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Simulation of Particle Damping under Centrifugal Loads

Riaz A. Bhatti, and Wang Yanrong

***Abstract*—**Particle damping is a technique to reduce the structural vibrations by means of placing small metallic particles inside a cavity that is attached to the structure at location of high vibration amplitudes. In this paper, we have presented an analytical model to simulate the particle damping of two dimensional transient vibrations in structure operating under high centrifugal loads. The simulation results show that this technique remains effective as long as the ratio of the dynamic acceleration of the structure to the applied centrifugal load is more than 0.1. Particle damping increases with the increase of particle to structure mass ratio. However, unlike to the case of particle damping in the absence of centrifugal loads where the damping efficiency strongly depends upon the size of the cavity, here this dependence becomes very weak. Despite the simplicity of the model, the simulation results are considerably in good agreement with the very scarce experimental data available in the literature for particle damping under centrifugal loads.

***Keywords*—**Impact damping, particle damping, vibration control, vibration suppression.

I. Introduction

PARTICLE impact damping (PID) is a passive mean of vibration control in which small metallic or ceramic particles are placed inside a cavity that is attached to the main structure at the place of high vibration amplitudes. Vibration energy is dissipated as a result of frictional losses and momentum exchange, when particles make inelastic collisions with the walls of the cavity and other particles. This method is particularly suitable in extreme environments [1] as there is a very little chance of surface degradation or loss of material properties as opposed to the case of more traditional ways of vibration damping, for example, constrained layer damping, frictional devices etc. The damping is achieved as long as the particles are capable of moving and making impacts. However, in environments such as encountered in a gas turbine engine, there are not only very high temperatures involved but the structures have to go through tremendous centrifugal loads also. So, the challenging question arises whether the particles will be able to move and make impacts under such high centrifugal loads or not? Eric M. Flint [2] has shown by performing a series of experiments on particle damping under varying centrifugal loads that this damping technique indeed works in such harsh environments. So far, no effort has come forward that can prove analytically or numerically that the particle damping is a viable solution for vibration control under high centrifugal loads. We believe that the particles cannot move by sliding as the sliding friction is very high because of high centrifugal force involved. However, we do believe that particles are still capable of moving but through pure rolling motion as the effective coefficient of rolling friction is usually smaller than coefficient of sliding or Coulomb friction. Based on this basic idea, we present a simple analytical model for particle damping of two dimensional transient vibrations in structures operating under centrifugal loads. The model takes into account both the normal as well as oblique impacts, so the impact friction has been taken care of. The model is implemented by writing a code in Matlab and the simulation results are compared with the reported experimental results [2]-[3]. Looking at the simplicity of the model, the results are very encouraging as the agreement between the theory and the experiment is remarkable. We believe that the method developed can provide a good initial guess for implementation of particle damping under centrifugal loads to minimize the expensive trial and error experimental testing.

R. A. Bhatti is currently a PhD. degree student at School of Jet Propulsion, Beijing University of Aeronautics and Astronautics, 37-Xueyuan Road, Beijing, China (phone: 0086-13240410683; e-mail: riazabhatti@gmail.com).

Y. Wang is a professor in School of Jet Propulsion, Beijing University of Aeronautics and Astronautics, 37-Xueyuan Road, Beijing, China (e-mail: yrwang@buaa.edu.cn).

II. Mathematical Modeling

A. Basic Model and Governing Differential Equations

The model consists of a two degrees of freedom (2 dof), uncoupled spring-mass system with an internal rectangular cavity of length ℓ_1 and width ℓ_2 as shown in Fig. 1. The cavity holds a single particle of effective mass m_p and radius r. A constant centrifugal force *N* is acting on the system in a direction that is perpendicular to the plane of vibration as shown in the Fig. 1. The origin of coordinates lies at the center of the cavity. When the system is disturbed from its static equilibrium position, it starts vibrating in its fundamental mode. The effective (modal) stiffness *K* and effective mass *M* of the 2 dof system depend upon the condition whether the effective particle m_p is moving or not. Due to presence of large centrifugal force *N*, we assume that the particle is never able to overcome the frictional force (Coulomb force) developed so and hence cannot slide.

