Solution of Interval-valued Manufacturing Inventory Models With Shortages

Susovan Chakrabortty, Madhumangal Pal and Prasun Kumar Nayak

Abstract—A manufacturing inventory model with shortages with carrying cost, shortage cost, setup cost and demand quantity as imprecise numbers, instead of real numbers, namely interval number is considered here. First, a brief survey of the existing works on comparing and ranking any two interval numbers on the real line is presented. A common algorithm for the optimum production quantity (Economic lot-size) per cycle of a single product (so as to minimize the total average cost) is developed which works well on interval number optimization under consideration. Finally, the designed algorithm is illustrated with numerical example.

Keywords—EOQ, Inventory, Interval Number, Demand, Production, Simulation

I. INTRODUCTION

RECENTLY much attention has been focused on EOQ models with fuzzy carrying cost, fuzzy shortage cost, fuzzy setup cost, fuzzy demand etc; this means that elements of carrying cost, shortage cost, setup costs and demand are fuzzy numbers [8], [14], [19]. However EOQ model has played an important role in the field of control theory, when we apply the EOQ model to some practical problems which we encounter in real situation, it is difficult to know the values of carrying cost, shortage cost, setup cost and demand quantity exactly; we can only know the values approximately. Generally, uncertainties are considered as randomness and are handled by probability theory in conventional inventory models. Usually, researchers considered parameters either as constant or dependent on time or probabilistic in nature. But we cannot estimate the probability distribution due to lack of historical data. The research on fuzzy EOQ models has been developed by Park [11], Vujosevic [13], Kacprzyk and Staniewski [9], Mahato and Goswami [5], [6], Lin and Yao [3] has explored EOQ model with fuzzy lead time, fuzzy demand and fuzzy cost coefficients. However, in reality, it is not always easy to specify the membership function or probability distribution in an inexact environment. Since, the optimal total average cost of the model should be interval-valued no studies have yet been attempted for interval valued manufacturing inventory models with shortages, which will be examined in this paper. We choose the interval numbers instead of the fuzzy numbers due to the following facts.

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- If the parameters were assumed to be triangular fuzzy numbers, then membership function of the total cost can be calculated easily. However if the membership function of the fuzzy variable is complex, for example, when a trapezoidal fuzzy number and a Gaussian fuzzy number coexist in a model, it is hard to obtain the membership function of the total cost. Therefore, these membership functions play a significant role in these methods. However, in practice one may not be able to get exact membership function for fuzzy values. Since precise information is required, the lack of accuracy will affect the quality of the solution obtained.

- An interval number can be throughout an extension of the concept of a real number and also a subset of a real line $\Re$ Moore [18]. H.J.Zimmermann shows that α-cut of a fuzzy number is an interval number. As the coefficients of an interval signifies the extent of tolerance (or a region) that the parameter can possibly take.

- To define a fuzzy number, three parameters are required. For an interval number, two parameters are used. The notation of interval numbers has the advantage of being simple and at the same time a better model to represent the values in the situation like "is lies between α and β". So the interval numbers [18], serve better our required purpose.

Thus, the interval number theory, rather than the traditional probability theory and fuzzy set theory, is well suited to the inventory problem. According to the decision maker's point of view under changeable conditions, we may replace the real numbers by the interval valued numbers to formulate the problems more appropriately.

We organize the paper as follows : In section II, we give some basic definitions, notations and comparison on interval numbers. In section III, we give the model formulation. In section IV, we give the solution procedure and in section V a numerical example is presented to indicate the performance of the proposed method.

II. INTERVAL NUMBER

An interval number proposed by Moore [18], is considered as an extension of a real number and as a real subset of the real line $\Re$.

