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Sliding Mode Control of a Bus Suspension System

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***Abstract*—**The vibrations, caused by the irregularities of the road surface, are to be suppressed via suspension systems. In this paper, sliding mode control for a half bus model with air suspension system is presented. The bus is modelled as five degrees of freedom (DoF) system. The mathematical model of the half bus is developed using Lagrange Equations. For time domain analysis, the bus model is assumed to travel at certain speed over the bump road. The numerical results of the analysis indicate that the sliding mode controllers can be effectively used to suppress the vibrations and to improve the ride comfort of the busses.

***Keywords*—**Sliding mode control, bus model, air suspension.

I. INTRODUCTION

BASICALLY, the vehicle model consists of the vehicle body (sprung mass) and unsprung masses which include wheels, brakes, suspension linkages and other components. A suspension system is located between the sprung and unsprung masses. The suspension systems are classified into three groups: passive, semi-active [1], [2] and active [3]. Passive suspension systems consist of ordinary springs and dampers whose coefficients are fixed. Semi-active suspension systems, generally, consist of passive springs and dampers with variable coefficients [4], [5]. Finally, active suspension systems include actuators which provide force to suppress vibrations, caused by the irregularities of the road surface. For many years, various control strategies have been proposed to improve the ride comfort and the road holding such as PID controller [6], Fuzzy Logic Controller [7], [8] and Backstepping Controller [9]. It is not possible to avoid undesired disruptive effect and uncertainty such as pressure, temperature, modelling errors in real systems. Therefore, robust control strategies are required to apply on non-linear systems such as Sliding Mode Control [10], [11]. In this paper, Sliding Mode Controller is presented for a half bus model with two air suspension systems to suppress the vibrations and to improve the ride comfort.

II. VEHICLE MODEL

A. Half-Bus Model

The physical system of the half-bus model is presented in Fig. 1. The half-bus model has four DoF. The three of them are in the vertical direction and one of them in angular direction. Here, y is body bounce and θ is body pitch motions; M is the mass of the vehicle body; I is body inertia; V is the speed of the vehicle. m_f and m_r are the mass of the front and rear unsprung masses respectively; their vertical displacements are y_f and y_r. The b_f and b_r are the damping coefficients of the front and rear suspensions; k_f and k_r are the spring constants of the front and rear suspensions; k_{tf} and k_{tr} are the stiffness constants of the front and rear tires, respectively. The road surface input of the front and rear tires are z_f and z_r. The u_f and u_r are the control inputs of the front and rear suspension systems.

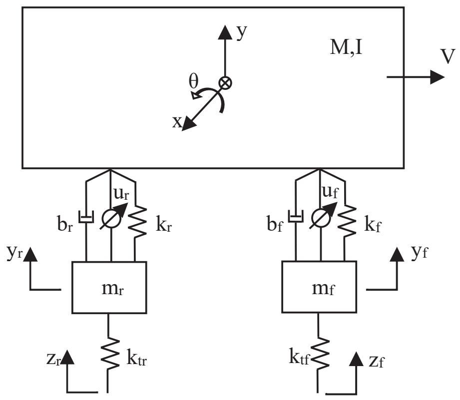

Fig. 1 The half bus model

The equations of motion of the half-bus model are:

$$\begin{aligned} M\ddot{y} = {} & k_f(y - a\sin\theta - y_f) + k_r(y + b\sin\theta - y_r) \\ & + b_f(\dot{y} - a\dot{\theta}\cos\theta - \dot{y}_f) + b_r(\dot{y} + b\dot{\theta}\cos\theta - \dot{y}_r) \\ & - u_f - u_r \end{aligned} \tag{1}$$

$$\begin{aligned} I\ddot{\theta} = {} & k_f\cos\theta\, a(-y + a\sin\theta + y_f) \\ & + k_r\cos\theta\, b(y + b\sin\theta - y_r) \\ & + b_f\cos\theta\, a(-\dot{y} + a\dot{\theta}\cos\theta + \dot{y}_f) \\ & + b_r\cos\theta\, b(\dot{y} + b\dot{\theta}\cos\theta - \dot{y}_r) \\ & + u_f a - u_r b \end{aligned} \tag{2}$$

$$\begin{aligned} m_f\ddot{y}_f = {} & k_f(y_f - y + a\sin\theta) + k_{tf}(y_f - z_f) \\ & + b_f(\dot{y}_f + a\dot{\theta}\cos\theta - \dot{y}) + u_f \end{aligned} \tag{3}$$

$$\begin{aligned} m_r\ddot{y}_r = {} & k_r(y_r - y - b\sin\theta) + k_{tr}(y_r - z_r) \\ & + b_r(\dot{y}_r + b\dot{\theta}\cos\theta - \dot{y}) + u_r \end{aligned} \tag{4}$$

