Some Equalities Connected with Fuzzy Soft Matrices

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Abstract**—**The aim of this paper is to use matrix representation of Fuzzy soft sets for proving some equalities connected with Fuzzy soft sets based on set-operations.

Keywords**—**Equality, Fuzzy soft matrix, Fuzzy soft sets, operations.

I. Introduction

WE cannot always use the classical methods to solve complicated problems in economics, engineering, social sciences, medical sciences etc. because different types of uncertainties are present in these theories. These days so many theories are available to deal with such type of uncertainties, such as, theory of fuzzy sets [1], theory of intuitionistic fuzzy sets [2], [3], theory of vague sets [4], theory of rough sets [5]. Every theory has its own problems because of the inadequacy of the parameterization tool of the theories. In 1999, Molodtsov [6] initiated the concept of fuzzy set to deal with uncertainties which is free from the above difficulties and later in 2001 Maji et al. [7] gave a hybrid model known as fuzzy soft set which is a combination of soft set and fuzzy set. They introduce the concept of union, intersection, complement etc. of Fuzzy Soft Sets and discussed their properties. These results were further revised and improved by Ahmad and Kharal [8]. Manash Jyoti Borah et al. [9] introduced the notions of symmetric, transitive, reflexive, equivalence fuzzy soft relations and discussed some related properties.

The theory of matrix is commonly used in the broad areas of science and engineering. However classical matrix theory sometimes fails to solve the problems involving uncertainties, occurring in an imprecise environment. In [10], Yong Yang and Chenli Ji, using a matrix representation [10], [11] of Fuzzy Soft Set, discussed its properties and applied the same in certain decision making problems. In [12] the notion of Fuzzy Soft Matrices has been extended.

In this paper, by defining some more set-operations on FSM_{mxn} analogous to [13], [14] defined for intuitionistic fuzzy sets, an attempt has been made to derive some new results based on these operations on FSM_{mxn}.

II. Preliminaries

In this section, we give definitions and notions (refer [12]), to be used in our subsequent work.

Definition 1**:** *Fuzzy Soft Set*- Let X be an initial universal set and E be a set of parameters. Let $\tilde{P}(X)$ denotes the power set of all Fuzzy subsets of X. Let $A \subseteq E$. A pair (F, A) is called Fuzzy Soft Set over X, where F is a mapping given by $F : A \to \tilde{P}(X)$.

Definition 2**:** *Fuzzy Soft Class* - The pair (X, E) denotes the collection of all Fuzzy Soft Sets on X with attributes from E and is called Fuzzy Soft Class.

Definition 3**:***Fuzzy Soft Matrices* - Let $X = \{x_1, x_2, ..., x_m\}$ be the universal set and $E = \{e_1, e_2, ..., e_n\}$ be the set of parameters. Let $A \subseteq E$ and (F, A) be a Fuzzy Soft Set in the Fuzzy Soft Class (X, E). Then we represent the Fuzzy Soft Set (F, A) in the matrix form as:

$$A_{mn} = \lfloor a_{ij} \rfloor_{mxn} \text{ or simply by } A = \lfloor a_{ij} \rfloor$$

where

$$a_{ij} = \begin{cases} \mu_j(x_i) & \text{if } e_j \in A \\ 0 & \text{if } e_j \notin A \end{cases}$$

Here $\mu_j(x_i)$ represent the membership of x_i in the Fuzzy Set $F(e_j)$. We would identify a Fuzzy Soft Set with its Fuzzy Soft matrix and vice versa. The set of all mxn Fuzzy Soft Matrices will be denoted FSM_{mxn} over X.

Definition4**:** *Set operations on* FSM_{mxn}- Let $A = \lfloor a_{ij} \rfloor_{mxn}$ and $B = \lfloor b_{ij} \rfloor_{mxn}$ be two Fuzzy Soft matrices over the universal set X.

Some set operations on FSM_{mxn} are defined as follows:

$$A \cup B = C = \lfloor c_{ij} \rfloor_{mxn} \quad (1)$$

where

$$c_{ij} = \max(a_{ij}, b_{ij}), \text{ for all } i \text{ and } j.$$

$$A \cap B = C = \lfloor c_{ij} \rfloor_{mxn} \quad (2)$$

where

$$c_{ij} = \min(a_{ij}, b_{ij}), \text{ for all } i \text{ and } j.$$

$$A \Diamond B = C = \lfloor c_{ij} \rfloor_{mxn} \quad (3)$$

where

$$c_{ij} = a_{ij} + b_{ij} - a_{ij} b_{ij}, \text{ for all } i \text{ and } j.$$

$$A \cdot B = C = \lfloor c_{ij} \rfloor_{mxn} \quad (4)$$

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where

$c_{ij} = a_{ij} . b_{ij}$ for all *i* and *j*.