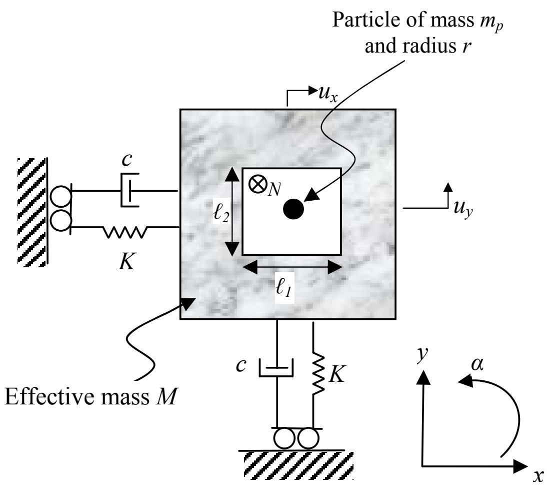

Fig. 1 A schematic of 2 degrees of freedom particle damping system under centrifugal load

However, the inertial force acting on the particle can be of such value that it can overcome the force developed due to rolling friction. Therefore, the particle can move by undergoing pure rolling motion and hence has the chance to make impact with the walls of the cavity. We further assume that due to presence of centrifugal force, the particle can never leave the floor (plane of the paper) of the cavity and its axis of rotation always lies in the plane of the paper. Initially, let us suppose that the particle is lying at the center of the cavity. Let the structure is released from rest with the initial displacements u_{xo} and u_{yo} from the static equilibrium position in the x and y directions respectively. For subsequent motion of the particle m_p and the structure, we will have two cases.

Case 1: Inertial force acting on $m_p \leq$ Force due to rolling friction.

In this case the particle remains stick to the structure and moves in the same way as the structure moves in both x and y directions. If in this case the effective mass and effective stiffness of the system is denoted by M_1 and K_1 respectively then its circular frequency will be given by $\omega_1 = \sqrt{\frac{K_1}{M_1}}$. If the displacements in x and y directions are given by u_x and u_y respectively then governing differential equations of motion both for the particle and the structure are given by:

$$\ddot{u}_i + 2\zeta\omega_1\dot{u}_i + \omega_1^2 u_i = 0 \text{ where } i = x, y \tag{1}$$

In the above equation, the over dots show the time derivatives and ζ is the intrinsic damping ratio of the material.

Case 2: Inertial force acting on $m_p >$ Force due to rolling friction.

The particle will start rolling. In this case, the effective mass of the system and its effective stiffness will be denoted by M_2 and K_2 respectively and its circular frequency will be given by $\omega_2 = \sqrt{\frac{K_2}{M_2}}$. Note that $m_p = M_1 - M_2$. If the angular displacements of the particle in x and y directions are $\alpha_{x,p}$ and $\alpha_{y,p}$ respectively and its linear displacements in these directions are $u_{x,p}$ and $u_{y,p}$ respectively, then the differential equation governing the motion of m_p is given by:

$$I\ddot{\alpha}_{i,p} = rm_p\ddot{u}_i - r\frac{\mu_r}{r}N\,\mathrm{sgn}(\dot{u}_i - \dot{u}_{i,p}) \text{ where } i = x, y \tag{2}$$

In the above equation, μ_r is the coefficient of rolling friction and I is the moment of inertia of the particle and is given by $I = \frac{2}{5}m_p r^2$. Note that μ_r has the dimension of length. The differential equation governing the motion of M_2 is given by:

$$\ddot{u}_i + 2\zeta\omega_2\dot{u}_i + \omega_2^2 u_i = -\frac{\mu_r}{M_2 r}N\,\mathrm{sgn}(\dot{u}_i - \dot{u}_{i,p}) \quad i = x, y \tag{3}$$

The term $\mathrm{sgn}(\dot{u}_i - \dot{u}_{i,p})$ on right sides of (2) and (3) is included to ensure that the sign of μ_r is always such that it opposes the relative velocity of the particle and the structure. Also for pure rolling motion of the particle, we must have to satisfy the following two equations also:

$$\ddot{u}_{i,p} - \ddot{u}_i = -r\ddot{\alpha}_{i,p} \text{ where } i = x, y \tag{4}$$

$$\dot{u}_{i,p} - \dot{u}_i = -r\dot{\alpha}_{i,p} \text{ where } i = x, y \tag{5}$$

