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Septic B-spline collocation method for numerical solution of the Kuramoto-Sivashinsky equation

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Abstract—In this paper the Kuramoto-Sivashinsky equation is solved numerically by collocation method. The solution is approximated as a linear combination of septic B-spline functions. Applying the Von-Neumann stability analysis technique, we show that the method is unconditionally stable. The method is applied on some test examples, and the numerical results have been compared with the exact solutions. The global relative error and L_∞ in the solutions show the efficiency of the method computationally.

Keywords—Kuramoto-Sivashinsky equation; Septic B-spline; Collocation method; Finite difference.

I. Introduction

In this paper we consider the solution of the Kuramoto-Sivashinsky equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\alpha\frac{\partial^2 u}{\partial x^2}+\upsilon\frac{\partial^4 u}{\partial x^4}=0, \qquad x\in[a,b],\quad x\in[0,T], \tag{1}$$

with the initial condition

$$u(x,0)=f(x), \qquad x\in[a,b], \tag{2}$$

and boundary conditions

$$u(a,t)=g_0(t),\quad u(b,t)=g_1(t), \tag{3}$$

$$\frac{\partial u}{\partial x}(a,t)=0,\quad \frac{\partial u}{\partial x}(b,t)=0, \tag{4}$$

$$\frac{\partial^2 u}{\partial x^2}(a,t)=0,\quad \frac{\partial^2 u}{\partial x^2}(b,t)=0, \tag{5}$$

where α and υ are constants.

In recent years, many different methods have been used to estimate the solution of the Kuramoto-Sivashinsky, for example, see [1]-[5]. The generalized Kuramoto-Sivashinsky equation is solved in [6] and the numerical solution of the Kuramoto-Sivashinsky equation is solved by radial basis function (RBF) based mesh-free method in [7].
The paper is organized as follows. In Section 2, septic B-spline collocation method is explained. In Section 3, we develop an algorithm for the numerical solution of the Kuramoto-Sivashinsky equation. Section 4, is devoted to stability analysis of the method. In Section 5, examples are presented. Note that we have computed the numerical results by Mathematica (7) programming.

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II. Septic B-spline collocation method

The interval $[a,b]$ is partitioned in to a mesh of uniform length $h=x_{i+1}-x_i$ by the knots $x_i, i=0,1,\dots,N$ such that $a=x_0<x_1<x_2<\ldots<x_{N-1}<x_N=b$. The septic B-spline function $B_i(x)$ at these knots be given by

$$B_i(x)=\frac{1}{h^7}\begin{cases}(x-x_{i-4})^7, & x\in[x_{i-4},x_{i-3}),\\ (x-x_{i-4})^7-8(x-x_{i-3})^7, & x\in[x_{i-3},x_{i-2}),\\ (x-x_{i-4})^7-8(x-x_{i-3})^7+ & \\ 28(x-x_{i-2})^7, & x\in[x_{i-2},x_{i-1}),\\ (x-x_{i-4})^7-8(x-x_{i-3})^7+ & \\ 28(x-x_{i-2})^7-56(x-x_{i-1})^7, & x\in[x_{i-1},x_i),\\ (x_{i+4}-x)^7-8(x_{i+3}-x)^7+ & \\ 28(x_{i+2}-x)^7-56(x_{i+1}-x)^7, & x\in[x_i,x_{i+1}),\\ (x_{i+4}-x)^7-8(x_{i+3}-x)^7+ & \\ 28(x_{i+2}-x)^7, & x\in[x_{i+1},x_{i+2}),\\ (x_{i+4}-x)^7-8(x_{i+3}-x)^7, & x\in[x_{i+2},x_{i+3}),\\ (x_{i+4}-x)^7, & x\in[x_{i+3},x_{i+4}),\\ 0, & otherwise.\end{cases} \tag{6}$$