$$A @ B = C = \lfloor c_{ij} \rfloor_{mxn} \quad (5)$$

where

$c_{ij} = \frac{1}{2}\left(a_{ij} + b_{ij}\right)$, for all *i* and *j*.

$$A \$ B = C = \lfloor c_{ij} \rfloor \quad (6)$$

where

$c_{ij} = \sqrt{a_{ij} \; b_{ij}}$, for all *i* and *j*.

$$A \# B = C = \lfloor c_{ij} \rfloor \quad (7)$$

where

$c_{ij} = \frac{2a_{ij} \, b_{ij}}{a_{ij} + b_{ij}}$, for all *i* and *j*,

for which we will accept that if $a_{ij} = b_{ij} = 0,$ then

$$\frac{a_{ij} \, b_{ij}}{a_{ij} + b_{ij}} = 0.$$

$$A * B = \left[\frac{a_{ij} + b_{ij}}{2\,(a_{ij}\, b_{ij} + 1)}\right] \quad (8)$$

III. Equalities Connected with FSM_{MXN}

Theorem 1: If $A = \lfloor a_{ij} \rfloor$, $B = \lfloor b_{ij} \rfloor \in FSM_{mxn}$, then

$$\{(A \cup B) \# (A \cap B) \$ \{(A \cup B) @ (A \cap B)\} = A \$ B$$

Proof: $(A \cup B) \# (A \cap B) = \lfloor \max (a_{ij}, b_{ij}) \rfloor \# \lfloor \min (a_{ij}, b_{ij}) \rfloor$

$$= \left[\frac{2 \max (a_{ij}, b_{ij}) . \min (a_{ij}, b_{ij})}{\max (a_{ij}, b_{ij}) + \min (a_{ij}, b_{ij})}\right] = \left[\frac{2a_{ij} \, b_{ij}}{a_{ij} + b_{ij}}\right]$$

and

$(A \cup B) @ (A \cap B) = \lfloor \max (a_{ij}, b_{ij}) \rfloor @ \lfloor \min (a_{ij}, b_{ij}) \rfloor$

$$= \left[\frac{\max (a_{ij}, b_{ij}) + \min (a_{ij}, b_{ij})}{2}\right] = \left[\frac{a_{ij} + b_{ij}}{2}\right].$$

Thus

$\{(A \cup B) \# (A \cap B)\} \$ \{(A \cup B) @ (A \cap B)\}$

$$= \left[\frac{2a_{ij} \, b_{ij}}{a_{ij} + b_{ij}}\right] \$ \left[\frac{a_{ij} + b_{ij}}{2}\right] = \left[\sqrt{\frac{2a_{ij} \, b_{ij}}{a_{ij} + b_{ij}} . \frac{a_{ij} + b_{ij}}{2}}\right]$$

$$= \left[\sqrt{a_{ij} \, b_{ij}}\right] = A \$ B$$

Hence the result is proved.

Theorem 2: If $A = \lfloor a_{ij} \rfloor$, $B = \lfloor b_{ij} \rfloor \in FSM_{mxn}$, then

$$\langle \{(A \cup B) . (A \cap B)\} @ (A \lozenge B) \rangle \$ (A \# B) = (A \$ B)$$

Proof: $(A \cup B) . (A \cap B) = \lfloor \max (a_{ij}, b_{ij}) \rfloor . \lfloor \min (a_{ij}, b_{ij}) \rfloor$

$$= \lfloor \max (a_{ij}, b_{ij}) . \min (a_{ij}, b_{ij}) \rfloor = \lfloor a_{ij} \, b_{ij} \rfloor$$

and

$$\{(A \cup B) . (A \cap B)\} @ (A \lozenge B)$$

$$= \lfloor a_{ij} \, b_{ij} \rfloor @ \lfloor a_{ij} + b_{ij} - a_{ij} \, b_{ij} \rfloor = \left[\frac{a_{ij} \, b_{ij} + a_{ij} + b_{ij} - a_{ij} \, b_{ij}}{2}\right]$$

$$= \left[\frac{a_{ij} + b_{ij}}{2}\right]$$

Now consider

$\langle \{(A \cup B) . (A \cap B)\} @ (A \lozenge B) \rangle \$ (A \# B)$

$$= \left[\frac{a_{ij} + b_{ij}}{2}\right] \$ \left[\frac{2a_{ij} \, b_{ij}}{a_{ij} + b_{ij}}\right] = \left[\sqrt{\frac{a_{ij} + b_{ij}}{2} . \frac{2\, a_{ij} \, b_{ij}}{a_{ij} + b_{ij}}}\right]$$

$$= \left[\sqrt{a_{ij} \, b_{ij}}\right] = A \$ B$$

Hence the result is proved.

