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Solving Fuzzy Multi-Objective Linear Programming Problems with Fuzzy Decision Variables

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***Abstract*—**In this paper, a method is proposed for solving Fuzzy Multi-Objective Linear Programming problems (FMOLPP) with fuzzy right hand side and fuzzy decision variables. To illustrate the proposed method, it is applied to the problem of selecting suppliers for an automotive parts producer company in Iran in order to find the number of optimal orders allocated to each supplier considering the conflicting objectives. Finally, the obtained results are discussed.

***Keywords*—**Fuzzy multi-objective linear programming problems, triangular fuzzy numbers, fuzzy ranking, supplier selection problem.

I. Introduction

IN optimizing real world systems, one usually ends up with a linear or nonlinear programming problem [1]. The real life decision problems have two main properties. The first one is to have conflicting objectives in the problem structure, and the second one is the fuzziness in the description of problem parameters [2]. The first application of Fuzzy Sets Theory (FST) to decision-making processes was presented by Bellman and Zadeh [3], and the concept of fuzzy mathematical programming on general level was first proposed by Tanaka et al. [4]. The first formulation of fuzzy linear programming (FLP) is proposed by Zimmermann [5]. Various fuzzy linear programming techniques are surveyed in literature which are classified into two main classes: fuzzy linear programming [6]-[12] and possibilistic linear programming [13]-[18]. For the sake of simplicity, usually these techniques consider only crisp solutions of the fuzzy problems [19]. Generally speaking, in fuzzy linear programming problems, the coefficients of decision variables are fuzzy numbers, while decision variables are the crisp ones. This means that, in an uncertain environment, a crisp decision is made to meet some decision criteria [20]; thus, the decision-making process is constrained to crisp the decisions that hide the fuzzy aspect of the problem [19]. Then, some authors have developed the fuzzy linear programming problems in which all the parameters as well as the variables are represented by fuzzy numbers is known as FFLP problems [21]-[27]. These problems do not show enough understanding of the extension principle.

The paper is organized as follows: in Section II, some basic definitions of fuzzy set theory and arithmetic between two triangular fuzzy numbers are reviewed; in Section III formulation of FMOLPP with fuzzy right hand side and fuzzy decision variables are discussed; in Section IV a new method is proposed for solving FMOLPP presented in Section III; to illustrate the proposed method, it is applied to a real life allocation problem and the obtained results are discussed in Section V; conclusion is drawn in Section VI.

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II. Preliminaries

Definition 1. A fuzzy number $\tilde{A} = (m_A, w_A, \acute{w}_A)$ is a LR type if and only if:

$$A(x) = \begin{cases} L\left(\frac{m_A - x}{w_A}\right) & -\infty < x \le m_A \\ R\left(\frac{x - m_A}{\acute{w}_A}\right) & m_A \le x < +\infty \end{cases} \qquad w_A, \acute{w}_A \ge 0 \quad (1)$$

where m_A is the center, and w_A and $\acute{w}_A$ are the left and right bandwidths of $\tilde{A}$, respectively. This is a parametric form of fuzzy number $\tilde{A}$, so we can show it as a triangular shape as (2):

$$\tilde{A} \equiv (m, w_A, \acute{w}_A)_{LR} \quad (2)$$

Definition 2. Given two triangular fuzzy numbers $\tilde{A} = (a, w_A, \acute{w}_A)_{LR}$ and $\tilde{B} = (b, w_B, \acute{w}_{\boldsymbol{B}})_{LR}$, $\tilde{B} < \tilde{A}$ if and only if:

$$b<a \text{ and } w_B + \acute{w}_{\boldsymbol{B}} \ge w_A + \acute{w}_A \quad (3)$$

and $\tilde{B} = \tilde{A}$ if and only if

$$b = a \quad \text{and} \quad w_B + \acute{w}_{\boldsymbol{B}} = w_A + \acute{w}_A \quad (4)$$

Definition 3. A triangular fuzzy number $\tilde{A} = (a, w_A, \acute{w}_A)_{LR}$ is said to be non-negative fuzzy number if and only if $a - w_A \ge 0$.