Definition I: Let $\Re$ be the set of all real numbers. An interval number $\widetilde{A}$ is a closed interval defined by

$$\widetilde{A} = [a_L, a_R] = \{x \in \Re : a_L \le x \le a_R\} \qquad (1)$$

The numbers a_L, a_R are called respectively the lower and upper limits of the interval $\widetilde{A}$. An interval number $\widetilde{A}$ alternatively

represented in mean-width or center-radius form as

$$\widetilde{A} = \langle m(\widetilde{A}), w(\widetilde{A}) \rangle$$
$$= \{ x \in \Re : m(\widetilde{A}) - w(\widetilde{A}) \le x \le m(\widetilde{A}) + w(\widetilde{A}) \}$$

where $m(\widetilde{A}) = \frac{1}{2}(a_L + a_R)$ and $w(\widetilde{A}) = \frac{1}{2}(a_R - a_L)$ are respectively the mid-point and half-width of the interval $\widetilde{A}$. Actually, each real number can be regarded as an interval, such as, for all $x \in \Re$, x can be written as an interval $[x, x]$, which has zero length. The set of all interval numbers in $\Re$ is denoted by $I(\Re)$.

A. Basic interval arithmetic

Let $\widetilde{A} = [a_L, a_R] = \langle m_1, w_1 \rangle$ and $\widetilde{B} = [b_L, b_R] = \langle m_2, w_2 \rangle \in I(\Re)$, where m_1, w_1 and m_2, w_2 are respectively the mid-point and half-width of the interval $\widetilde{A}$ and $\widetilde{B}$. Then

$$\widetilde{A} + \widetilde{B} = [a_L + b_L, a_R + b_R] = \langle m_1 + m_2, w_1 + w_2 \rangle.$$

The multiplication of an interval by a real number $c \ne 0$ is defined as

$$c\widetilde{A} = \begin{cases} [ca_L, ca_R]; & \text{if } c > 0, \\ [ca_R, ca_L]; & \text{if } c < 0 \end{cases}$$
$$c\widetilde{A} = c\langle m_1, w_1 \rangle = \langle cm_1, |c|w_1 \rangle.$$

The difference of these two interval numbers is

$$\widetilde{A} - \widetilde{B} = [a_L - b_R, a_R - b_L].$$

The product of these two distinct interval numbers is given by

$$\widetilde{A}\widetilde{B} = [\min\{a_L.b_L, a_L.b_R, a_R.b_L, a_R.b_R\},$$
$$\max\{a_L.b_L, a_L.b_R, a_R.b_L, a_R.b_R\}]$$

The division of these two interval numbers with $0 \notin B$ is given by

$$\widetilde{A}/\widetilde{B} = \left[\min\left\{ \frac{a_L}{b_L}, \frac{a_L}{b_R}, \frac{a_R}{b_L}, \frac{a_R}{b_R} \right\}, \max\left\{ \frac{a_L}{b_L}, \frac{a_L}{b_R}, \frac{a_R}{b_L}, \frac{a_R}{b_R} \right\} \right].$$

The power of an interval $\widetilde{A} = [a_L, a_R]$ is given by

$$\widetilde{A}^k = \begin{cases} [1,1]; & \text{if } k = 0, \\ [a_L^k, a_R^k]; & \text{if } a_L \ge 0 \text{ or } k \text{ odd}, \\ [a_R^k, a_L^k]; & \text{if } a_R \le 0 \text{ or } k \text{ even}, \\ [a_1 b_1^{k-1}, \max\{a_L^k, a_R^k\}]; & \text{if } a_L \le 0 \le a_R, k \ge 0 \text{ even} \end{cases}$$

B. Comparison between interval numbers

Let $\widetilde{A} = [a_L, a_R] = \langle m_1, w_1 \rangle$, $\widetilde{B} = [b_L, b_R] = \langle m_2, w_2 \rangle$ be two interval numbers within $I(\Re)$. These two intervals may be one of the following types:

- Two intervals are completely disjoint(non-overlapping).
- Two intervals are nested, (fully overlapping).
- Intervals are partially overlapping.

A brief comparison on different interval orders is given in [1], [17].

Case 1 (Disjoint subintervals): Moore [18] defined

Fig. 1. Disjoint subintervals

transitive order relations over intervals as $\widetilde{A}$ is strictly less than $\widetilde{B}$ if and only if $a_R < b_L$ and this is denoted by $\widetilde{A} < \widetilde{B}$.

