

Sparsity-Based Unsupervised Unmixing of Hyperspectral Imaging Data Using Basis Pursuit

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$$\mathbf{Y} = \mathbf{AS} + \mathbf{E} \quad (3)$$

Abstract—Mixing in the hyperspectral imaging occurs due to the low spatial resolutions of the used cameras. The existing pure materials “endmembers” in the scene share the spectra pixels with different amounts called “abundances”. Unmixing of the data cube is an important task to know the present endmembers in the cube for the analysis of these images. Unsupervised unmixing is done with no information about the given data cube. Sparsity is one of the recent approaches used in the source recovery or unmixing techniques. The l_1 -norm optimization problem “basis pursuit” could be used as a sparsity-based approach to solve this unmixing problem where the endmembers is assumed to be sparse in an appropriate domain known as dictionary. This optimization problem is solved using proximal method “iterative thresholding”. The l_1 -norm basis pursuit optimization problem as a sparsity-based unmixing technique was used to unmix real and synthetic hyperspectral data cubes.

Keywords—Basis pursuit, blind source separation, hyperspectral imaging, spectral unmixing, wavelets.

I. INTRODUCTION

HYPERSPECTRAL imaging (HSI) acquires images using hundreds of contiguous wavelengths in potentially different electromagnetic bands [1]. Using HSI for remote sensing is not easy, as hyperspectral cameras typically have low spatial resolutions that usually render an acquired spectral pixel a mixture of spectra of pure materials, usually called endmembers. Unmixing of these mixtures is very important task for required application. Unsupervised unmixing does this task without any prior knowledge about the nature of the scene. Every spectral pixel in the hyperspectral data cube is separated into a set of endmembers [2]. We model any spectral pixel using a linear mixture model such that:

$$y_i = \sum_{j=1}^M s_j a_{ij} + e_i \quad (1)$$

where $s_j = [s_{j1}, \dots, s_{jL}]^T$ is the spectrum of the j^{th} material present in the scene and $a_{ij} \geq 0$ are its corresponding proportion (abundance) in the i^{th} pixel. e_i represents an additive perturbation (noise), and M indicates the number of endmembers. The abundances are subject to the two main constraints:

$$\begin{aligned} \text{Nonnegativity: } & a_{ij} \geq 0 \\ \text{Sum-to-one: } & \sum_{j=1}^M a_{ij} = 1 \end{aligned} \quad (2)$$

In the matrix form, (1) could be written as:

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The problem of spectral unmixing is to estimate the endmembers (sources) matrix \mathbf{S} , the abundance matrix $\mathbf{A} = [a_1; \dots; a_N]$, from the pixels spectra matrix $\mathbf{Y} = [y_1; \dots; y_N]$, where N is the number of spectral pixels in the data cube. This is considered as Blind Source Separation (BSS) problem [3].

In [2], the authors gave an overview on unmixing approaches of HSI. These unmixing approaches could be categorized into four main types; geometrical, statistical, spatial-spectral and sparsity-based approaches. The geometrical approaches exploit the fact that pixels spectra must lie in a simplex set formed by the endmembers [4]. The statistical approaches build Bayesian model enforcing the constraints using distribution-based likelihood and priors to estimate a posterior parameter probability [5]. The spatial-spectral contextual approaches exploit the spatial correlation between the pixels spectra in addition to the spectral features contained in the data cube [6]. The sparse regression approaches, our interest here in this paper, is one of the most recent approaches to solve the unmixing problem.

Sparsity has become an attractive approach in signal processing. It has many applications; restoration, feature extraction, source separation, compression ... etc. [7]. Sparse signal representation uses a suitable domain where most of its coefficients are zero. Depending on the nature of the signal, one could find an appropriate domain where it would be sparse. Sparsity could be used for signal compression, as it requires less memory for its storage. It could also result in simpler signal processing algorithms, e.g., signal denoising via simple thresholding operations in a domain where the signal is assumed to be sparse [8]. In addition, the computational cost to process a sparsely represented signal would be typically less than the cost to process its dense counterpart.

