad t \geq 0, t \neq t_k, \\ u_i(t_k^+) &= u_i(t_k^-) + J_{ik}(u_i(t_k^-)), \quad k \in N, \\ u_i(s) &= \phi_i(s), \quad -\tau < s \leq 0 \end{aligned} \right. \quad (28)$$

for $i \in \mathcal{I}$. For system (28), we have following corollary.

Corollary 1: Under assumptions (H1), (H2), (H3), (C1) and (C2), then system (28) has exactly one globally exponentially stable equilibrium point.

Remark 3. when $\sigma_{ij} = 0$, system (2.1) may reduce to the following model:

$$\left\{ \begin{aligned} \frac{u_i(t, x)}{\partial t} &= \sum_{r=1}^m \frac{\partial}{\partial x_r} (D_{ir} \frac{\partial u_i(t, x)}{\partial x_r}) - a_i u_i(t, x) \\ &\quad + \sum_{j=1}^n a_{ij} f_j(u_j(t, x)) \\ &\quad + \sum_{j=1}^n \alpha_{ij} g_j(u_j(t - \tau_{ij}(t), x)) \\ &\quad + \sum_{j=1}^n \beta_{ij} g_j(u_j(t - \tau_{ij}(t), x)) + I_i, \\ &\quad t \geq 0, t \neq t_k, x \in \mathcal{G}, \\ u_i(t_k^+, x) &= u_i(t_k^-, x) + J_{ik}(u_i(t_k^-, x)), \quad k \in N, \\ u_i(s, x) &= \phi_i(s, x), \quad -\tau < s \leq 0, x \in \mathcal{G}, \\ u_i(t, x) &= 0, \quad t > 0, x \in \partial \mathcal{G} \end{aligned} \right. \quad (29)$$

for $i \in \mathcal{I}$. For system (29), it is easy to obtain the following result:

Corollary 2: Under assumptions (H1) and (H2), if the following conditions hold:

(C1') there exist constant $\lambda > 0$ and vectors $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T > 0$ such that

$$\begin{aligned} 0 &> \left[\lambda - 2a_i - 2 \sum_{r=1}^m \frac{D_{ir}}{l_r^2} \right. \\ &\quad \left. + \sum_{j=1}^n (F_j |a_{ij}| + G_j |\alpha_{ij}| + G_j |\beta_{ij}|) \right] \xi_i \\ &\quad + \sum_{j=1}^n (F_j |a_{ij}| \xi_j + \sum_{j=1}^n (G_j |\alpha_{ij}| + G_j |\beta_{ij}|) e^{\lambda \tau_{ij}} \xi_j); \end{aligned}$$

$$(C2) \mu = \sup_{k \in N} \left\{ \frac{\ln \mu_k}{t_k - t_{k-1}} \right\} < \lambda, \text{ where } \mu_k = \max_{1 \leq i \leq n} \{1, \gamma_{ik}\}, k \in N;$$

then system (29) has exactly one globally stable equilibrium point, and its exponential convergence rate equals $\lambda - \mu$.

Remark 4. Note that Lemma 1 transforms the fuzzy AND (\wedge) and the fuzzy OR (\vee) operation into the SUM operation (\sum). So above results can be applied to the following classical impulsive stochastic fuzzy neural networks with time-varying delays and reaction-diffusion terms

$$\left\{ \begin{aligned} du_i(t, x) &= \left[\sum_{r=1}^m \frac{\partial}{\partial x_r} (D_{ir} \frac{\partial u_i(t, x)}{\partial x_r}) - a_i u_i(t, x) \right. \\ &+ \sum_{j=1}^n a_{ij} f_j(u_j(t, x)) \\ &+ \sum_{j=1}^n \alpha_{ij} g_j(u_j(t - \tau_{ij}(t), x)) + I_i \Big] dt \\ &+ \sum_{j=1}^m \sigma_{ij}(u_j(t, x), u_j(t - \tau_{ij}(t), x)) dw_j(t), \\ & \quad t \geq 0, t \neq t_k, x \in \mathcal{G}, \\ u_i(t_k^+, x) &= u_i(t_k^-, x) + J_{ik}(u_i(t_k^-, x)), \quad k \in N, \\ u_i(s, x) &= \phi_i(s, x), \quad -\tau < s \leq 0, x \in \mathcal{G}, \\ u_i(t, x) &= 0, \quad t > 0, x \in \partial \mathcal{G} \end{aligned} \right. \quad (30)$$

for $i \in \mathcal{I}$. For system (30), it is easy to obtain the following result.