Case 2 (Nested subintervals) : Let $\widetilde{A}$ and $\widetilde{B}$ be such that $a_L \le b_L < b_R \le a_R$. Then $\widetilde{B}$ is contained in $\widetilde{A}$ and it is denoted by $\widetilde{B} \subseteq \widetilde{A}$ which is the extension of the concept of the set inclusion [18]. we define the ranking order of $\widetilde{A}$ and

Fig. 2. Nested subintervals

$\widetilde{B}$ as

$$\widetilde{A} \vee \widetilde{B} = \begin{cases} \widetilde{A}, & \text{if the player is optimistic} \\ \widetilde{B}, & \text{if the player is pessimistic.} \end{cases}$$

The notation $\widetilde{A} \vee \widetilde{B}$ represents the maximum among the interval numbers $\widetilde{A}$ and $\widetilde{B}$. Similarly

$$\widetilde{A} \wedge \widetilde{B} = \begin{cases} \widetilde{B}, & \text{if the player is optimistic} \\ \widetilde{A}, & \text{if the player is pessimistic.} \end{cases}$$

The notation $\widetilde{A} \wedge \widetilde{B}$ represents the minimum among the interval numbers $\widetilde{A}$ and $\widetilde{B}$.

Case 3 (Partially overlapping subintervals) : The above mentioned order relations introduced by Moore [18] can not explain ranking between two overlapping closed intervals.
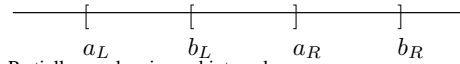

Fig. 3. Partially overlapping subintervals

Definition II: For $m_1 \le m_2$ and $w_1 + w_2 \ne 0$, an acceptability index to the premise $\widetilde{A} \prec \widetilde{B}$ is defined as follows [17]:

$$\Psi(\widetilde{A} \prec \widetilde{B}) = \frac{m_2 - m_1}{w_1 + w_2}, \qquad (2)$$

which is the value judgement or satisfaction degree of the decision makers (DM) that the interval $\widetilde{A}$ is not superior to $\widetilde{B}$ ($\widetilde{B}$ is not inferior to $\widetilde{A}$) in terms of value.

Thus, the max operator "$\vee$" for two intervals $\widetilde{A}$ and $\widetilde{B}$ is defined as follows [17]:

$$\widetilde{A} \vee \widetilde{B} = \begin{cases} \widetilde{B}, & \text{if } \Psi(\widetilde{A} \prec \widetilde{B}) > 0 \\ \widetilde{A}, & \text{if } \Psi(\widetilde{A} \prec \widetilde{B}) = 0; w_1 < w_2 \text{ and DM is pessimistic} \\ \widetilde{B}, & \text{if } \Psi(\widetilde{A} \prec \widetilde{B}) = 0; w_1 < w_2 \text{ and DM is optimistic.} \end{cases}$$

The following algorithm shows the $\widetilde{\max}$ which determines the maximum between two interval numbers.

FUNCTION $\widetilde{\max}(\widetilde{A}, \widetilde{B})$
if $\widetilde{A} = \widetilde{B}$ then maximum = $\widetilde{A}$;
else
if $\widetilde{A} = \langle m_1, w_1 \rangle$ and $\widetilde{B} = \langle m_2, w_2 \rangle$ are not
non-dominating then
if $((\widetilde{A} \prec \widetilde{B})$ or $(\widetilde{A} \prec_P \widetilde{B}))$ then
maximum = $\widetilde{B}$;
else
maximum = $\widetilde{A}$;
endif;
else
if $(w_1 < w_2)$ then
if the decision maker is optimistic maximum = $\widetilde{B}$;
if the decision maker is pessimistic maximum = $\widetilde{A}$;

endif;
endif;
endif;
return(maximum);
END Function.