The set of splines, $\{B_{-3},B_{-2},\dots,B_{N+2},B_{N+3}\}$ forms a basis for functions defined over $[a,b]$. We let U be a shape function that satisfies the boundary conditions (3), (4) and (5). U is expressed as a linear combination of $N+7$ shape function given by

$$U(x,t)=\sum_{i=-3}^{N+3}c_i(t)B_i(x), \tag{7}$$

where $c_i(t)$ are time-dependent quantities to be determined from the boundary conditions and collocation form of the differential equations, and the B_i are septic B-spline functions. Using approximate function (7) and Table 1, we have

$$u_i=c_{i-3}+120c_{i-2}+1191c_{i-1}+2416c_i+1191c_{i+1}+120c_{i+2}+c_{i+3}, \tag{8}$$

$$hu_i^{'}=-7c_{i-3}-392c_{i-2}-1715c_{i-1}+1715c_{i+1}+392c_{i+2}+7c_{i+3}, \tag{9}$$

$$h^2u_i^{''}=42c_{i-3}+1008c_{i-2}+630c_{i-1}-3360c_i+630c_{i+1}+1008c_{i+2}+42c_{i+3}, \tag{10}$$

$$h^3u_i^{'''}=-210c_{i-3}-16800c_{i-2}+3990c_{i-1}-3990c_{i+1}+1680c_{i+2}+210c_{i+3}, \tag{11}$$

$$h^4u_i^4=840c_{i-3}-7560c_{i-1}+13440c_i-7560c_{i+1}+840c_{i+3}. \tag{12}$$

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TABLE I. B_i ,$B_i^{'}$,$B_i^{''}$,$B_i^{'''}$ AND $B_i^{(4)}$ AT THE NODE POINTS.

x	B_i	$hB_i^{'}$	$h^2B_i^{''}$	$h^3B_i^{'''}$	$h^4B_i^{(4)}$
x_{i-4}	0	0	0	0	0
x_{i-3}	1	7	42	210	840
x_{i-2}	120	392	1008	1680	0
x_{i-1}	1191	1715	630	-3990	-7560
x_i	2416	0	-3360	0	13440
x_{i+1}	1191	-1715	630	3990	-7560
x_{i+2}	120	-392	1008	-1680	0
x_{i+3}	1	-7	42	-210	840
x_{i+4}	0	0	0	0	0

III. Construction of the Method

We discretize the time derivative of the Kuramoto-Sivashinsky equation using a finite-difference formula and applying the θ-weighted, where $0 \leq \theta \leq 1$. Using the finite difference method, we can write

$$\frac{\partial u_i}{\partial t} = \frac{u_i^{n+1} - u_i^n}{\delta t}, \qquad (13)$$

$$u_i = \theta u_i^{n+1} + (1-\theta)u_i^n, \qquad (14)$$

where ∂t is a time step size and $u_i^n = u(x_i, t^{n-1} + \partial t)$. Hence Eq. (1) can be written as:

$$\frac{\partial u_i}{\partial t} = \frac{u_i^{n+1}-u_i^n}{\delta t} + \theta\Big((u\frac{\partial u}{\partial x})^{n+1} + \alpha(\frac{\partial^2 u}{\partial x^2})^{n+1} + \upsilon(\frac{\partial^4 u}{\partial x^4})^{n+1}\Big) + (1-\theta)\Big((u\frac{\partial u}{\partial x})^n + \alpha(\frac{\partial^2 u}{\partial x^2})^n + \upsilon(\frac{\partial^4 u}{\partial x^4})^n\Big). \qquad (15)$$

The nonlinear term in Eq. (15) can be approximated by using the following formula [8]:

$$(u\frac{\partial u}{\partial x})^{n+1} = (u)^n(\frac{\partial u}{\partial x})^{n+1} + (u)^{n+1}(\frac{\partial u}{\partial x})^n - (u\frac{\partial u}{\partial x})^n. \qquad (16)$$