Definition 4. Given two fuzzy numbers $\tilde{A} = (a, w_A, \acute{w}_A)_{LR}$ and $\tilde{B} = (b, w_B, \acute{w}_{\boldsymbol{B}})_{LR}$ with continuous nondecreasing function over $[0, \infty)$, fuzzy Arithmetic operations are defined as (5)-(9):

$$A + B = \left(a + b, w_A + w_B, \acute{w}_A + \acute{w}_{\boldsymbol{B}}\right)_{LR} \quad (5)$$

$$A - B = \left(a - b, w_A + \acute{w}_{\boldsymbol{B}}, \acute{w}_A + w_{\boldsymbol{B}}\right)_{LR} \quad (6)$$

$$(a, w_A, \acute{w}_A)_{LR}.(b, w_B, \acute{w}_{\boldsymbol{B}})_{LR} \approx (a.b, aw_{\boldsymbol{B}} + bw_A, a\acute{w}_{\boldsymbol{B}} + b\acute{w}_A)_{LR} \quad (7)$$

$$(a, w_A, \acute{w}_A)_{LR}.(b, w_B, \acute{w}_{\boldsymbol{B}})_{LR} \approx (a.b, -a\acute{w}_{\boldsymbol{B}} + bw_A, -aw_{\boldsymbol{B}} + b\acute{w}_A)_{LR} \quad (8)$$

$$c.(a, w_A, \acute{w}_A)_{LR} = (ca, cw_A, c\acute{w}_A)_{LR} \quad (9)$$

III. A Multi Objective Linear Programming Problem with Triangular Fuzzy Variables and Right Hand Side Number

A multi objective linear programming problem with triangular fuzzy variables and right hand side number is defined as (10):

$$\max(\min)\ \tilde{Z}_P = c_{p1}(x_{1m}, w_1, \acute{w}_1) + c_{p2}(x_{2m}, w_2, \acute{w}_2) + \cdots + c_{pn}(x_{nm}, w_n, \acute{w}_n) \quad \forall P = 1, \dots, p$$
$$s.t.\ a_{i1}(x_{Mm}, w_1, \acute{w}_1) + a_{i2}(x_{mm}, w_2, \acute{w}_2) + \cdots + a_{in}(x_{nm}, w_n, \acute{w}_n) \leq [b_i, b_{iw}, b_{i\acute{w}}]\ i = 1, \dots, M$$
$$(x_{jm}, w_j, \acute{w}_j) \geq 0 \quad j = 1, \dots, n \qquad (10)$$

where

$$\tilde{x} = (x_m, w, \acute{w}), b = (b, b_w, b_{\acute{w}}), x_m = core\ (\tilde{x}), b = core\ (\tilde{b})$$

and $w, \acute{w}, b_w, b_{\acute{w}}$ are the bandwidths of $\tilde{x}$ and $\tilde{b}$ respectively. $\tilde{x}\epsilon E^n, \tilde{b}\epsilon E^M, A = [a_{ij}]^{M\times n}\epsilon R^{M\times n}, C\epsilon R^n$.

Definition 5. We say that fuzzy vector $\tilde{x} = (x_{jm}, w_j, \acute{w}_j)$ is a fuzzy feasible solution to the problem (10) if $\tilde{x}$ satisfies the constraints of the problem.

Definition 6. A fuzzy feasible solution $\tilde{x}^*$ is a fuzzy optimal solution of (10), if for all fuzzy feasible solution $\tilde{x}$ for (10), we have $c\tilde{x}^* \gtrsim (\lesssim) c\tilde{x}$

IV. Proposed Method to Find the Fuzzy Optimal Solution of the Problem 3-1

In this section, a new method is proposed to find the fuzzy optimal solution of the (3.1) type of FMOLPP:

Step 1. Consider each objective function of the model subject to all constraints exist in the original problem separately and define P single objective problems.