Similarly, the min operator "∧" for two intervals $\widetilde{A}$ and $\widetilde{B}$ is defined as follows [17]:

$$\wedge \widetilde{B} = \begin{cases} \widetilde{B}, & \text{if } \Psi(\widetilde{B} \prec \widetilde{A}) > 0 \\ \widetilde{A}, & \text{if } \Psi(\widetilde{B} \prec \widetilde{A}) = 0; w_1 > w_2 \text{ and DM is pessimistic} \\ \widetilde{B}, & \text{if } \Psi(\widetilde{B} \prec \widetilde{A}) = 0; w_1 > w_2 \text{ and DM is optimistic.} \end{cases}$$

FUNCTION $\widetilde{\min}(\widetilde{A}, \widetilde{B})$
if $\widetilde{A} = \widetilde{B}$ then minimum = $\widetilde{A}$;
else
if $\widetilde{A} = \langle m_1, w_1 \rangle$ and $\widetilde{B} = \langle m_2, w_2 \rangle$ are not
non-dominating then
if $((\widetilde{A} \prec \widetilde{B})$ or $(\widetilde{A} \prec_P \widetilde{B}))$ then
minimum = $\widetilde{A}$;
else
minimum = $\widetilde{B}$;
endif;
else
if $(w_1 > w_2)$ then
if the decision maker is optimistic minimum = $\widetilde{B}$;
if the decision maker is pessimistic minimum = $\widetilde{A}$;
endif;
endif;
endif;
return(minimum);
END Function.

III. MODEL FORMULATION

The purpose of the EOQ model is to find the optimal order quantity of inventory items at each time such that the sum of the order cost, the carrying cost and the shortage cost, i.e., total cost is minimal.

Notations : For the sake of clarity, the following notations are used throughout the paper.

T, the interval between production cycle ;
t_1, the time after which the production is stopped ;
$\widetilde{D} = [d_L, d_R]$, demand rate per unit time;
$\widetilde{Q}$, fixed lot size per cycle;
$\widetilde{S_1}$, the inventory level at the end of t_1;
$\widetilde{S_2}$, the shortage level at the end of t_3;
$\widetilde{C}(Q)$, total cost in the plan period;
$\widetilde{C_1} = [C_{1L}, C_{1R}]$, the inventory carrying cost or holding cost per unit item per unit time;
$\widetilde{C_2} = [C_{2L}, C_{2R}]$, the inventory shortage cost per unit item per unit time;
$\widetilde{C_3} = [C_{3L}, C_{3R}]$, the ordering or setup cost per unit item;

Assumptions : We have the following assumptions:

- Production rate or replenishment rate is finite, say, K units per unit item where $K > D$.
- Shortages are allowed and fully backlogged.
- Lead time is zero.

- The inventory planning horizon is infinite and the inventory system involves only one item and one stocking point.
- Carrying cost $(\widetilde{C_1})$, shortages cost $(\widetilde{C_2})$, ordering cost $(\widetilde{C_3})$ and demand $(\widetilde{D})$ are assume to be interval numbers.

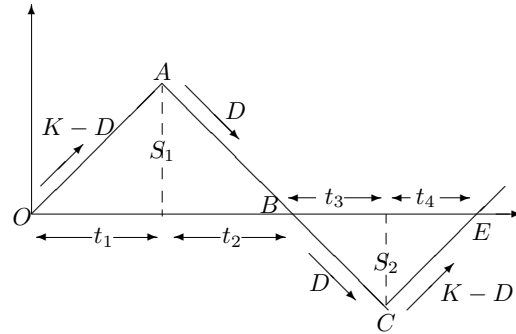A typical behavior of the EOQ manufacturing inventory

Fig. 4. EOQ manufacturing inventory model with shortage

model with uniform demand and with shortage is depicted in Figure 4. In this model, we can easily observe that the inventory carrying cost C_1 as well as shortages cost C_2 will be involved only when $0 \le S_1 \le Q$.