B. Air Spring

The air springs are operated by compressed air in. Air springs have been widely used in the suspension systems of the busses and off-road vehicles such as agricultural vehicles, construction vehicles. In order to improve the air spring adaptability to a road and working conditions, the air springs

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can be operated with the different spring coefficients [12]. Therefore, auxiliary chambers are appended to the air spring volume. These auxiliary chambers are connected through a valve for adding or separating extra volume to the air springs [13]. The physical model of the air spring with auxiliary chamber is shown as Fig. 2.

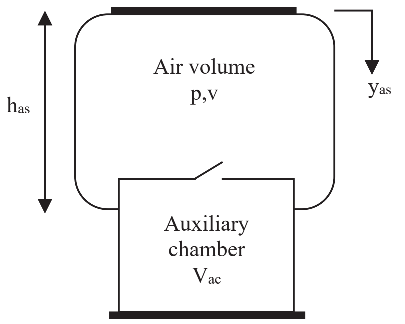

Fig. 2 Air spring model with auxiliary chamber (Adapted from [13])

p is the pressure and v is the volume of air spring at any time t; V_{ac} is volume of auxiliary chamber; h_{as} is height of air spring; y_{as} is displacement of the air spring, as shown in Fig. 2. Accordingly, the spring constant of the air spring is

$$k_i = nM_i g A_{as}(h_{as}A_{as} + n_{ac}V_{ac})^n [(h_{as}A_{as} + n_{ac}V_{ac}) - A_{as}y_i]^{-(n+1)} \tag{5}$$

where i=1, 2 and n is the polytrophic exponent; M_i is the body mass; g is the acceleration of gravity; A_{as} is the area of air volume; n_{ac} is the number of auxiliary chamber; y_i is displacement of the air spring.

III. Sliding Mode Control

Sliding mode controller, which can be applied to non-linear systems, is a variable structure control method. This control method presents robustness against parameter uncertainties and unmodelled dynamics. Sliding Mode Controller can also be applied to unstable systems. A controlled non-linear dynamic system is described as

$$\underline{\dot{x}} = \underline{f}(x) + [B]\underline{u} \tag{6}$$

Here, $x=[x_1, x_2 \ldots x_{10}]^T$ is the state variables, f(x) is the non-linear vector of state equations, [B] is the control forces matrix and $u=[u_1, u_2]^T$ is the control input vector. The aim of the Sliding Mode Controller is to hold the system on a sliding surface. This surface is defined by

$$S = \{\underline{x} : \underline{\sigma}(x,t) = 0\} \tag{7}$$

The sliding surface equation for controlling the system can be selected as

$$\underline{\sigma}(x,t) = [G]\Delta\underline{x} \tag{8}$$

where

$$\Delta\underline{x} = \underline{x}_{ref} - \underline{x} = \underline{e} \tag{9}$$

$$\underline{\sigma}(x,t) = [G]\Delta\underline{x} = [G](\underline{x}_{ref} - \underline{x}) \tag{10}$$

$$\underline{\sigma}(x,t) = \underline{\Phi}(t) - [G]\underline{x} \tag{11}$$

Δx is the difference between the reference values (x_{ref}) and the system responses and [G] is the slope of sliding surface. According to Lyapunov Stability Criteria, the Lyapunov function must be positive definite and its derivate must be negative semi-definite for the stability of the system. The Lyapunov function and its derivate are described by

$$v(\underline{\sigma}) = \frac{\underline{\sigma}^T(x,t)\underline{\sigma}(x,t)}{2} \rangle 0 \tag{12}$$

$$\frac{dv(\underline{\sigma})}{dt} = \underline{\sigma}^T(x,t)\underline{\dot{\sigma}}(x,t) \le 0 \tag{13}$$

where $\dot{v} = 0$ and $\underline{\dot{\sigma}}(x,t) = 0$

$$\frac{d\underline{\sigma}(x,t)}{dt} = \frac{d\underline{\Phi}(t)}{dt} - [G]\underline{\dot{x}} = 0 \tag{14}$$

