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Sinusoidal Roughness Elements in a Square Cavity

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***Abstract*—**Numerical studies were conducted using Lattice Boltzmann Method (LBM) to study the natural convection in a square cavity in the presence of roughness. An algorithm based on a single relaxation time Bhatnagar-Gross-Krook (BGK) model of Lattice Boltzmann Method (LBM) was developed. Roughness was introduced on both the hot and cold walls in the form of sinusoidal roughness elements. The study was conducted for a Newtonian fluid of Prandtl number (Pr) 1.0. The range of Ra number was explored from 10^3 to 10^6 in a laminar region. Thermal and hydrodynamic behavior of fluid was analyzed using a differentially heated square cavity with roughness elements present on both the hot and cold wall. Neumann boundary conditions were introduced on horizontal walls with vertical walls as isothermal. The roughness elements were at the same boundary condition as corresponding walls. Computational algorithm was validated against previous benchmark studies performed with different numerical methods, and a good agreement was found to exist. Results indicate that the maximum reduction in the average heat transfer was 16.66 percent at Ra number 10^5.

***Keywords*—**Lattice Boltzmann Method Natural convection, Nusselt Number Rayleigh number, Roughness.

I. Introduction

BOUYANCY induced natural convection is one of the most active area of research for years. This phenomenon of heat transfer is found in near to far environment of our universe [1]. The heat transfer through natural convection has become one of the most significant mode because of its inherent reliability from cooling of electronics equipment to large scale nuclear reactors [2].

Fluid flows found in the present day study, either in bounded or unbounded geometries come across some degree of surface roughness. Example of such flows are found in; oceanography, geophysics and geology, solar collectors, meteorology, energy systems for buildings, and atmosphere [3]. Surface roughness is considered to be the most effective way to augmenting heat transfer. Enhancement of heat transfer in both laminar and turbulent region of flow using roughness spans over decades [4]. But major studies of the heat transfer during natural convection in the presence of the roughness were performed with rectangular, V-shaped, and square roughness elements. Furthermore, the reported results of such studies showed somewhat conflicting conclusion [4]. Elsherbiny et al. [5] performed experimental studies to observe the role of V-shaped grooves machined on hot plate on natural convection. They considered an enclosure of aspect ratio (W/L) 12 or larger. They used the air as a working fluid with three cases of amplitude 1, 2.5, and 4 given by a ratio of plate spacing (L) to height (H) of V-grooves (L/H). Range of Rayleigh number was explored from 10 to $4x10^6$ with inclination angle of 0 to 60 degree. They concluded that the presence of grooves delayed the onset of natural convection, and degraded the heat transfer up to 50 percent.

Bajorek and LIyod [6] conducted experiments to observe the effects of two partitions on heat transfer during natural convection. They placed two partitions on insulated walls of a square cavity. Both the partitions were in phase. They used air and carbon dioxide as working fluid. They explored thermal and hydrodynamic behavior for Ra number from $1.7x10^5$ to $3.0x10^6$. A reduction in the heat transfer was observed from 12 to 21 percent.

Amin [2] studied the role of rectangular roughness elements. He used a computational code NACHOS based on finite element method in the rectangular enclosures. He [2] considered rectangular enclosures of aspect ratio (W/H) 0.4 to 2.4. He introduced the rectangular roughness elements on bottom adiabatic wall with side walls as isothermal. Newtonian fluids of Pr number 0.72 to 4.52 were considered with a range of Ra number from $1.85x10^2$ to $1.85x10^7$. He observed a maximum reduction in the heat transfer equal to 44 % within an enclosure of aspect ratio 2.4. This was observed at Ra number $1.85x10^4$ for a fluid of Pr number 4.52.

Some significant studies are described above performed using the roughness elements. Most of studies were conducted with rectangular, V-shaped, and square roughness elements. Therefore, present study was focused on the role of sinusoidal roughness elements during natural convection in a square cavity. The numerical studies were conducted using a single relaxation time Bhatnagar-Gross-Krook (BGK) model of LBM in two-dimensions.