In this model, each production cycle time T consists of two parts t_{12} and t_{34} which are further subdivision into t_1, t_2, t_3, t_4 where

- inventory is building up at a constant rate $K - D$ units per unit time during the interval $[0, t_1]$,
- at time $t = t_1$, the production is stopped and the stock level decreases due to meet up the customer's demand only up to the time $t = t_1 + t_2$,
- shortages are accumulated at a constant rate of D units per unit time during the time t_3,
- shortages are being filled up immediately at a constant rate $K - D$ units per unit time during the time t_4.
- The production cycle then repeat itself after the time $T = t_1 + t_2 + t_3 + t_4$.

Again, let at the end of t_1, the inventory level is S_1 and at the end of time $t = t_1 + t_2$ the level becomes nil. Now shortages start and suppose that shortages are built up of quantity S_2 at time $T = t_1 + t_2 + t_3$ and then these shortages be filled up upto the time $T = t_1 + t_2 + t_3 + t_4$. Therefore,

$$S_1 = (K - D)t_1 = Dt_2. \tag{3}$$

Again we have,

$$S_2 = Dt_3 = (K - D)t_4. \tag{4}$$

Since $T = t_1 + t_2 + t_3 + t_4$, we get

$$T = \frac{K(t_1 + t_4)}{D}. \tag{5}$$

The inventory carrying cost during t_{12} is given by

$$C_1 \times \text{ area of } \triangle OAB = C_1 \times \left(\frac{1}{2} \times S_1 \times (t_1 + t_2) \right)$$

$$= \frac{1}{2} C_1 \frac{K(K - D)}{D} t_1^2, \text{ from, (3)}$$

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The inventory shortage cost during t_{34} is given by

$$C_2 \times \text{ area of } \triangle BCE = C_2 \times \left(\frac{1}{2} \times S_2 \times (t_3 + t_4) \right)$$

$$= \frac{1}{2} C_2 \frac{K - D}{KD} (DT - Kt_1)^2, [\text{ from, (4) and (5)}]$$

The inventory ordering cost during T is C_3. Hence the total cost in the plan period $[0, T]$ can be expressed as

$$X = C_3 + \frac{1}{2} C_1 \frac{K(K - D)}{D} t_1^2 + \frac{1}{2} C_2 \frac{K - D}{KD} (DT - Kt_1)^2$$

Therefore total average cost $C(T, t_1)$ is given by

$$C(T, t_1) = \frac{X}{T} = \frac{C_3}{T} + \frac{(K - D)}{2D} \left[C_1 \frac{t_1^2}{T} + C_2 \frac{(DT - Kt_1)^2}{KT} \right] \tag{6}$$

By using calculus, we optimize $C(T, t_1)$ and we get optimum values of T, t_1, and C as,

$$T^* = \sqrt{\frac{2KC_3(C_1 + C_2)}{C_1 C_2 D(K - D)}}, t_1^* = \sqrt{\frac{2DC_2C_3}{C_1(C_1 + C_2)K(K - D)}},$$

$$C^* = \sqrt{\frac{2C_1 C_2 C_3 D(K - D)}{K(C_1 + C_2)}}.$$

Then using these we get the optimum values of Q, t_2, t_3, S_1 and S_2.

Usually, in mathematical programming we deal with the real numbers which are assumed to be fixed in value. In usual models- Carrying cost (C_1), set up cost (C_3), demand (D) are always fixed in value. But in real life, business cannot be properly formulated in this way due to uncertainty. For example, inventory carrying cost may be different in rainy season compared to summer or winter seasons (costs of taking proper action to prevent deteriorations of items in different seasons and also the labour charges in different seasons are different). Ordering cost, being dependent on the transportation facilities may also vary from season to season. Changes in the price of fuels, mailing charges, telephonic charges may also make the ordering cost fluctuating. Unit purchase cost is highly dependent on the costs of raw materials and labour charges, which may fluctuate over time. In such cases demand and other costs are assumed to be interval valued. But in interval oriented system we cannot use the calculus method for optimization.