Equation (6) is written in (14)

$$\frac{d\underline{\sigma}(x,t)}{dt} = \frac{d\underline{\Phi}(t)}{dt} - [G](\underline{f}(x) + [B]\underline{u}) = 0 \tag{15}$$

Here u is the equivalent control force which is only valid on the sliding surface. Therefore, u_{eq} can be written as

$$\underline{u}_{eq} = [GB]^{-1}\left(\frac{d\underline{\Phi}(t)}{dt} - [G]\underline{f}(x)\right) \tag{16}$$

There are many differences between the real system and the modeled system. Thus, the calculated equivalent control can be completely different the actual equivalent control input. In this study, it is suggested that a low-pass filter is used to calculate the equivalent control by averaging the total control force. The equivalent control force is

$$\underline{\hat{u}}_{eq} = \frac{1}{\tau s + 1}\underline{u} \tag{17}$$

The equivalent control force is only valid on the sliding surface thereby it must be described the control force u which can be reach the system to the sliding surface. For chattering

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free Sliding Mode Controller, the Lyapunov function and its derivate are selected as

$$\underline{v} = \frac{\sigma^T \sigma}{2} > 0 \tag{18}$$

$$\dot{\underline{v}} = -\underline{\sigma}^T \Gamma \dot{\underline{\sigma}} < 0 \tag{19}$$

By equating (19) and derivate of (18)

$$\dot{\underline{\sigma}} + \Gamma \underline{\sigma} = 0 \tag{20}$$

From (20) and (15)

$$\underline{u} = \hat{\underline{u}}_{eq} + \left[GB\right]^{-1} \Gamma \underline{\sigma} \tag{21}$$

where Γ is a positive constant and u is the total control force.

IV. Numerical Results

In order to evaluate the performance of Sliding Mode Controller, the half-bus model is assumed to travel over a bump road input. The bump road input is seen in Fig. 3.

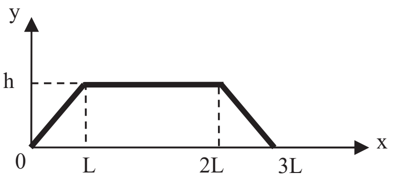

Fig. 3 Bump road input

The bump parameters h and L are given in Appendix. The road input to the rear wheel is the same as the front but delayed by Δt

$$\Delta t = (a+b)/V \tag{22}$$

Here, V is the velocity of the vehicle; (a+b) is the distance between front and rear axles.

A. Time Responses of the Half-Bus Model

The vehicle body bounce and pitch motion and corresponding accelerations are given in Figs. 4 (a)-(d) for controlled and uncontrolled systems. It is clear in these figures that the magnitudes of the body bounce and pitch motion are significantly decreased. Also, the displacements of the controlled suspension system vanish faster than the uncontrolled one. It is shown that the Sliding Mode Controller is significantly improved the ride comfort of the bus.