II. Description of Work

Computational studies to investigate the effects of sinusoidal roughness elements on natural convection were performed by using computational algorithm based on single relaxation time Bhatnagr-Gross and Krook (BGK) model of LBM. Two-dimensional simulations for a geometry shown in Fig. 1 with corresponding boundary conditions were carried out in a laminar flow region for a Newtonian fluid of Pr number 1.0. The range of Ra number was investigated from 10^3 to 10^6. The dimensionless number called average Nusselt number (Nu), Prandtl number (Pr) and Rayleigh number (Ra) were calculated by using following relations. The average Nu was calculated along vertical walls and through entire domain of the fluid.

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$$Nu_{av} = 1 + \frac{< U.\theta > H}{k\Delta\theta} \quad (1)$$

$$Nu_{av} = \frac{H}{L}\int_0^L \left(\frac{\partial\theta}{\partial x}\right)dy \quad (2)$$

$$Ra_L = \frac{g\beta\Delta TH^3}{\upsilon\alpha} \quad (3)$$

$$P_r = \frac{\upsilon}{\alpha} \quad (4)$$

$$\theta = \frac{T - T_{ref}}{T_h - T_c} \quad (5)$$

Lattice Boltzmann method which was originated from lattice gas automata has a proven ability to be an alternative to traditional numerical schemes based on finite volume, finite difference, and finite elements methods [7], [8]. Main advantages of LBM are; easy to make algorithm, easy to treat complex boundaries, local computing, and no solution of Laplace equation at every time step [9]. LBM was first introduced in 1980s, and has demonstrated significant performance to simulate single and multiphase flow, condensation and evaporation, and buoyancy induced flows with complex geometries. The fundamental equation which simulates Navier-Stokes and energy equations based on hypothetical particle given by Boltzmann equation is shown in (6);

$$f_i(x+e_i\Delta t,t+\Delta t)= f_i(x,t)-\frac{\left(f_i(x,t)-f_i^{eq}(x,t)\right)}{\tau}+\Delta tF_{ext} \quad (6)$$

'F_{ext}' is external force and 'τ' is relaxation time. In the present study, nine velocity (D2Q9) and five temperature (D2Q5) directions were considered in two-dimensional study of the natural convection flow and heat transfer. Boussinesq approximations were used to incorporate the effects of buoyancy in external force term. All other properties were considered constant. The value of relaxation factors for Navier-Stokes and energy equations are calculated using kinematic viscosity and thermal diffusivity.

Proper boundary conditions are essential to have accurate numerical results and to achieve fast convergence. For velocities on wall, bounce back boundary conditions were implemented on all walls and solid nodes present in the cavity. The isothermal boundary conditions were implemented on hot and cold walls respectively as illustrated by Sukop and Thorne [9]. The horizontal walls were considered adiabatic or insulated.

III. BENCHMARKING

An algorithm based on a single relaxation time Bhatnagr-Gross and Krook (BGK) model of LBM was validated against previous benchmark solutions. In the first step, grid independence study was carried out and comparison was made as shown in Table I. A comparison showed that as the number of grid points was increased, values of the average Nusselt number improved. A mesh of 700x350 was reasonable and hence was selected for Ra number 10^4 and large mesh was adopted for larger values of Ra number.

TABLE I
GRID INDEPENDENCE STUDY FOR AVERAGE NU

Ra	H	Present	Davis [10] % Error
	200	2.2241	0.8426
10^4	250	2.2306	0.5528
	350	2.2381	0.2185

In the second step, results of the average Nu were compared with benchmark solution of [10] as shown in Fig. 2. The geometry for comparison purpose is same as shown in Fig. 1 but no roughness elements were present on the hot and cold walls.