A. Interval valued EOQ model

Let us assume interval valued demand by $\widetilde{D} = [d_L, d_R]$, carrying cost by $\widetilde{C_1} = [c_{1L}, c_{1R}]$, shortage cost by $\widetilde{C_2} = [c_{2L}, c_{2R}]$ and set up cost by $\widetilde{C_3} = [c_{3L}, c_{3R}]$, where first term within the bracket denote lower limit and 2nd term within the bracket denote the upper limit of the variable. Replacing D by $[d_L, d_R]$, C_1 by $[c_{1L}, c_{1R}]$, C_2 by $[c_{2L}, c_{2R}]$ and C_3 by $[c_{3L}, c_{3R}]$ in equation (6) we have,

$$\widetilde{C}(T, t_1) = \frac{1}{T} [c_{3L}, c_{3R}] + \frac{K}{2T} [c_{1L}, c_{1R}][K - d_R, K - d_L]$$

$$\left[\frac{1}{d_R}, \frac{1}{d_L} \right] t_1^2 + \frac{1}{2KT} [c_{2L}, c_{2R}] \left[\frac{1}{d_R}, \frac{1}{d_L} \right] ([d_L, d_R]T - Kt_1)^2 \tag{7}$$

Addition and other composition rules (seen in the section II in this paper) on interval numbers are used in this equation.

But in interval oriented system we cannot use the calculus method for optimization of $\widetilde{C}(T, t_1)$. If we take $T = [T_L, T_R]$ and $t_1 = [t_{1L}, t_{1R}]$ then (7) becomes

$$\widetilde{C}(T, t_1) = \left[\frac{1}{T_R}, \frac{1}{T_L} \right] [c_{3L}, c_{3R}] + \frac{K}{2} \left[\frac{1}{T_R}, \frac{1}{T_L} \right]$$

$$[c_{1L}, c_{1R}] . [K - d_R, K - d_L][t_{1L}^2, t_{1R}^2] + \frac{1}{2K} \left[\frac{1}{T_R}, \frac{1}{T_L} \right]$$

$$[c_{2L}, c_{2R}] \left[\frac{1}{d_R}, \frac{1}{d_L} \right] ([d_L, d_R][T_L, T_R] - K [t_{1L}, t_{1R}])^2 \tag{8}$$

In the next section, we have presented a new method (Multi-section Technique) dependent on interval computing technique to solve the unconstrained optimization problems. By using multi-section method, we are to find $t_1^* = [t_{1L}^*, t_{1L}^*]$, and $T^* = [T_L^*, T_L^*]$ for which $\widetilde{C}(T, t_1)$ have the optimal (minimum) value.

IV. SOLUTION PROCEDURE:

The idea of multi-section comes out from the concept of bisection [15], which searches for a solution by repeatedly dividing the range of variable in to multiple parts where more than one bisection are done at a single iteration cycle. First of all, calculate the value of the interval valued cost function at each cell and on the basis of the comparison of intervals (as described in the section II(B) of this paper) finds the optimal(minimal) value of the cost function.

Let us consider a bound unconstrained optimization (maximization or minimization) problem with fixed coefficients as follows:

$$z = f(x), \quad l \leq x \leq u,$$

where $x = (x_1, x_2, \ldots, x_n), l = (l_1, l_2, \ldots, l_n), u = (u_1, u_2, \ldots, u_n)$, n represents the number of decision variables, the jth decision variable x_j; $(j = 1, 2, \ldots, n)$ lies in the prescribed interval $[l_j, u_j]$. Hence, the search space of the above problem is as follows:

$$S = x \in \Re^n : l_j \leq x_j \leq u_j, j = 1, 2, \ldots, n.$$

Suppose, an industry divides the sales season into λ periods. Now our object is to split the accepted region(reduced)region (for the first time, it is the given search space or assumed if the search space is not given)into finite number of distinct equal subregions $R_1, R_2, \ldots, R_\lambda$ to select the subregion containing the best function value.