During course of its motion, the particle m_p can make an inelastic impact with any of the cavity walls. Table I gives the criterion for the particle to make an impact with a particular wall of the cavity. If after impact, the ratio of the velocity of the particle relative to M_2 to the velocity of M_2 is less or equal to a tolerance ε (we chose $\varepsilon = 10^{-2}$ as with this value the error in calculation of damping was negligible but number of impacts reduced appreciably) then we call it a sticking condition i.e. the particle and the structure move together in the same way as governed by (1). Table I also gives the separation criteria for the particle when it sticks to a particular wall after an impact. In Table I, a_x and a_y stand for acceleration of structure in x and y directions respectively. After separation the motion of m_p and M_2 is governed by (2) to (4).

B. Modeling of Impacts

In order to model the impact between the particle m_p and the effective mass M_2, we invoke the conservation

TABLE I
IMPACT AND SEPARATION CRITERIA FOR THE PARTICLE.

Wall of the cavity	Impact criteria	Separation criteria after sticking
Left wall	$u_{x,p} \le u_x - \ell_1/2 + r$	$-m_p a_x > abs[\mu_r \operatorname{sgn}(\dot{u}_x - \dot{u}_{x,p})N]/r$
Right wall	$u_{x,p} \ge u_x + \ell_1/2 - r$	$m_p a_x > abs[\mu_r \operatorname{sgn}(\dot{u}_x - \dot{u}_{x,p})N]/r$
Bottom wall	$u_{y,p} \le u_y - \ell_2/2 + r$	$-m_p a_y > abs[\mu_r \operatorname{sgn}(\dot{u}_y - \dot{u}_{y,p})N]/r$
Top wall	$u_{y,p} \ge u_x + \ell_2/2 - r$	$m_p a_y > abs[\mu_r \operatorname{sgn}(\dot{u}_y - \dot{u}_{y,p})N]/r$

of linear momentum in normal and tangential directions (with respect to the walls of the cavity) together with the coefficient of restitution e and coefficient of dynamic friction μ. Note that our 2 degrees of freedom, uncoupled system is momentarily coupled at an impact event. The Conservation of linear momentum in normal direction gives:

$$m_p\dot{u}^+_{n,p} + M_2\dot{u}^+_n = m_p\dot{u}^-_{n,p} + M_2\dot{u}^-_n \tag{6}$$

Conservation of linear momentum in tangential direction gives:

$$m_p\dot{u}^+_{t,p} + M_2\dot{u}^+_t = m_p\dot{u}^-_{t,p} + M_2\dot{u}^-_t \tag{7}$$

In (6) and (7), the subscript *n* and *t* stand for normal direction and tangential direction respectively. Likewise, superscripts "-" and "+" indicate the velocities before and after the impact respectively. Note that subscripts *n* and *t* will be replaced with *x* and *y* respectively when an impact occurs with left or right wall and they will be replaced with subscripts *y* and *x* respectively when an impact occurs with top or bottom wall (in Fig. 1). The coefficient of restitution e ($0 \le e \le 1$) is defined as the negative of the ratio of the relative velocity between the effective mass M_2 and the particle after the impact to the relative velocity before the impact:

$$\dot{u}^+_n - \dot{u}^+_{n,p} = -e(\dot{u}^-_n - \dot{u}^-_{n,p}) \tag{8}$$

The fourth and the last equation to model the impact is given by [4]:

$$m_p\dot{u}^+_{t,p} + \mu M_2\dot{u}^+_n = m_p\dot{u}^-_{t,p} + \mu M_2\dot{u}^-_n \tag{9}$$