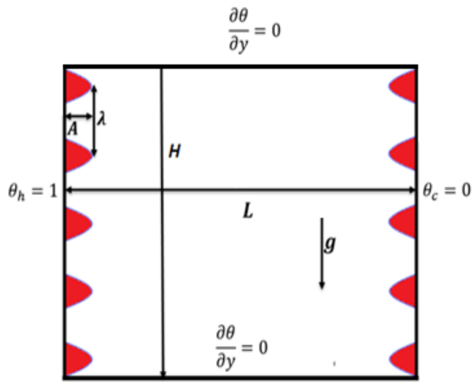

Fig. 1 A Square Cavity with Sinusoidal Roughness

A comparison study was performed for grid independence and for a range of Ra number with previous benchmark solution. Based on the comparison above, numerical results of present computational code can be considered as mesh independent and reliable within a good accuracy.

Simulations for square cavity with the roughness present on the both the hot and cold walls were performed up to Ra number 106. Simulations were performed using Intel (R) core (TM) i7-4820k CPU@3.7GHz with 32GB RAM.

IV. RESULTS AND DISCUSSION

Numerical results for a square cavity with sinusoidal roughness elements present on both the hot and cold walls in two-dimensional system are shown in Fig. 3. Many simulations were carried out to analyze the behavior of the fluid and the heat transfer in the smooth as well as in the rough cavity. But only some significant results are reported here. Results for the average value of Nu are compared with those of smooth cavity. A comparison of average Nu of both cases i.e. smooth and rough showed that amount of heat transfer was significantly degraded, when the roughness was present on the both walls simultaneously. The decrease in heat transfer was more at large Ra number as compared to small Ra. The deviation of the average Nu up to Ra number 10^4 is small. The deviation started increasing as Ra number was increased beyond 10^4. The maximum degradation in the average heat transfer in the presence of the sinusoidal roughness elements was calculated to be 16.66 percent at Ra number 10^5.

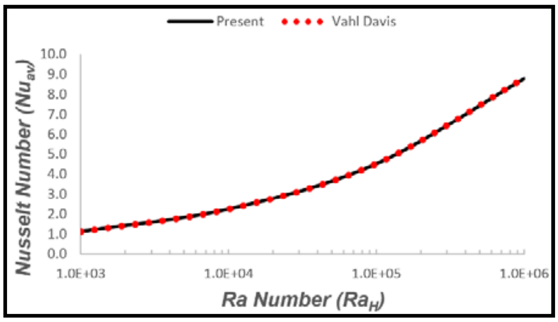

Fig. 2 A Comparison of Present values of Average Nu with Vahl Davis [10]

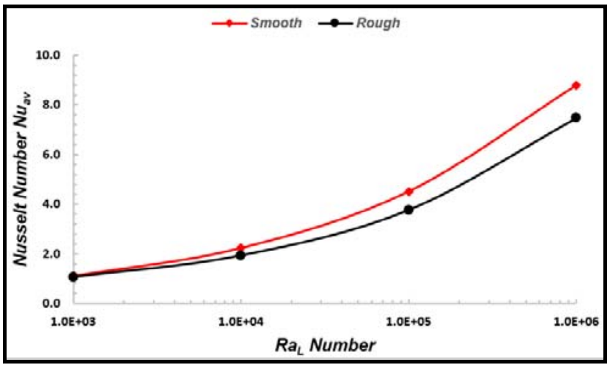

Fig. 3 Average Values of Nu for Smooth and Rough Cavity

An effort was made to explain some possible factors affecting the average heat transfer in the presence of roughness, which are also reported by the previous studies [4], [11], [12] with different shapes of the roughness elements. First, the presence of the roughness causes a decrease in average distance between two isothermal or vertical walls of the cavity. This decrease in the distance cause a decrease in the average or effective value of Ra number and hence heat transfer. Secondly, a possible factor affecting the average heat transfer may be the reduction in the fluid volume due to the roughness as compared to smooth cavity. Furthermore, the presence of the roughness elements causes an addition in the surface area for heat transfer as compared to smooth. But this addition of area in fact causes obstruction to velocity flow. This obstruction in the velocity may cause a significant decrease in the average heat transfer.

V. Streamlines and Isotherms

The presence of the sinusoidal roughness elements significantly affects the fluid flow and heat transfer in the cavity. Streamlines and isotherms are compared with smooth cavity results at same Ra numbers in order to make a better comparison.