Let $f(R_i) = [\underline{f_i}, \overline{f_i}]; i = 1, 2, \ldots, \lambda$ be the interval valued objective function $f(x)$ in the ith subregion R_i, where $\underline{f_i}, \overline{f_i}$ denote the upper and lower bounds of $f(x)$ in R_i, computed by the application of finite interval arithmetic. Now, comparing all the interval-valued values of objective function, $f(x)$ in $R_i(i = 1, 2, \ldots, \lambda)$ with the help of interval order relations mentioned in earlier section, the subregion containing best objective function value is accepted. Again, this accepted subregion is divided into other smaller distinct subregions $R_i'(i = 1, 2, \ldots, \lambda)$ by the aforesaid process and applying the same acceptance criteria, we get the reduced subregion.

This process is terminated after reaching the desired degree of accuracy and finally, we get the best value of the objective function and the corresponding values of the decision variables in the form of closed intervals with negligible width. To solve the problem, the optimal solution or an approximation of it has been obtained by applying the following steps.

A. Algorithm Interval Optimal Control

Input:n(number of variables, here $n = 2$), λ(number of divisions), l_j(lower bounds), u_j(upper bounds), where, $j = 1, 2, \cdots, n$.
Output: T^*, t_1^*, $\widetilde{S_1}^*$, $\widetilde{Q}^*$, $\widetilde{C}^*$
Step 1: Initialize l_{min} and u_{min} ,the lower and upper value of interval valued cost function.
Step 2: //calculation of step lengths//
    For $i = 0$ to $n - 1$
    calculate $h_i = (u_i - l_i)/\lambda$
    Set $l_i = a_i$
    end for

Step 3: //Division of region S into equal subregions R_i//
Step 3.1: For $j = 0$ to $\lambda - 1$
    Calculate $l_0 = a_i + j * h_i$ and
    $u_0 = a_i + (j + 1) * h_i$
Step 3.2: For $j1 = 0$ to $\lambda - 1$
    Calculate $l_1 = a_i + j1 * h_i$ and
    $u_1 = a_i + (j1 + 1) * h_i$
Step 3.3: //Call the function f_l and f_u//.
    By using basic interval arithmetic calculate f_l and
    f_u, lower and upper value of $\widetilde{C}(T, t_1)$ respectively
Step 3.4: Applying pessimistic order relation between ' any two interval numbers $[fl, fu]$ and $[l_{\min}, u_{\min}]$
    choose the optimal interval number.
    end $j1$ loop
    end j loop
Step 3.5: choose the subregion R^{opt} among $R_i (i = 1, 2, \ldots, \lambda)$ which has better objective function value by comparing the interval values $f(R_i), i = 1, 2, 3, \ldots, \lambda$ to each other.
Step 4: //calculation of widths//.
Step 4.1: For $i1 = 0$ to $n - 1$
    Calculate widths $w_{i1} = u_{i1} - l_{i1}$
Step 4.2: While $w_{i1} > \varepsilon$
    break
Step 4.3: Set $R^{opt} \leftarrow R_i$
    Return to step 1.2
    end for
    endwhile.
    Output
    END MULTISECTION

V. NUMERICAL EXAMPLE

In this section,we illustrate that the solution procedure proposed in the above Algorithm can be easily implemented on a computer and we show that, with such an implementation, the optimal solutions can be obtained. To serve our purpose, we

have written a computer program using C++ on a PENTIUM 4 personal computer.

Consider a interval valued EOQ inventory system with shortages in which the carrying cost $(\tilde{C}_1) = [0.15, 0.18]$, shortage cost $\tilde{C}_2 = [20, 25]$ and the ordering or setup cost $(\tilde{C}_3) = [500, 700]$, the demand quantity $\tilde{D} = [1500, 1600]$ and production rate $K = 4000$. The approach for computing the best found value in each subregion of the given search region of the test problem has been coded in $C++$ programming language. The solution is $t_1^* = 1.1387, T^* = 2.9552$, optimal cost $\tilde{C}^* = [330.8202, 553.1889]$, the optimum $\widetilde{S_1}^* = [2732.8872, 2846.7575]$ and the optimum $\widetilde{Q}^* = [4432.8000, 4728.3200]$. If we consider the fuzzy model as described by Mahata and Goswami [6] then the solution is optimal cost $\tilde{C}^{**} = [368.3304, 502.0832]$. According to pessimistic point of view $\widetilde{C}^* \wedge \tilde{C}^{**} = \widetilde{C}^* = [330.8202, 553.1889]$. which is better than the fuzzy model. Also the major advantage of our proposed method over [6] lies in the fact that, at any intermediate stage the change of value of any parameter needs no further bulk calculations.