Equation (9) is the consequence of the definition of coefficient of dynamic friction μ given by $\mu = P_t / P_n$. The quantities P_n and P_t are the impulses developed by the normal and tangential force components at the contact point respectively [4]. Please note that the coefficient μ in (7) and (9) is introduced to model the impacts only. Solving (6), (7), (8) and (9) simultaneously, we get the velocities after the impact:

$$\begin{aligned}
\dot{u}^+_{n,p} &= \dot{u}^-_{n,p} + \overline{m}(1+e)(\dot{u}^-_n - \dot{u}^-_{n,p})/m_p \\
\dot{u}^+_n &= \dot{u}^-_n - \overline{m}(1+e)(\dot{u}^-_n - \dot{u}^-_{n,p})/M_2 \\
\dot{u}^+_{t,p} &= \dot{u}^-_{t,p} + \mu\overline{m}(1+e)(\dot{u}^-_n - \dot{u}^-_{n,p})/m_p \\
\dot{u}^+_t &= \dot{u}^-_t - \mu\overline{m}(1+e)(\dot{u}^-_n - \dot{u}^-_{n,p})/M_2
\end{aligned} \tag{10}$$

In the above equation, $\overline{m} = m_pM_2/(m_p + M_2)$. Although, μ is an arbitrary constant, the tangential force and its impulse usually are found to be dissipative, such as from friction and cannot add energy to the system. The tangential impulse opposes the initial relative tangential motion. Therefore, μ can be either positive or negative and its sign is given by: $\operatorname{sgn}(\mu) = \operatorname{sgn}[(\dot{u}^-_t - \dot{u}^-_{t,p})/(\dot{u}^-_n - \dot{u}^-_{n,p})]$. Also, for positive energy loss, the value of μ is subject to the following condition [4]:

$$abs(\mu) \le abs[(\dot{u}^-_t - \dot{u}^-_{t,p})/\{(\dot{u}^-_n - \dot{u}^-_{n,p})(1+e)\}] \tag{11}$$

The simulation runs through time in small time steps Δt. In order to track the motion of the particle and the structure accurately, we chose Δt such that there are 40 time steps in a complete vibration cycle. At the end of each time step, the sate of the system is used as the initial condition for the next time step. Note that contribution to velocities due to rotation of the particle before and after an impact does not appear explicitly in our derivation. This is due to the reason that rotation does not occur in the plane of vibration.

C. Damping

In the context of particle damping, the damping is defined in terms of specific damping capacity (SDC) Ψ [5]:

$$\Psi = (\sum_{i=1}^{s} \Delta T^i)/T \tag{12}$$

Here s is total number of impacts occurred during one vibration cycle and T is maximum energy in that cycle. ΔT^i is the energy dissipated in the i_{th} impact. A cycle is defined to be the duration between two successive maxima V of the structure's effective mass velocity, *v(t)*, curve. Note that $v(t) = \sqrt{\dot{u}_x^2 + \dot{u}_y^2}$. The kinetic energy T will be maximum at the start of each cycle and is given by: $T = \frac{1}{2} M_1 V_i^2$. The energy dissipated in the i_{th} cycle can be written as: $\sum \Delta T^i = \frac{1}{2} M_1 (V_i^2 - V_{i+1}^2)$. A typical velocity vs time curve for a structure going through transient vibrations under centrifugal loads and damped by a particle damper is shown in Fig. 2. To make the data analysis more meaningful, we need to define few more dimensionless quantities. If g is the acceleration due to gravity, then dimensionless clearance of the cavity in x and y directions is defined as $\Delta_x = \omega_1^2 (\ell_1 - 2r)/g$ and $\Delta_y = \omega_1^2 (\ell_2 - 2r)/g$ [5] respectively. Likewise we define dimensionless acceleration amplitude as $\Gamma = V\omega_1 / g$ and mass ratio as m_p / M_2.