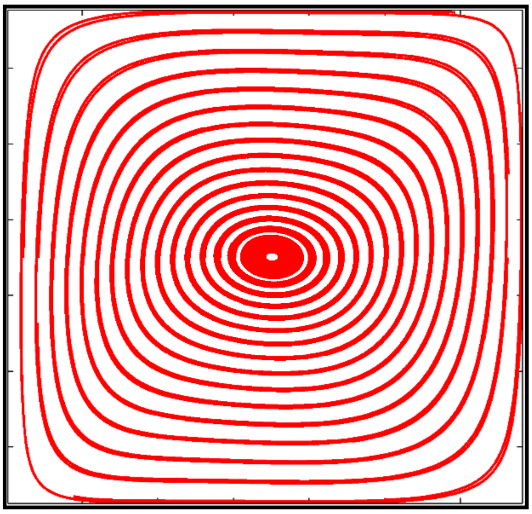

Fig. 4 Streamlines for Ra-10^4 for Smooth Cavity

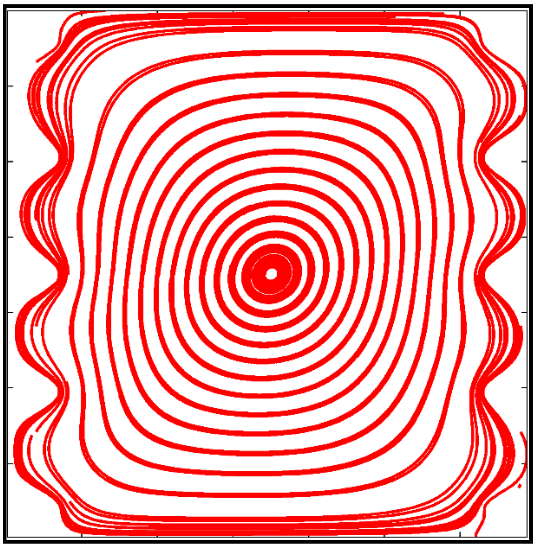

Fig. 5 Streamlines for Ra-10^4 for Rough Cavity

In a square cavity, with isothermal vertical walls and insulated horizontal walls, fluid flows upward along vertical hot wall and downward along cold wall. The central part of the cavity changes its shape with variation of Ra number. For small Ra number, mode of the heat transfer is dominated by conduction as compared to convection. The fluid, which is close to the isothermal hot wall having less density compared to fluid upwards, started rising upward. This upward rising fluid then moves towards the cold wall along insulated wall. Figs. 4 and 5 showed that streamlines behavior for Ra number 10^4 for smooth and rough cavity respectively. The streamlines behave in the same manner in both cavities. Central part circulation is also same in shape and size. The central part shrunk in size in case of the rough cavity due to the presence of the roughness elements. When the Ra number was increased beyond the 10^4, this increase in the Ra number produced an increase in the buoyancy force and hence results in the increase in the velocity of fluid. The central part of cavity remained dominated by conduction mode, and convection effects shrink towards isothermal cold and hot walls. This is clear from Figs. 6 and 7 for both smooth and rough cavities. The local circulations in the core area remained same for both the smooth and rough cavities. Streamlines were shifted toward the isothermal walls with increase in Ra number. The straight streamlines behavior presented the dominance of the conduction phenomenon as compared to the convection.