Theorem 2: Under assumptions (H1), (H2) and (H3), if the following conditions hold:

$$(C\bar{1}) \text{ there exist constant } \lambda > 0 \text{ and vectors } \xi = (\xi_1, \xi_2, \dots, \xi_n)^T > 0 \text{ such that}$$

$$\begin{aligned} 0 > & \left[\lambda - 2a_i - 2 \sum_{r=1}^m \frac{D_{ir}}{l_r^2} \right. \\ & + \sum_{j=1}^n (F_j |a_{ij}| + G_j |\alpha_{ij}|) + G_j |\beta_{ij}| \Big] \xi_i \\ & + \sum_{j=1}^n (F_j |a_{ij}| + s_{ij}) \xi_j \\ & + \sum_{j=1}^n (G_j |\alpha_{ij}| + G_j |\beta_{ij}| + w_{ij}) e^{\lambda \tau_{ij}} \xi_j; \end{aligned}$$

$$(C2) \mu = \sup_{k \in N} \left\{ \frac{\ln \mu_k}{t_k - t_{k-1}} \right\} < \lambda, \text{ where } \mu_k = \max_{1 \leq i \leq n} \{1, \gamma_{ik}\}, k \in N;$$

then system (30) has exactly one globally exponentially stable equilibrium point, and its exponential convergence rate equals $\lambda - \mu$.

IV. AN ILLUSTRATIVE EXAMPLE

In order to illustrate the feasibility of our above-established criteria in the preceding sections, we provide a concrete example. Although the selection of the coefficients and functions in the example is somewhat artificial, the possible application of our theoretical theory is clearly expressed.

Consider the following impulsive fuzzy neural networks with time-varying delays and reaction-diffusion terms:

$$\left\{ \begin{aligned} du_i(t, x) &= \left[\sum_{r=1}^3 \frac{\partial}{\partial x_r} (D_{ir} \frac{\partial u_i(t, x)}{\partial x_r}) - a_i u_i(t, x) \right. \\ &+ \sum_{j=1}^2 a_{ij} f_j(u_j(t, x)) \\ &+ \sum_{j=1}^2 \alpha_{ij} g_j(u_j(t - \tau_{ij}(t), x)) \\ &+ \sum_{j=1}^2 \beta_{ij} g_j(u_j(t - \tau_{ij}(t), x)) + I_i \Big] dt \\ &+ \sum_{j=1}^2 \sigma_{ij}(u_j(t, x), u_j(t - \tau_{ij}(t), x)) dw_j(t), \\ & \quad t \geq 0, t \neq t_k, x \in \mathcal{G}, \\ u_i(t_k^+, x) &= u_i(t_k^-, x) + (-1 + \sqrt{e}) \sin(u_i(t_k^-, x)), \\ & \quad t_0 = 0, t_k - t_{k-1} = 5, k \in N, \\ u_i(s, x) &= \phi_i(s, x), \quad -\tau < s \leq 0, x \in \mathcal{G}, \\ u_i(t, x) &= 0, \quad x \in \partial \mathcal{G} \end{aligned} \right. \quad (31)$$

for $i = 1, 2$, where $\mathcal{G} = \{(x_1, x_2, x_3)^T \mid |x_r| < l_r = 1, r = 1, 2, 3\}$, and

$$\begin{aligned} D_{11} &= 0.01, & D_{12} &= 0.03, & D_{13} &= 0.01, \\ D_{21} &= 0.01, & D_{22} &= 0.02, & D_{23} &= 0.02, \\ a_1 &= 2.5, & a_2 &= 3, & a_{11} &= 0.25, \\ a_{12} &= -0.5, & a_{21} &= -0.5, & a_{22} &= 0.5, \\ \alpha_{11} &= 0.25, & \alpha_{12} &= -0.25, & \alpha_{21} &= -0.25, \\ \alpha_{22} &= 0.5, & \beta_{11} &= 0.25, & \beta_{12} &= 0.25 \\ \beta_{21} &= -0.35, & \beta_{22} &= 0.25, & I_1 &= 0.75, \\ I_2 &= -0.75, & \tau_{11}(t) &= |\sin t|, & \tau_{12}(t) &= 1 + 0.5 \sin t, \\ \tau_{21}(t) &= |\cos t|, & \tau_{22}(t) &= 1.5 \cos^2 t; \end{aligned}$$