Sparse signal restoration assumes that the unknown signal is sparse in an appropriate domain. Therefore, signal sparsity could be used as prior information to obtain an estimate of the signal, even if the number of available measurements is smaller than the dimension of the unknown signal.

Most of the sparse spectral unmixing techniques are implemented in a semisupervised fashion. This is done by assuming that the pixels spectra are a linear combination of some pure spectral signatures known before. These signatures are obtained in labs using a field spectroradiometer. Unmixing then aims to find the optimal subset of signatures out of a very large spectral library that can best fit each mixed pixel [9]. Sparse spectral unmixing also could be solved blindly

(unsupervised) by non-negative matrix factorization (NMF) [10], but these methods are not our interest here in this paper.

Sparse regression formulates the unmixing problem as an l_0 -norm minimization problem. This optimization problem is NP-hard and nonconvex. This problem could be solved approximately by greedy algorithms like the orthogonal matching pursuit (OMP) [11], or solved by an approximate or relaxed l_1 -norm minimization problem exactly. This relaxed version with a regularization parameter λ is termed basis pursuit (BP) or *lasso* minimization problem [12]. In this paper, we implement the second approach, convex relaxations or the basis pursuit problem.

The basis pursuit optimization problems take the advantage of a sparse representation of the data in overcomplete dictionaries, to separate sources based on convex relaxations methods [12]. l_1 -norm basis pursuit optimization problems could be used in the spectral unmixing problem.

Sparsity has been recognized recently as an efficient source separation approach. Sparsity can be obtained using wavelets [13], curvelets [14] (analytic dictionaries) or using sparse coding dictionary algorithms [15] (learned dictionaries). Due to the physical nature of spectra which have a small number of peaks corresponding to resonant absorption properties, the wavelets are chosen as a predefined or analytic dictionary for sparsifying pixels spectra.

This paper is arranged as follows; Section II illustrates the l_1 -norm basis pursuit algorithm. Results of applying the l_1 -norm basis pursuit algorithm to unmix real and synthetic hyperspectral data cubes are shown in Section III.

II. THE L_1 -NORM BASIS PURSUIT ALGORITHM FOR SPECTRAL UNMIXING PROBLEM

A. The Sparse Signal Representation and Basis Pursuit Definitions

A signal x is considered sparse in an overcomplete frame which is known as dictionary $\Phi = [\varphi_1, \dots, \varphi_T]$, if it can be well represented as a superposition of few of the T elementary dictionary elements or atoms φ_i as shown in Fig. 1.

$$x = \Phi\alpha = \sum_{i=1}^T \alpha[i]\varphi_i \quad (4)$$

where α is called the coefficients of x in Φ whose columns φ_i are normalized to a unit l_2 -norm. The dictionary Φ could be a concatenation of K subdictionaries Φ_k such that $\Phi = [\Phi_1, \dots, \Phi_K]$.

The sparse source recovery problem can be written as follows knowing that x is sparse:

$$\min_x \|x\|_0 \quad s.t. \quad \|y - Ax\|_2 \leq \varepsilon \quad (5)$$

This is the l_0 -norm minimization problems equation which is computationally hard due to its nonconvex optimization nature. The corresponding l_1 -norm relaxation of the l_0 -norm equation can be written in the form:

$$\min_x \|x\|_1 \quad s.t. \quad \|y - Ax\|_2 \leq \varepsilon \quad (6)$$

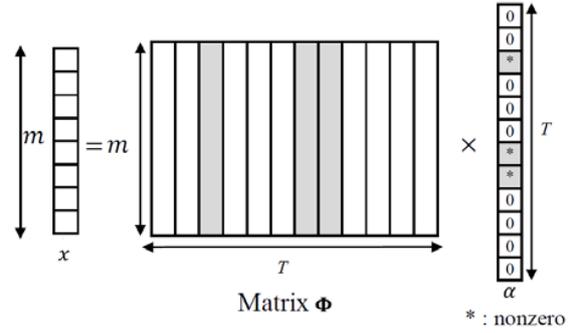


Fig. 1 The sparse signal representation

Using an appropriate Lagrange multiplier factor λ , we can also rewrite the above problem as an unconstrained minimization:

$$\min_x \frac{1}{2} \|y - Ax\|_2^2 + \lambda \|x\|_1 \quad (7)$$

The above l_1 -norm regularized problem is widely known as the Basis Pursuit [12] in signal processing and as *lasso* among statisticians.

B. The l_1 -Norm Basis Pursuit Problem for Spectral Unmixing Problem

In the multichannel case as our interest of HSI, an extension of sparse signal representation to multichannel data can be done. As the hyperspectral data cube typically has a large number of spectra pixels, they could be considered as different observations or channels. The sparse sources recovery for the multichannel representation will be:

$$\min_{\alpha} \|\alpha\|_1 \quad s.t. \quad \mathbf{Y} = \mathbf{A}\alpha\Phi^T \quad (8)$$

where \mathbf{Y} is the $N \times L$ measurement matrix, \mathbf{A} is the $N \times M$ abundance matrix, α is the $M \times T$ sources sparse coefficients matrix and Φ is the $L \times T$ dictionary matrix. Note that the sources \mathbf{S} are sparsified in dictionary Φ with coefficients matrix α using $\mathbf{S} = \alpha\Phi^T$.

The l_1 -norm basis pursuit minimization problem concepts could be used to solve the spectral unmixing problem. The sources or endmembers recovery optimization equation could be written similar to the relaxed l_1 -norm basis pursuit minimization problem (7), as:

$$\min_{\mathbf{A}, \alpha} \frac{1}{2} \|\mathbf{Y} - \mathbf{A}\alpha\Phi^T\|_2^2 + \lambda \|\alpha\|_1 \quad (9)$$

It could be proved that the solution for α can be obtained by proximal method or iterative thresholding [7].

C. The Unmixing Algorithm

To solve (9), orthogonal dictionary Φ should be chosen to sparsify the pixels spectra obtaining the matrix $\beta = \mathbf{Y}\Phi$, the matrix where each of its rows stores the coefficients of each pixel y_i .

In [16], the authors argued using redundant dictionary Φ instead of orthogonal dictionary such that the spectra pixels

have the sparsest representation. Their argument was based on the fact that the sparse decomposition algorithm must preserve linearity, meaning that the sparsest decomposition of the spectra obtained must be equal to the linear combination of the sparsest decomposition of the sources.

The optimization problem (9) then becomes:

$$\min_{\mathbf{A}, \alpha} \frac{1}{2} \|\boldsymbol{\beta} - \mathbf{A}\alpha\|_2^2 + \lambda \|\alpha\|_1 \quad (10)$$

Here, the data \mathbf{Y} have to be transformed once in Φ which is computationally much cheaper. Indeed, as $N \gg M$, it turns out that (10) is a multichannel overdetermined least squares error fit with l_1 -sparsity penalization. The obtained solution is an iterative and alternate estimation of α and \mathbf{A} [7]:

- *Update the coefficients:* when \mathbf{A} is fixed, α is obtained using:

$$\alpha = \text{Thresh}_{\delta}(\mathbf{A}^{\dagger} \boldsymbol{\beta}) \quad (11)$$

where \mathbf{A}^{\dagger} is the pseudo-inverse of the current estimate \mathbf{A} of the mixing matrix, Thresh_{δ} is a thresholding operator, and the threshold δ decreases with increasing iteration count.