Step 2. Using arithmetic operations, defined in Definition 4, the single objective fuzzy linear programming problem, obtained in Step 1, is converted into (11):

$$\max(\min)(Z_{Pm}, Z_{Pw}, Z_{P\acute{w}}) = \left(\sum_{j=1}^{n} c_j x_{jm}, \sum_{j=1}^{n} c_j w_j, \sum_{j=1}^{n} c_j \acute{w}_j\right) \quad \forall P = 1, \dots, p \qquad (11)$$
$$s.t.\ \left(\sum_{j=1}^{n} a_{ij} x_{jm}, \sum_{j=1}^{n} a_{ij} w_j, \sum_{j=1}^{n} a_{ij} w_j\right) \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} (b_{im}, b_{iw}, b_{i\acute{w}}) \quad \forall i = i = 1, \dots, M$$
$$\tilde{x} = (x_{jm}, w_j, \acute{w}_j)\ is\ non-negative \quad j = 1, \dots, n$$

Step 3. Using the fuzzy ranking method defined in Definition 2, the problem (11) is converted to a crisp LP problem as (12):

$$\max(\min)(Z_{Pm}) = \sum_{j=1}^{n} c_j x_{jm}$$
$$\min(\max)(Z_{pw} + Z_{p\acute{w}}) = \sum_{j=1}^{n} c_j w_j + c_j \acute{w}_j \quad \forall P = 1, \dots, p$$
$$s.t. \sum_{j=1}^{n} a_{ij} x_{jm} \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b_{im} \quad \forall i = 1, \dots, M$$
$$\sum_{j=1}^{n} a_{ij}(w_j + \acute{w}_j) \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b_{iw} + b_{i\acute{w}} \quad \forall i = 1, \dots, M \qquad (12)$$
$$x_{jm} - w_j \geq 0 \quad j = 1, \dots, n$$

Step 4. Find the optimal solution of all P single objective problems $\tilde{x}_p^*$ using the CLP presented in Step 3, and then calculate $Z_P^* = (Z_{Pm}^*, Z_{Pw}^*, Z_{P\acute{w}}^*)$ for each $P = 1, \dots, p$

Step 5. Define a new single objective function subject to crisp constraints defined in Step 3 as (13):

$$\min \sum_{p=1}^{P} w_P[(Z_{Pm}, Z_{Pw}, Z_{P\acute{w}}) - (Z_{Pm}^*, Z_{Pw}^*, Z_{P\acute{w}}^*)]$$
$$s.t. \sum_{j=1}^{n} a_{ij} x_{jm} \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b_{im} \quad \forall i = 1, \dots, M$$
$$\sum_{j=1}^{n} a_{ij}(w_j + \acute{w}_j) \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b_{iw} + b_{i\acute{w}} \quad \forall i = 1, \dots, M \qquad (13)$$
$$x_{jm} - w_j \geq 0 \quad j = 1, \dots, n$$

where w_P, $P = 1, \dots, p$ is the weight of objective function P determined by DM.

Step 6. Find the optimal solution of Step 5 which is an efficient solution of the original FMOLP problem.

V. Application of the Proposed Model to the Supplier Selection Problem

In this section, the proposed approach is applied to a supplier selection and order allocation problem for an automotive part producer company.

The company needs about 2400 pieces of a special kind of automotive part per month. 10 suppliers in the car industry are able to supply these parts. The company aims to select at least four of them and to determine the optimal amount of orders allocated to each supplier. Three different objectives need to be met as follows:

Minimizing the total production time;

1- Minimizing the total amount of scrap;

2- Maximizing the delivery reliability percent of the parts.

Total variable cost should not exceed about 13000 for each month. Considering the imprecise demand per month, the triangular fuzzy number [2400, 200, 200] is used in the right hand side of the constraint related to it. In regard to the imprecise demand, the total variable cost would be imprecise and is presented by the triangular fuzzy number [13000, 1000, 1000] in the constraint related to variable cost.