III. Simulation Results and Analysis

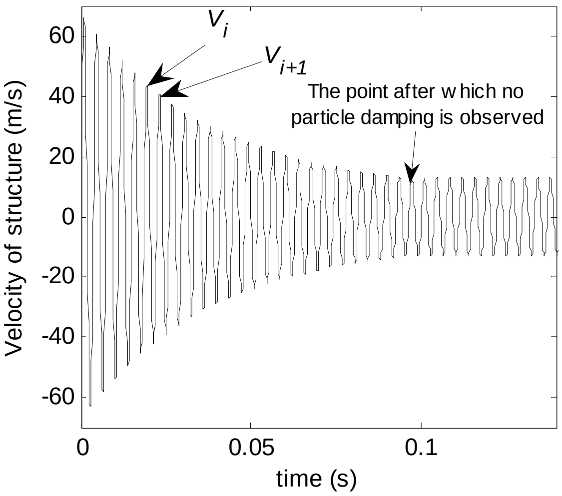

Fig. 2 Velocity vs time curve of the structure at 10000 RPM. The structure is damped by a particle damper

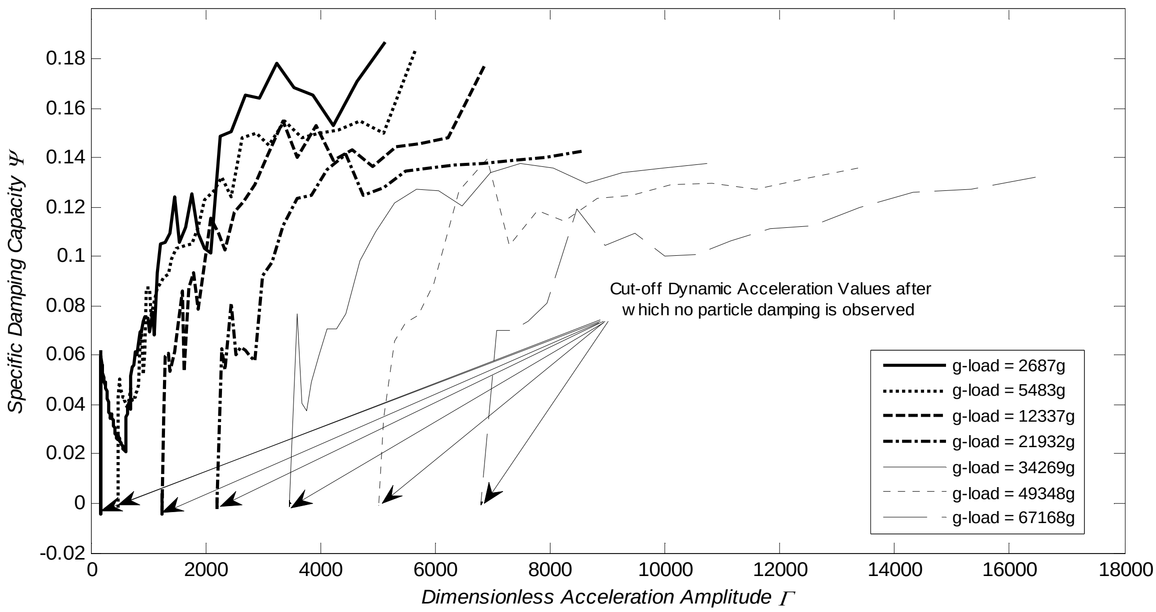

Fig. 3 Effect of centrifugal or g-loading on the performance of particle damper. mass ratio= 0.6%, r =1mm, e =0.5, μ=0.3, μ_r=0.1mm

A. *Effect of Centrifugal Loading on Particle Damper*

In this section we will discuss the effect of centrifugal or g-loading on the performance of the particle damper. It is worth mentioning here that as the centrifugal loading is increased, the fundamental frequency of the structure also increases. For convenience, we have chosen a plate like structure. The length, width and thickness of the plate is 20 cm, 8 cm and 1cm. To find the fundamental mode frequency, we used the ANSYS software and it came out to be 198 Hz. To find the fundamental frequency in the presence of g-loading, we first performed the static analysis with centrifugal loading and then used the results to perform the modal analysis. The analysis results are given in Table II. Fig. 3 shows the damping performance results of the particle damper under centrifugal loads as predicted by our model. It can be seen from this figure that as the centrifugal loading is increased, the value of the dynamic acceleration (shown by cut-off point in Fig. 3) of the plate at which the particle damper ceases to perform also increases. We call this value of dynamic acceleration as the cut-off point. The ratio of the dynamic acceleration at cut-off to the applied centrifugal load is around 0.1. This ratio along with the cut-off dynamic acceleration values are also given in Table II. In fact, the cut-off value of the acceleration is the value after which the particle does not have enough energy to overcome the centrifugal force, so, it stops moving and cannot make impacts. These results are in accordance with the published experimental data [3]. The specific damping capacity curves in Fig. 3 also show that as the centrifugal loading is increased, the damping performance slightly decreases. As shown in this figure, for g-loading range of 2687g to 67168g, the specific damping capacity drops from 18% to 14%. Intuitively this seems to be correct because, as the centrifugal loading is increased, it is difficult for the particle to move and hence make impacts. From this point of view, the results are also according to the published data [2].