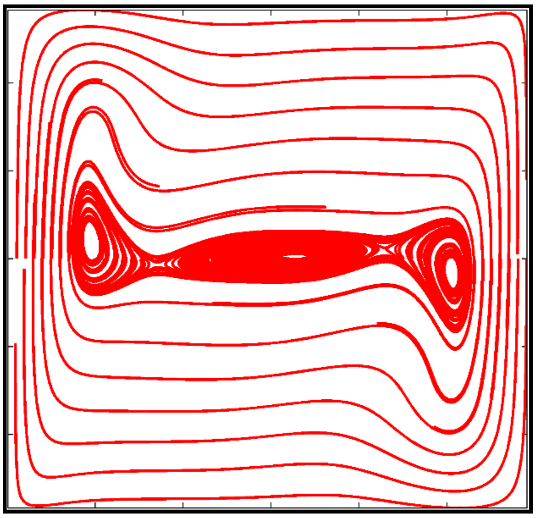

Fig. 6 Streamlines for Ra-10^6 for Smooth Cavity

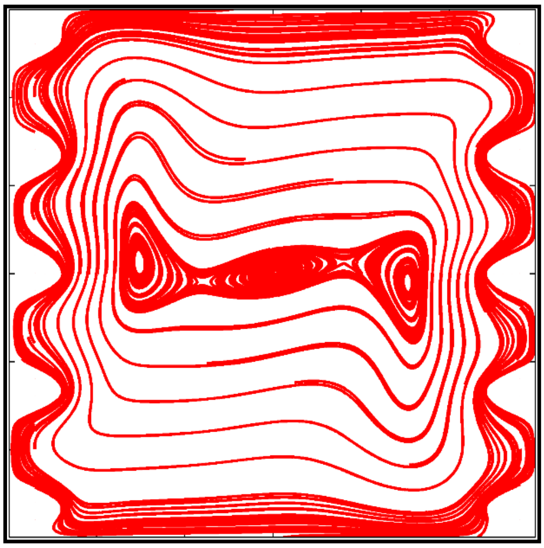

Fig. 7 Streamlines for Ra-10^6 for Rough Cavity

Thermal behavior of the Newtonian fluid was also affected by the presence of the roughness elements. At lower Ra number < 10^4, fluid behavior was same as in case of smooth cavity. Therefore, isothermal behavior is only shown for Ra number equal to 10^4 and 10^6. At Ra number equal to 10^4, isotherms behavior in the smooth and rough cavities was almost same except some distortion due to the presence of the roughness elements. This is very clear from Figs. 8 and 9. The effects of the roughness elements are more pronounced at lower half of the hot wall and upper half of the cold wall. The stratification in the isotherms for both cases is same.