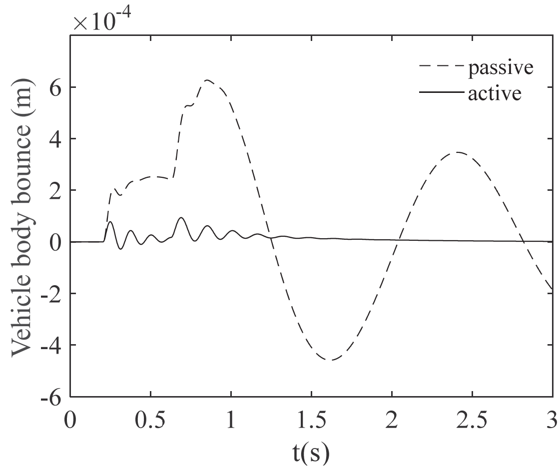

Fig. 4 (a) Time response of the vehicle body bounce

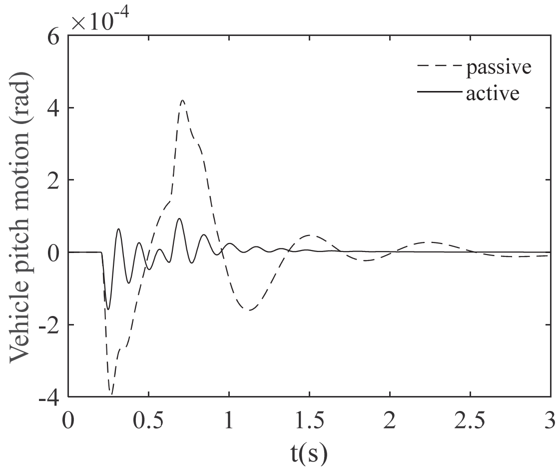

Fig. 4 (b) Time response of the vehicle pitch motion

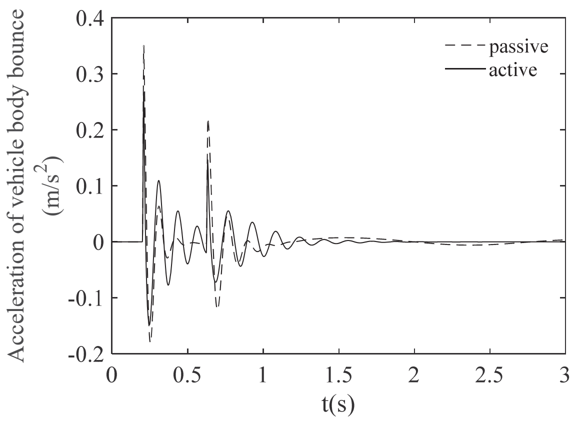

Fig. 4 (c) Time response of the vehicle body bounce acceleration

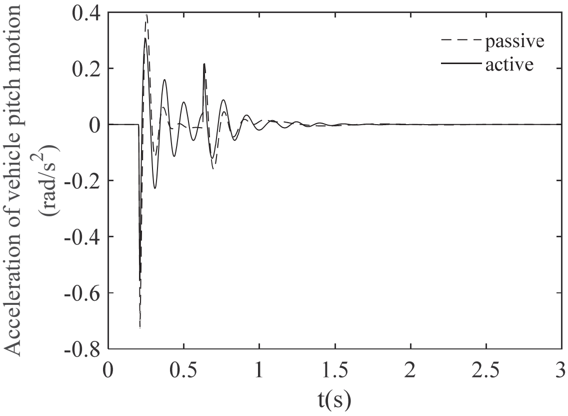

Fig. 4 (d) Time response of the vehicle pitch motion acceleration

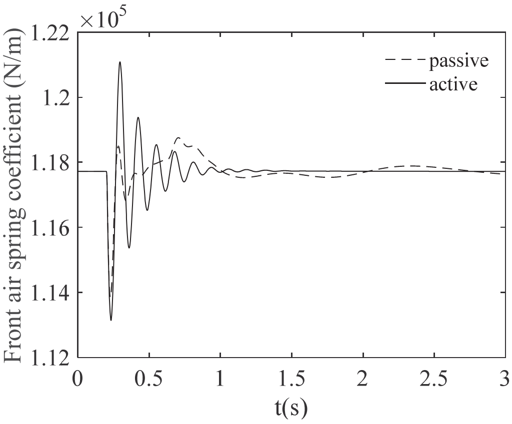

Fig. 5 (a) Time response of the front air spring coefficient

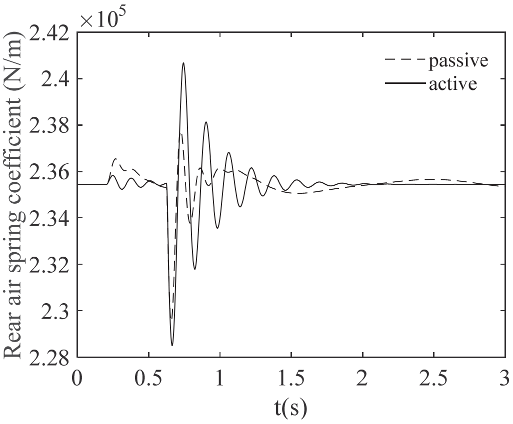

Fig. 5 (b) Time response of the rear air spring coefficient

Time histories for the air spring coefficient are presented in Figs. 5 (a), (b). The air spring coefficients of the controlled suspension system stabilize faster than the uncontrolled suspension system.