TABLE II
EFFECT OF CENTRIFUGAL LOADING ON PARTICLE DAMPING

Centrifugal Loading (g)	Fundamental frequency (Hz)	Cut-off acceleration (g)	Ratio of Dynamic acceleration to g-loading
2687	208.9	191	0.07
5483	218.8	493	0.09
12337	241.3	1303	0.11
21932	269.5	2211	0.10
34269	301.7	3465	0.10
49348	336.7	5028	0.10
67168	373.5	6819	0.10

B. Effect of Mass Ratio

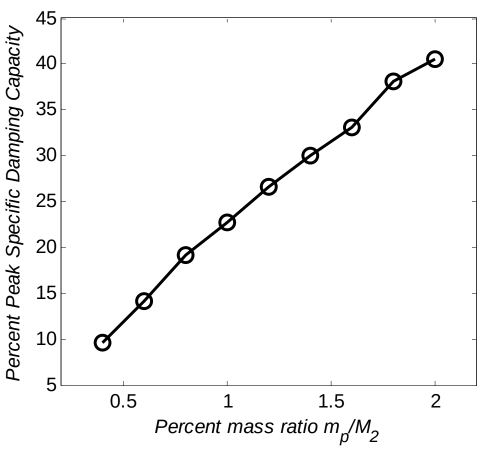

Fig. 4 Effect of changing mass ratio on damping performance of particle damper under centrifugal load of 22000 g. Δ_H=145, Δ_V=116, r =1mm, e = 0.5, μ = 0.3, μ_r= 0.1mm

In this section we present the model predictions regarding the effect of mass ratio i.e. the ratio of the mass of the particle to the mass of the structure. Results are shown in Fig. 4. These results are given for 10000 RPM or equivalently 22000g centrifugal loading condition. Results show that as the mass ratio increases, the percent peak specific damping capacity also increases in almost a linear fashion. The reason for this behavior is that as the mass ratio increases, the momentum between the particle and the structure during an inelastic collision also increases and this leads to a greater damping value. The results are very much in accordance with the publish data [5]-[11].

C. Effect of Cavity Size

Model predictions regarding the effect of cavity size on performance of particle damper are shown in Fig. 5.

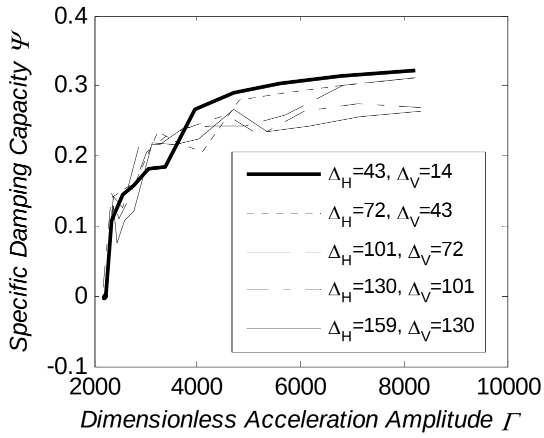

Fig. 5 Effect of changing cavity size on damping performance of a particle damper. centrifugal-load =22000g, r =1mm, e = 0.5, μ = 0.3, μ_r= 0.1mm, mass ratio= 1.2 %