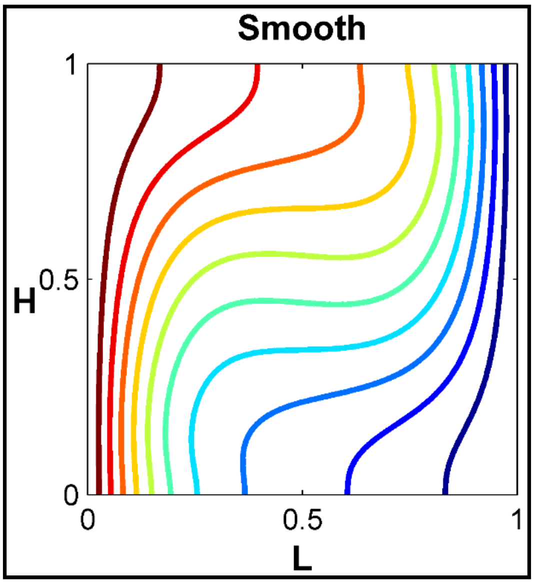

Fig. 8 Isotherms for Ra-10^4 for Smooth Cavity

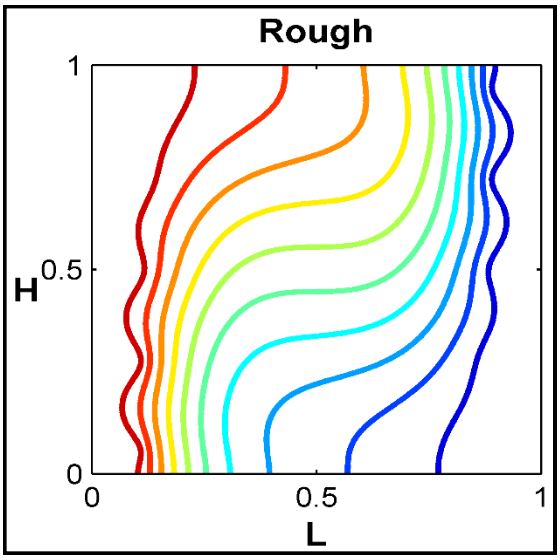

Fig. 9 Isotherms for Ra-10^4 for Rough Cavity

When Ra number was increase to 10^6, the isotherms behavior in the cavities changed as compared to that of Ra number 10^4. Isotherms started shifting toward the hot and cold walls of the smooth cavity. This behavior of shifting towards the isothermal walls is almost same in the rough and smooth cavity. But in case of roughness elements present on the isothermal walls, there is a constriction of the isotherms. This constriction was not observed for isotherms at Ra number 10^4.

Moreover, with the increase in the Ra number, isotherms in the core or central part of the cavity are almost straight. This straightness in the isotherms behavior shows the dominance of the conduction in the core area.

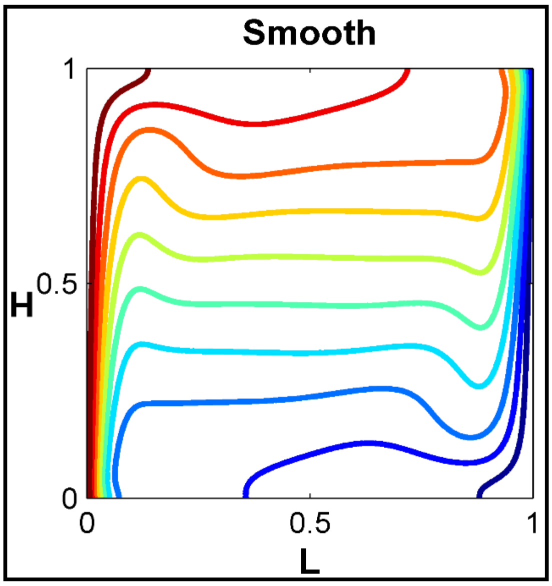

Fig. 10 Isotherms for Ra-106 for Smooth Cavity

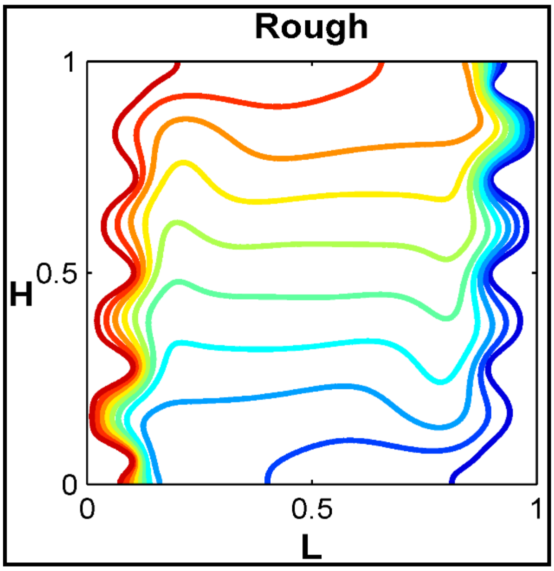

Fig. 11 Isotherms for Ra-106 for Rough Cavity

VI. Conclusion

Computational studies were conducted for a square cavity with smooth and sinusoidal roughness elements for a Newtonian fluid of Pr number 1.0 using single relaxation time BGK model of LBM in two-dimensions. Numerical results indicate that hydrodynamic and thermal behavior is significantly affected due to the presence of sinusoidal roughness elements on hot and cold walls. Sinusoidal roughness elements cause a reduction in the average heat transfer in the cavity up to 16.66% as compared to smooth. Most significant phenomenon playing its role in the reduction of heat transfer as compared to smooth cell seems to be reduction in the fluid volume, decrease in the effective distance between two walls, and obstruction in the velocity of the fluid in the cavity.

Nomenclature

lu	Lattice unit
ρ	Density
β	Thermal coefficient
T_w	Wall or surface temperature
T_{ref}	Reference or Mean temperature
Δt	Time Step
υ	Kinematic viscosity
α	Thermal diffusivity
f	Density Distribution function

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