$$\begin{aligned} & \text{trace} \left[(\sigma_i(u, v) - \sigma_i(u^*, u^*))^T (\sigma_i(u, v) - \sigma_i(u^*, u^*)) \right] \\ & \leq \sum_{j=1}^n s_{ij} (u_j - u_j^*)^2 + \sum_{j=1}^n w_{ij} (v_j - u_j^*)^2, \end{aligned}$$

where

$$\begin{aligned} s_{11} &= 0.2, & s_{12} &= 0.2, & s_{21} &= 0.1 \\ s_{22} &= 0.2, & w_{11} &= 0.2, & w_{12} &= 0.1, \\ w_{21} &= 0.1, & w_{22} &= 0.2 \\ f_1(u) &= f_2(u) = g_1(v) = g_2(v) = \frac{|v+1| - |v-1|}{2}. \end{aligned}$$

It is easy to verify that assumptions (H1), (H2) and (H3) are satisfied, and it is easy to calculate that

$$\begin{aligned} F &= G = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}, \\ (\tau_{ij})_{2 \times 2} &= \begin{pmatrix} 1 & 1.5 \\ 1 & 1.5 \end{pmatrix}, \quad \gamma_{ik} = e, \quad i = 1, 2, k \in N, \\ \mu_k &= \max\{1, e\} = e, \quad \mu = \sup_{k \in N} \frac{\ln e}{t_k - t_{k-1}} = 0.2. \end{aligned}$$

Solving the following optimization problem

$$\left\{ \begin{array}{l} \max \lambda, \\ 0 > \left[\lambda - 2a_1 - 2 \sum_{r=1}^3 \frac{D_{1r}}{T_{1r}} + \sum_{j=1}^2 ((|a_{1j}| + G_j |\alpha_{1j}|) \right. \\ \quad \left. + G_j |\beta_{1j}|) \right] \xi_1 + \sum_{j=1}^2 (|a_{1j}| F_j + s_{1j}) \xi_j \\ \quad + \sum_{j=1}^2 (G_j |\alpha_{1j}| + |\beta_{1j}| G_j + w_{1j}) e^{\lambda \tau_{1j}} \xi_j \\ 0 > \left[\lambda - 2a_2 - 2 \sum_{r=1}^3 \frac{D_{2r}}{T_{2r}} + \sum_{j=1}^2 ((|a_{2j}| + G_j |\alpha_{2j}|) \right. \\ \quad \left. + G_j |\beta_{2j}|) \right] \xi_2 + \sum_{j=1}^2 (|a_{2j}| F_j + s_{2j}) \xi_j \\ \quad + \sum_{j=1}^2 (G_j |\alpha_{2j}| + |\beta_{2j}| G_j + w_{2j}) e^{\lambda \tau_{2j}} \xi_j \\ \lambda > 0, \quad \xi = (\xi_1, \xi_2)^T > 0. \end{array} \right.$$

we obtain that $\xi = (1461190, 1771331)^T > 0$, $\lambda \approx 0.479257 > 0.1 = \mu$. From Theorem 1, the equilibrium point of system (31) is globally exponentially stable, and its exponential convergence rate $\lambda - \mu \approx 0.279257$.

V. CONCLUSIONS

A class of impulsive stochastic FCNNs with time-varying delays and reaction-diffusion terms has been formulated and investigated. The general sufficient conditions have been obtained to ensure the existence, uniqueness and exponential stability of the equilibrium point for impulsive stochastic FCNNs with time-varying delays and reaction-diffusion terms. In particular, the estimate of the exponential convergence rate is also provided, which depends on the system parameters, boundary conditions, delays and impulses. An illustrate example is given to show the effectiveness of obtained results. In addition, the sufficient conditions what we obtained are easily verified. This has practical benefits, since easily verifiable conditions for the global exponential stability are important in the design and applications of neural networks.

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