Corollary 1: If $A = \lfloor a_{ij} \rfloor$, $B = \lfloor b_{ij} \rfloor \in FSM_{mxn}$, then

$$\{(A \cup B) \# (A \cap B)\} \$ \{(A \cup B) @ (A \cap B)\}$$
$$= \langle \{(A \cup B) . (A \cap B)\} @ (A \lozenge B) \rangle \$ (A \# B) = (A \$ B).$$

Proof: It follows from Theorems 1 & 2.

Theorem 3: If $A = \lfloor a_{ij} \rfloor$, $B = \lfloor b_{ij} \rfloor \in FSM_{mxn}$, then

$$(A \lozenge B) \cup (A @ B) @ (A \cdot B) \cap (A @ B) = (A @ B)$$

Proof:

$$(A \lozenge B) \cup (A @ B) = \lfloor a_{ij} + b_{ij} - a_{ij} \, b_{ij} \rfloor \cup \left[\frac{a_{ij} + b_{ij}}{2}\right]$$

$$= \left[\max \left(a_{ij} + b_{ij} - a_{ij} \, b_{ij}, \frac{a_{ij} + b_{ij}}{2}\right)\right] = \lfloor a_{ij} + b_{ij} - a_{ij} \, b_{ij} \rfloor$$

and

$$(A \cdot B) \cap (A @ B) = \lfloor a_{ij} b_{ij} \rfloor \cap \left[\frac{a_{ij} + b_{ij}}{2}\right]$$

$$= \left[\min \left(a_{ij} \, b_{ij}, \frac{a_{ij} + b_{ij}}{2}\right)\right] = \lfloor a_{ij} b_{ij} \rfloor$$

Now Consider

$$\{(A \lozenge B) \cup (A @ B)\} @ \{(A \cdot B) \cap (A @ B)\}$$
$$= \lfloor a_{ij} + b_{ij} - a_{ij} b_{ij} \rfloor \ @ \ \lfloor a_{ij} b_{ij} \rfloor = \left[\frac{a_{ij} + b_{ij} - a_{ij} b_{ij} + a_{ij} b_{ij}}{2} \right]$$
$$= \left[\frac{a_{ij} + b_{ij}}{2} \right] = (A @ B),$$

which proves the result

***Theorem 4*:** If $A = \lfloor a_{ij} \rfloor$, $B = \lfloor b_{ij} \rfloor \in FSM_{mxn}$, then

$$\{(A \lozenge B) \cap (A @ B)\} \$ \{(A \lozenge B) \cap (A \# B)\} = (A \$ B).$$

***Proof*:** $(A \lozenge B) \cap (A @ B) = \lfloor a_{ij} + b_{ij} - a_{ij} b_{ij} \rfloor \cap \left[\frac{a_{ij} + b_{ij}}{2} \right]$

$$= \left[\min \left(a_{ij} + b_{ij} - a_{ij} b_{ij}, \frac{a_{ij} + b_{ij}}{2} \right) \right] = \left[\frac{a_{ij} + b_{ij}}{2} \right]$$

and

$$(A \lozenge B) \cap (A \# B) = \left[\min \left(a_{ij} + b_{ij} - a_{ij} b_{ij}, \frac{2 a_{ij} b_{ij}}{a_{ij} + b_{ij}} \right) \right]$$
$$= \left[\frac{2 a_{ij} b_{ij}}{a_{ij} + b_{ij}} \right]$$

Thus

$$\{(A \lozenge B) \cap (A @ B)\} \$ \{(A \lozenge B) \cap (A \# B)\}$$
$$= \left[\frac{a_{ij} + b_{ij}}{2} \right] \$ \left[\frac{2 a_{ij} b_{ij}}{a_{ij} + b_{ij}} \right] = \sqrt{\frac{a_{ij} + b_{ij}}{2} \cdot \frac{2 a_{ij} b_{ij}}{a_{ij} + b_{ij}}} = \left[\sqrt{a_{ij} b_{ij}} \right]$$
$$= A \$ B$$

Hence the result is proved.