Figs. 6 (a) and (b) present the time response of the control force for the front and rear suspension systems. It seen that the front and the rear suspensions apply a maximum control force about 3000-4000N to suppress the effect of the road disturbance.

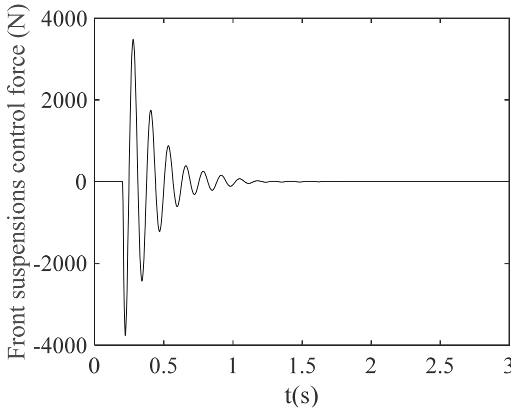

Fig. 6 (a) Time response of the control force for the front suspension system

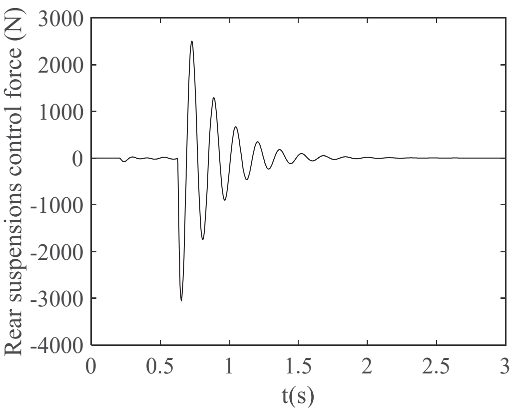

Fig. 6 (b) Time response of the control force for the rear suspension system

B. Frequency Response of the Half-Bus Model

Another way to investigate the ride comfort is the frequency responses. Figs. 7 (a) and (b) present the frequency responses of the vehicle body bounce and pitch motions for both controlled and uncontrolled systems. It is observed that there are two resonance frequencies belonging to the vehicle body bounce and the unsprung masses, respectively in the uncontrolled system. The amplitudes of the resonance frequencies belonging to the vehicle body motions of controlled system decrease. In spite of decreasing the amplitudes of the resonance frequencies of vehicle body motions, controller is not effective in decreasing the amplitude of resonance frequency of unsprung mass. Because the controller is only designed to improve ride comfort. In other words, while the controllers apply the force to improve the ride comfort, the same controllers apply the force in negative direction on unsprung masses.

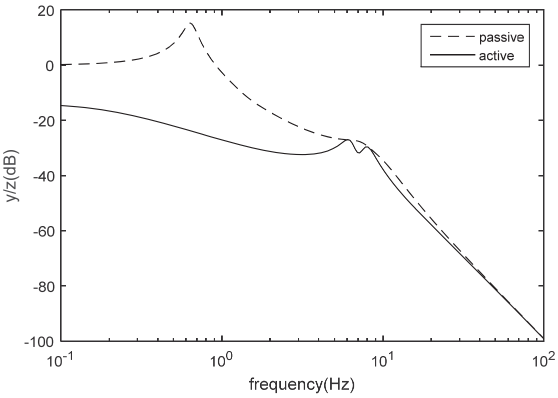

Fig. 7 (a) Frequency response of the vehicle body bounce

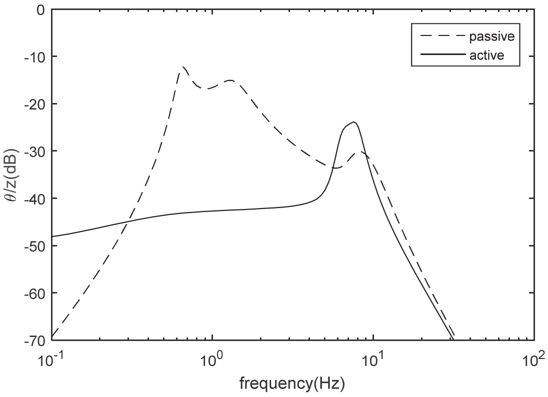

Fig. 7 (b) Frequency response of the pitch motion.