Based on the numerical example considered above, we now study sensitivity of $t_1^*, T^*, \widetilde{Q}^*, \widetilde{S_1}^*$ and $\widetilde{C}^*$ to changes in the values of the system parameters $\widetilde{C}_1, \widetilde{C}_2, \widetilde{C}_3$ and $\widetilde{D}$. The

TABLE I
EFFECT OF CHANGES IN THE VARIOUS PARAMETERS OF THE INVENTORY MODEL

Value of the parameter	% change	Change in				
		t_1^*	T^*	$\widetilde{Q}^*$	$\widetilde{S_1}^*$	$\widetilde{C}^*$
$m(\widetilde{C}_1)$	+50	-12.18	-11.97	-11.97	-12.18	+23.07
	+25	-11.60	-11.47	-11.47	-11.61	+11.55
	-25	+24.43	+24.24	+24.24	+24.43	-12.48
	-50	+73.26	+72.73	+72.73	+73.26	-26.32
$m(\widetilde{C}_2)$	+50	+1.52	+1.35	+1.35	+1.52	+0.32
	+25	+1.45	+1.35	+1.35	+1.45	+0.22
	-25	-0.28	-0.13	-0.12	-0.28	-0.15
	-50	-2.75	-2.32	-2.32	-2.76	-0.52
$m(\widetilde{C}_3)$	+50	+29.43	+29.43	+29.43	+29.43	+23.20
	+25	+12.12	+12.12	+12.12	+12.12	+11.82
	-25	-12.12	-12.12	-12.12	-12.12	-13.29
	-50	-12.12	-12.12	-12.12	-12.12	-26.36
$m(\widetilde{D})$	+50	+41.91	-5.34	+41.92	-2.98	+0.43
	+25	+19.94	-4.04	+19.95	+0.97	+2.08
	-25	-10.85	+18.85	-10.86	+3.25	-4.86
	-50	+18.73	+137.37	+18.69	+56.29	+7.09

sensitivity analysis is performed by changing mid value of each parameters by $+50\%, +25\%, -25\%$ and -50%; taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown in the Table I. From the Table I, it is seen that

- $t_1^*, T^*, \widetilde{Q}^*$ and $\widetilde{S_1}^*$ are fairly sensitive while $\widetilde{C}^*$ are less sensitive to changes in the value of the carrying cost $\widetilde{C}_1$.
- Each of $t_1^*, T^*, \widetilde{Q}^*, \widetilde{S_1}^*$ and $\widetilde{C}^*$ are not much sensitive to

changes in the value of the shortage cost $\widetilde{C}_2$.

- Percentage of the changes of $t_1^*, T^*, \widetilde{Q}^*, \widetilde{S_1}^*$ are same and all of them with $\widetilde{C}^*$ are moderately sensitive to changes in the value of the setup cost $\widetilde{C}_3$.

- $t_1^*, \widetilde{Q}^*$ is moderately sensitive and $T^*, \widetilde{S_1}^*$ and $\widetilde{C}^*$ are very less sensitive sensitive except $T^*, \widetilde{S_1}^*$ are very high sensitive for -50% changes in the demand rate $\widetilde{D}$.

VI. Conclusion

In this paper, we have presented an inventory model with shortage, where carrying cost, the ordering or setup cost and demand are assumed as interval numbers instead of crisp or probabilistic in nature. We have considered the nature of these quantities as interval numbers to make the inventory model more realistic. At first, we have formulated a solution procedure to optimize a general function with coefficients as interval valued numbers using interval arithmetic. Using multi-section technique, we have derived the solution of the model. The algorithm has been tested using numerical example. Lastly, to study the effect of the determined quantities on changes of different parameters, a sensitivity analysis is also presented.

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