***Theorem* 5:** If $A = \lfloor a_{ij} \rfloor$, $B = \lfloor b_{ij} \rfloor \in FSM_{mxn}$, then

$$\{(A \lozenge B) \cup (A \# B)\} @ \{(A \cdot B) \cap (A \# B)\} = (A @ B).$$

Proof:

$$(A \lozenge B) \cup (A \# B) = \lfloor a_{ij} + b_{ij} - a_{ij} b_{ij} \rfloor \cup \left[\frac{2 a_{ij} b_{ij}}{a_{ij} + b_{ij}} \right]$$
$$= \left[\max \left(a_{ij} + b_{ij} - a_{ij} b_{ij}, \frac{2 a_{ij} b_{ij}}{a_{ij} + b_{ij}} \right) \right] = \lfloor a_{ij} + b_{ij} - a_{ij} b_{ij} \rfloor$$

and

$$(A \cdot B) \cap (A \# B) = \lfloor a_{ij} b_{ij} \rfloor \cap \left[\frac{2 a_{ij} b_{ij}}{a_{ij} + b_{ij}} \right] = \left[\min \left(a_{ij} b_{ij}, \frac{2 a_{ij} b_{ij}}{a_{ij} b_{ij}} \right) \right]$$
$$= \lfloor a_{ij} b_{ij} \rfloor$$

Now

$$\{(A \lozenge B) \cup (A \# B)\} @ \{(A \cdot B) \cap (A \# B)\}$$
$$= \lfloor a_{ij} + b_{ij} - a_{ij} b_{ij} \rfloor \ @ \ \lfloor a_{ij} b_{ij} \rfloor = \left[\frac{a_{ij} + b_{ij} - a_{ij} b_{ij} + a_{ij} b_{ij}}{2} \right]$$
$$= \left[\frac{a_{ij} + b_{ij}}{2} \right] = A @ B$$

Thus the result is proved.

***Theorem* 6:** If $A = \lfloor a_{ij} \rfloor$, $B = \lfloor b_{ij} \rfloor \in FSM_{mxn}$, then

$$\{(A \lozenge B) \cup (A \$ B)\} @ \{(A \cdot B) \cap (A \$ B)\} = (A @ B)$$

Proof:

$$(A \lozenge B) \cup (A \$ B) = \lfloor a_{ij} + b_{ij} - a_{ij} b_{ij} \rfloor \cup \left\lfloor \sqrt{a_{ij} b_{ij}} \right\rfloor$$
$$= \left\lfloor \max \left(a_{ij} + b_{ij} - a_{ij} b_{ij}, \sqrt{a_{ij} b_{ij}} \right) \right\rfloor$$
$$= \lfloor a_{ij} + b_{ij} - a_{ij} b_{ij} \rfloor$$

and

$$(A . B) \cap (A \$ B) = \lfloor a_{ij} b_{ij} \rfloor \cap \left\lfloor \sqrt{a_{ij} b_{ij}} \right\rfloor = \left\lfloor \min \left(a_{ij} b_{ij}, \sqrt{a_{ij} b_{ij}} \right) \right\rfloor$$
$$= \lfloor a_{ij} b_{ij} \rfloor$$

Now

$$\{(A \lozenge B) \cup (A \$ B)\} @ \{(A . B) \cap (A \$ B)\}$$
$$= \lfloor a_{ij} + b_{ij} - a_{ij} b_{ij} \rfloor \ @ \ \lfloor a_{ij} b_{ij} \rfloor = \left[\frac{a_{ij} + b_{ij} - a_{ij} b_{ij} + a_{ij} b_{ij}}{2} \right]$$
$$= \left[\frac{a_{ij} + b_{ij}}{2} \right] = (A @ B)$$

Thus the result is proved.

Theorem 7: If $A = \lfloor a_{ij} \rfloor$, $B = \lfloor b_{ij} \rfloor \in FSM_{mxn}$, then

$$(A \cup B) * (A \cap B) = A * B.$$

***Proof* :**

$$(A \cup B) * (A \cap B) = \lfloor \max(a_{ij}, b_{ij}) \rfloor * \lfloor \min(a_{ij}, b_{ij}) \rfloor$$
$$= \left[\frac{\max(a_{ij}, b_{ij}) + \min(a_{ij}, b_{ij})}{2\{\max(a_{ij}, b_{ij}) . \min(a_{ij}, b_{ij}) + 1\}} \right]$$
$$= \left[\frac{a_{ij} + b_{ij}}{2(a_{ij} b_{ij} + 1)} \right] = A * B$$

Hence the result is proved.

IV. Conclusion

Manashm, Jyoti Borah et al. [12] defined some operations on FSM such as union intersection complement etc. and discussed their properties. In the present paper this study has further been extended by defining some more operations on it. Further work in this direction is required to study whether the notion put forward in this paper yields a fruitful result.

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