Data needed to build the model are provided by the company in Table I:

TABLE I
DATA RELATED TO EACH COMPONY

company	delivery reliability percent	rate of scrap per part	variable cost per part	production time per part
1	0.9	0.333	6.009	239.16
2	0.6	0.166	5.57	282.14
3	0.5	0.166	5.853	224
4	0.9	0.416	5.152	221.25
5	1.3	0.5	5.985	231.25
6	1.1	0.75	6.444	339.8
7	0.4	0.83	5.05	217.5
8	0.6	0.95	4.898	211.66
9	0.9	0.5	7.28	336.25
10	0.6	0.333	7.37	348

According to problem (10), model (14) is proposed to satisfy the objectives of the company considering its demands and constraints:

$\text{Min}\,\tilde{Z}_1 = 239.16(x_{1m}, w_1, w_1) + 282.14(x_{2m}, w_2, w_2) + 224(x_{3m}, w_3, w_3) + 221.25(x_{4m}, w_4, w_4) + 231.25(x_{5m}, w_5, w_5) + 339.8(x_{6m}, w_6, w_6) + 217.5(x_{7m}, w_7, w_7) + 211.66(x_{8m}, w_8, w_8) + 336.25(x_{9m}, w_9, w_9) + 348(x_{10m}, w_{10}, w_{10})$

$\text{Min}\,\tilde{Z}_2 = 0.333(x_{1m}, w_1, w_1) + 0.166(x_{2m}, w_2, w_2) + 0.166(x_{3m}, w_3, w_3) + 0.416(x_{4m}, w_4, w_4) + 0.5(x_{5m}, w_5, w_5) + 0.75(x_{6m}, w_6, w_6) + 0.83(x_{7m}, w_7, w_7) + 0.95(x_{8m}, w_8, w_8) + 0.5(x_{9m}, w_9, w_9) + 0.333(x_{10m}, w_{10}, w_{10})$

$\text{Max}\,\tilde{Z}_3 = 0.9(x_{1m}, w_1, w_1) + 0.6(x_{2m}, w_2, w_2) + 0.5(x_{3m}, w_3, w_3) + 0.9(x_{4m}, w_4, w_4) + 1.3(x_{5m}, w_5, w_5) + 1.1(x_{6m}, w_6, w_6) + 0.4(x_{7m}, w_7, w_7) + 0.6(x_{8m}, w_8, w_8) + 0.9(x_{9m}, w_9, w_9) + 0.6(x_{10m}, w_{10}, w_{10})$

Subject to:

$6.009(x_{1m}, w_1, w_1) + 5.57(x_{2m}, w_2, w_2) + 4.898(x_{3m}, w_3, w_3) + 5.152(x_{4m}, w_4, w_4) + 5.985(x_{5m}, w_5, w_5) + 6.444(x_{6m}, w_6, w_6) + 5.05(x_{7m}, w_7, w_7) + 5.853(x_{8m}, w_8, w_8) + 7.28(x_{9m}, w_9, w_9) + 7.37(x_{10m}, w_{10}, w_{10}) \leq [13000\ \ 1000\ \ 1000]$

(14)

$(x_{1m}, w_1, w_1) + (x_{2m}, w_2, w_2) + (x_{3m}, w_3, w_3) + (x_{4m}, w_4, w_4) + (x_{5m}, w_5, w_5) + (x_{6m}, w_6, w_6) + (x_{7m}, w_7, w_7) + (x_{8m}, w_8, w_8) + (x_{9m}, w_9, w_9) + (x_{10m}, w_{10}, w_{10}) \leq [2400\ \ 200\ \ 200]$

$x_{jm} - w_j \geq 100 y_j$

$x_{jm} - w_j \leq 2600 y_j \quad j = 1, \dots, 10$

$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} \geq 5$

$x_{jm} \geq 0\ and\ integer \quad w_j \geq 0\ \ and\ integer \quad j = 1, \dots, 10$

$y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10} = 0\ or\ 1$

where, $\tilde{x}_j = fuzzy\ number\ of\ orders\ allocated\ to\ jth\ supplier$ $y_j = 0\ or\ 1$, $the\ binary\ variable\ of\ selection\ of\ jth\ supplier\ or\ not$, $\tilde{Z}_1$ = the total production time, $\tilde{Z}_2$ = total amount of scrap, $\tilde{Z}_3$ = total delivery reliability percent of all parts.