V. Conclusion

In this study, an active suspension system is presented to investigate the effect on ride comfort for a half-bus model with air suspensions. The vibrations, caused by the irregularities of the road surface, are suppressed using Sliding Mode Controller. In order to evaluate the performance of the Sliding Mode Controller, the half-bus model is assumed to travel over a bump road input. As a result, the time and frequency responses indicate the Sliding Mode Control improves the ride comfort of the passengers.

Appendix

TABLE I
Vehicle Parameters

M	*18000 kg*	I	*33127,3 kgm*^{2}
m_f	*286 kg*	m_r	*473 kg*
b_f	*9860 Ns/m*	b_r	*9520 Ns/m*
k_{tr}	*870000 N/m*	k_{tf}	*920000 N/m*
a	*3,853 m*	b	*2,022 m*
V	*20 m/s*		

TABLE II
Air Spring Parameters

n	*1,2*	A_{as}	*0,0314 m*^{2}
V_{ac}	*0,0031416 m*^{2}	h_{as}	*0,4 m*
g	*9,81 m/s*^{2}	n_{ac}	*2*

TABLE III
Bump Road Parameters

h	*0,035m*	L	*0,025 m*

References

[1] Yao, G. Z., Yap, F. F., Chen, G., Li, W. H., Yeo, S.H., 2002, “MR Damper and Its Application for Semi-Active Control of Vehicle Suspension System”, *Mechatronics*, 12, 963-973.

[2] Du, H., Sze, K. Y., Lam, J., 2005, “Semi-Active H∞ Control of Vehicle Suspension with Magneto-rheological Dampers”, *Journal of Sound and Vibration*, 283, 981-996.

[3] Guo, L. X., Zhang, L. P., 2012, “Robust H∞ Control of Active Vehicle Suspension Under Non-Stationary Running”, *Journal of Sound and Vibration*, 331, 5824-5837.

[4] Crosby, M.J., Karnopp, D.C., 1973, “The Active Damper a New Concept for Shock and Vibration Control”, *the Shock and Vibration Bulletin*, 43, 119-133.

[5] Karnopp, D., Crosby, M.J., Harwood, R.A., 1974, “Vibration Control Using Semi-Active Force Generators”, *Journal of Engineering for Industry*, 619-626.

[6] Ekoru, J. E. D., Pedro, J. O., 2013, “Proportional-integral-derivate Control of Nonlinear Half-Car Electro-Hydraulic Suspension Systems”, *Journal of Zhejiang University - Science A (Applied Physics & Engineering),* 14(6), pp. 401-416.

[7] Turkkan, M., Yagiz, N., 2013, “Fuzzy Logic Control for Active Bus Suspension System”, *Journal of Physics*, 410, 1-4.

[8] Taskin, Y., Hacioglu, Y., Yagiz N., 2007, “Use of Fuzzy Logic Control to Improve the Ride Comfort of Vehicles”, *Strojniski Vestnik-Journal of Mechanical Engineering*, 53 (4), 233-240.

[9] Yagiz, N., Hacioglu, Y., 2008, “Backstepping Control of a Vehicle with Active Suspensions”, *Control Engineering Practice*, 16, 1457-1467.

[10] Emelyanov, S.V., 1967, *Variable Structure Control Systems*, USSR, Moscow.Y.

[11] Yagiz, N., Yuksek, I., 2001, “Sliding Mode Control of Active Suspensions for a Full Vehicle Model”, *International Journal of Vehicle Design*, 26, Nos. 2/3, 264-276.

[12] Bal, H., 2006, “Theoretical Investigation of the Performances of the Passive and Semi-active Suspension Systems with Air Spring and Comparison with Mechanical Suspensions”, *Gazi University, Institute of Science and Technology*, M. Sc. Thesis.

[13] Shiong, Z., Jiasheng, W., Ying, Z., 2008, “Research on Theoretical Calculation Model for Dynamic Stiffness of Air Spring with Auxiliary Chamber, Vehicle Power and Propulsion Conference”, *IEEE Vehicle Power and Propulsion Conference*, 3-5 September 2008, Harbin China.

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