The results show that for lower values of acceleration amplitude, the damping curves are overlapping. However, for large values of dimensionless acceleration amplitude, the damping slightly decreases i.e. for almost 5 times increase in cavity, there is a decrease of just 5% in damping. Therefore, as a whole we can say that change of the cavity size does not significantly change the damping performance. At present, no published data are available that comment on this aspect of particle damping under centrifugal loads. However, lot of literature is available that shows that in absence of centrifugal loading, if cavity size is increased, the damping performance increases [5]-[8]. This is especially true for a particle damper working in a vertical plane along the direction of the gravity. For a particle damper working in a horizontal plane transverse to the direction of gravity, damping performance does not strongly depend upon the cavity size [12]. In a vertical damper, the gravity is one of the controlling parameters for particle motion where as in a horizontal particle damper; the particles are free to move. In both cases, increase in cavity size provides more space to the particle to make rigorous collisions. In the present case however, the particle are bound to move on the floor of the cavity under tremendous influence of centrifugal force. So bigger cavity size does not really matter, rather, smaller cavity size perhaps contributes more towards damping by added friction. Anyhow, if the model prediction is true then this result is very important as in a centrifugal environment, there is usually a very little space for construction of large cavities.

D. Effect of Initial Displacement

Fig. 6 shows the dependence of specific damping capacity on initial displacement of vibration. Vertical dotted line is drawn such that all specific damping capacity curves pass through it. It can be seen that for a particular value of dimensionless acceleration amplitude, the specific damping capacity is not same rather it depends (albeit slightly) on the initial amplitude. An excellent explanation of this behaviour was given by Friend and Kinra [5]. Interested reader is suggested to consult this reference. We have presented these results to support the validity of our model.

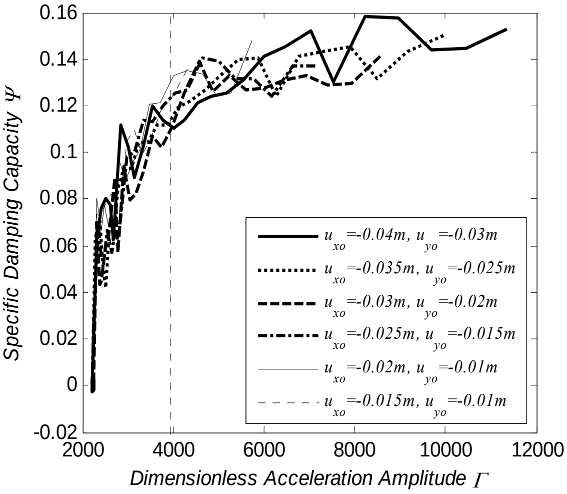

Fig. 6 Effect of initial amplitude on damping performance of a particle damper working under centrifugal loads. Centrifugal load=22000g, r =1mm, e = 0.5, μ =0.3, μ_r = 0.1mm, mass ratio= 0.6 %

E. Effect of Radius of Particle

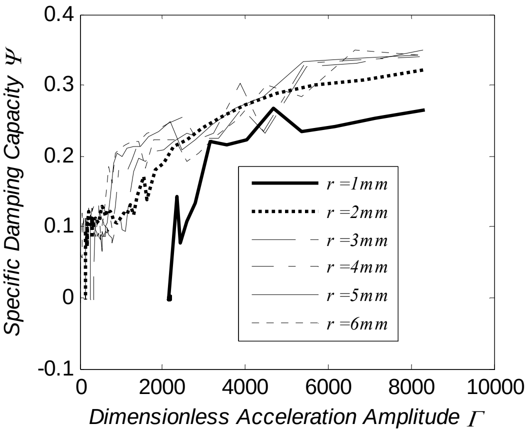

Fig. 7 Effect of radius of the particle on performance of the particle damper. Centrifugal load = 22000g, e = 0.5, μ = 0.3, μ_r =0.1mm, mass ratio = 1%

Numerical results showing the dependence of damping performance of the two dimensional particle damper on size or radius of the particle are shown in Fig. 7. These results are complementing the results of the section in which effect of cavity size is discussed. Discussion done over there is also valid for this section. Actually, for a fixed size of the cavity, an increase in the size of the particle results in the reduction of the clearance between the particle and the cavity wall.

IV. Conclusion

The two dimensional particle damping model developed to suppress the transient vibrations in structures operating under high centrifugal loads, explains the essential features of particle damping under centrifugal loads. Simulation results regarding the effect of cavity size on performance of particle damper working under centrifugal loads require bench testing for its further support.

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