Constrain (1): total demand of parts,

Constrain (2): total variable cost.

Constrain (3): if jth supplier is selected, the number of its order allocation will be neither less than 100 nor more than 2600.

According to (12) the crisp MOLP is represented in (15).

$\text{maxZ} = 135.66\ x_{1m} + 185.14x_{2m} + 157x_{3m} + 159.25x_{4m} + 91.25x_{5m} + 384.8x_{6m} + 512.25x_{7m} + 506.66x_{8m} + 316.25x_{9m} + 334.5x_{10m}$

$\text{minW} = 675.66w_1 + 545.14w_2 + 457w_3 + 699.25w_4 + 871.25w_5 + 384.8w_6 + 752.5w_7 + 866.66w_8 + 856.25w_9 + 694.5w_{10}$

s.t:

$6.009x_{1m} + 5.57x_{2m} + 4.898x_{3m} + 5.152x_{4m} + 5.985x_{5m} + 6.444x_{6m} + 5.05x_{7m} + 5.853x_{8m} + 7.28x_{9m} + 7.37x_{10m} \leq 13000$

$6.009w_1 + 5.57w_2 + 4.898w_3 + 5.152w_4 + 5.985w_5 + 6.444w_6 + 5.05w_7 + 5.853w_8 + 7.28w_9 + 7.37w_{10} \geq 1000$

(15)

$x_{1m} + x_{2m} + x_{3m} + x_{4m} + x_{5m} + x_{6m} + x_{7m} + x_{8m} + x_{9m} + x_{10m} \leq 2400$

$w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_8 + w_9 + w_{10} \geq 200$

$x_{jm} - w_j \geq 100y_j \qquad j = 1,2, \dots, 10$

$x_{jm} - w_j \leq 2600y_j \qquad j = 1,2, \dots, 10$

$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} \geq 5$

$x_{jm} \geq 0 \text{ and integer} \quad w_j \geq 0 \text{ and integer} \quad j = 1, \dots, 10$

The final result obtained from solving the model is presented below:

$\tilde{x}_1 = (100,0,0)$ $\tilde{x}_2 = (100,0,0)$ $\tilde{x}_4 = (1587,0,0)$ $\tilde{x}_5 = (613,200,200)$

$\tilde{Z}^*_1 = (517078.14,46250,46250)$

$\tilde{Z}^*_2 = (1016.592,100,100)$

$\tilde{Z}^*_3 = (2375.2,260,260)$

Results show that suppliers 1, 2, 4 and 5 are the best options for the company, and the amount of orders allocated to them and the acceptable tolerance of decisions according to the fuzziness in the company are determined. So, the impreciseness of the decisions is presented precisely, and there is no need to do sensitivity analysis of the inexact right hand side numbers. The owners of the company confirmed the results as reasonable.

VI. Conclusion

In this paper, a new method was proposed to find the fuzzy optimal solution of FMOLPP with fuzzy right hand side and fuzzy decision variables. By using the proposed method, the fuzzy optimal solution of FMOLPP which occurs in real life situation, can be easily obtained. With the aid of this method, decision makers can obtain fuzzy decisions to reflect the inherent fuzziness of a decision problem and the need to do sensitivity analysis after obtaining a crisp solution decreases. To illustrate the proposed method, a real life order allocation problem to some suppliers was solved and the results satisfied the owners of the company.

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