- *Update the mixing matrix:* when α is fixed, \mathbf{A} is obtained by a least-squares estimate using:

$$\mathbf{A} = \boldsymbol{\beta} \alpha^T (\alpha \alpha^T)^{-1} \quad (12)$$

The two stages iterative process leads to the solution of (10). After convergence and obtaining the coefficient matrix α , the spectra of the sources \mathbf{S} could be obtained using the same dictionary Φ using $\mathbf{S} = \alpha \Phi^T$.

III. RESULTS

Applying the l_1 -norm basis pursuit algorithm as a sparsity-based unmixing approach to unmix the hyperspectral data cubes using wavelets as a predefined dictionary to sparsify the pixels spectra is investigated in this section.

The wavelet transform is an excellent basis to sparsely represent 1-D smooth signals having a small number of irregular points. The wavelet family was chosen as our potential dictionary of choice because spectral pixels are 1-D smooth signals and the physical nature of spectra indicates that spectra have a small number of peaks corresponding to resonant absorption peaks.

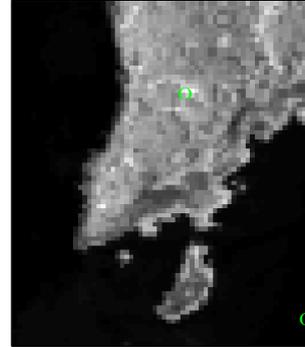
Two types of datasets were used, the first one is actual hyperspectral cube downloaded from AVIRIS website [17], and the second one is synthetic cube made from few selected materials from ASTER spectral library [18].

A. The Actual AVIRIS Cube

A data cube was downloaded from the AVIRIS website [17]. This cube consists of images using 224 wavelengths (365 nm to 2497 nm). A subimage (75 x 65 pixels) was selected for our study. The ground truth of the selected scene was obtained from Google Maps showing that it contains water and trees. The reference endmembers spectra have been obtained using one of the geometrical HSI unmixing approaches known as N-

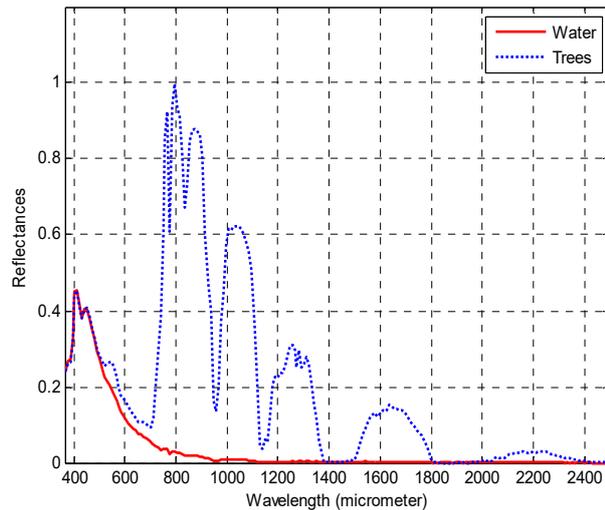
FINDR [19]. MATLAB implementation available online for the N-FINDR method was used. The obtained endmembers, shown in Fig. 2, correspond to water and trees.

The l_1 -norm basis pursuit algorithm illustrated in section II with 50 iterations to estimate of α and \mathbf{A} alternately as a tool to unmix the given data was applied. Fig. 2 shows the obtained estimated endmembers.



(a)

AVIRIS Cube Estimated Endmembers Spectra using NFINDER algorithm



(b)

Fig. 2 Results of N-FINDR algorithm. (a) The position of endmembers. (b) The corresponding endmembers spectra

B. The Synthetic Cube

A mixed cube was synthesized from three materials spectra, grass, concrete, and asphalt, picked up from the ASTER spectral library [18]. The cube was established using random abundances with the constraints given in (2). Additive Gaussian noise was added to the synthetic cube spectra with $\text{SNR} = 20$ db. The l_1 -norm basis pursuit algorithm illustrated in section II with 50 iterations to estimate of α and \mathbf{A} alternately was applied to the synthetic cube. Fig. 4 shows the obtained estimated endmembers.

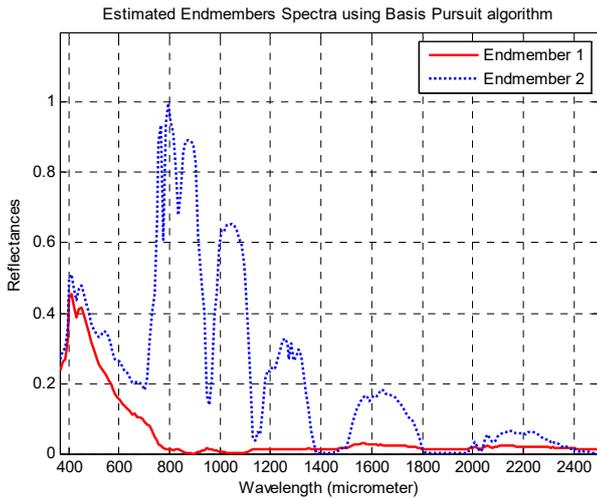
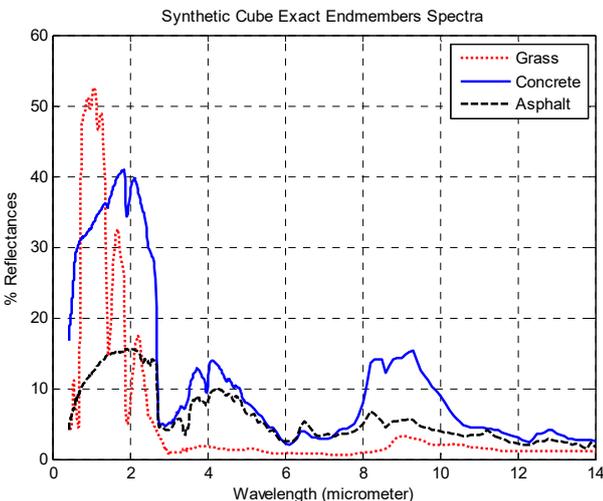


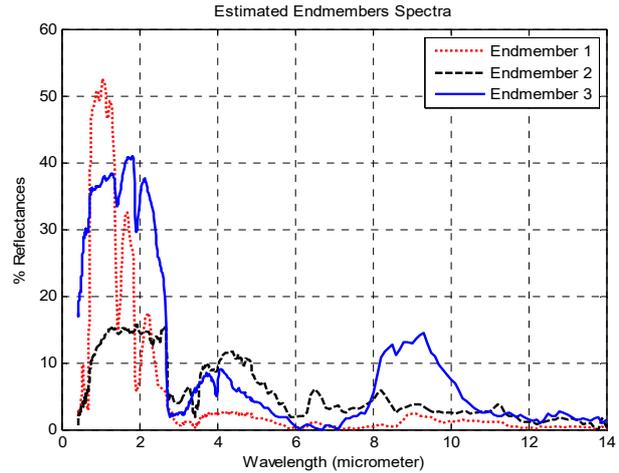
Fig. 3 The estimated endmembers spectra using l_1 -norm basis pursuit with 50 iterations only

IV. CONCLUSION

Unmixing of hyperspectral images data cubes is a very important task for different applications. This unmixing process could be done using different geometrical, statistical, spatial-spectral, sparsity-based approaches. The l_1 -norm basis pursuit algorithm was implemented as a sparsity-based unmixing algorithm for hyperspectral data cubes. The endmembers sparsity was used as prior information to estimate of these unknown endmembers. This optimization problem is solved using proximal method or iterative thresholding. The unmixing l_1 -norm basis pursuit algorithm using wavelets was applied to real and synthetic data cubes, and the results were presented.



(a)



(b)

Fig. 4 (a) The exact endmembers spectra, (b) the estimated spectra with 50 iterations

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