Substituting the approximate solution U for u and putting the values of the nodal values U and the value $\theta = 0.5$, we obtain

$$\acute{a}_i c_{i-3}^{n+1} + \acute{b}_i c_{i-2}^{n+1} + \acute{m}_i c_{i-1}^{n+1} + \acute{d}_i c_i^{n+1} + \acute{e}_i c_{i+1}^{n+1} + \acute{f}_i c_{i+2}^{n+1} + \acute{g}_i c_{i+3}^{n+1} = \Psi_i^n, \ \ i = 0, \ldots, N, \qquad (17)$$

where

$$\Psi_i^n = \breve{a}_i c_{i-3}^n + \breve{b}_i c_{i-2}^n + \breve{m}_i c_{i-1}^n + \breve{d}_i c_i^n + \breve{e}_i c_{i+1}^n + \breve{f}_i c_{i+2}^n + \breve{g} c_{i+3}^n, \ \ i = 0, \ldots, N, \qquad (18)$$

with

$$\begin{cases} \acute{a}_i = 1 + x - 7y + 42z + 840w, \\ \acute{b}_i = 120 + 120x - 392y + 1008z, \\ \acute{m}_i = 1191 + 1191x - 1715y + 630z - 7560w, \\ \acute{d}_i = 2416 + 241x - 3360z + 13440w, \\ \acute{e}_i = 1191 + 1191x + 1715y + 630z - 7560w, \\ \acute{f}_i = 120 + 120x + 392y + 1008z, \\ \acute{g}_i = 1 + x + 7y + 42z + 840w, \\ \breve{a}_i = \breve{g}_i = 1 - 42z - 840w, \\ \breve{b}_i = \breve{f}_i = 120 - 1008z, \\ \breve{m}_i = \breve{e}_i = 1191 - 630z + 7560w, \\ \breve{d}_i = 2416 + 3360z - 13440w, \end{cases} \qquad (19)$$

where $x = \frac{\delta t}{2}(\frac{\partial u}{\partial x})^n$, $y = \frac{\delta t}{2}\frac{u^n}{h}$, $z = \frac{\delta t}{2}\frac{\alpha}{h^2}$, $w = \frac{\delta t}{2}\frac{\upsilon}{h^4}$.

The system (17) consists of $N+1$ equations in the $N+7$ unknowns $C = (c_{-3}, c_{-2}, \ldots, c_{N+2}, c_{N+3})^T$. To obtain a unique solution for C, we must use the boundary conditions. From the boundary conditions we can write

$$\begin{cases} c_{-3}^{n+1} + 120c_{-2}^{n+1} + 1191c_{-1}^{n+1} + 2416c_0^{n+1} + 1191c_1^{n+1} \\ +120c_2^{n+1} + c_3^{n+1} = g_0(t^{n+1}), \\ \\ -7c_{-3}^{n+1} - 392c_{-2}^{n+1} - 17151c_{-1}^{n+1} + 1715c_1^{n+1} + 392c_2^{n+1} \\ +7c_3^{n+1} = 0, \\ \\ 42c_{-3}^{n+1} + 1008c_{-2}^{n+1} + 630c_{-1}^{n+1} - 3360c_0^{n+1} + 630c_1^{n+1} \\ +1008c_2^{n+1} + 42c_3^{n+1} = 0, \end{cases} \qquad (20)$$

and

$$\begin{cases} c_{N-3}^{n+1} + 120c_{N-2}^{n+1} + 1191c_{N-1}^{n+1} + 2416c_N^{n+1} + 1191c_{N+1}^{n+1} \\ +120c_{N+2}^{n+1} + c_{N+3}^{n+1} = g_1(t^{n+1}), \\ \\ -7c_{N-3}^{n+1} - 392c_{N-2}^{n+1} - 17151c_{N-1}^{n+1} + 1715c_{N+1}^{n+1} + 392c_{N+2}^{n+1} \\ +7c_{N+3}^{n+1} = 0, \\ \\ 42c_{N-3}^{n+1} + 1008c_{N-2}^{n+1} + 630c_{N-1}^{n+1} - 3360c_N^{n+1} + 630c_{N+1}^{n+1} \\ +1008c_{N+2}^{n+1} + 42c_{N+3}^{n+1} = 0. \end{cases} \qquad (21)$$

The B-spline method in matrix form can be written as follows

$$AC = Q, \qquad (22)$$

where
A=

$$\begin{pmatrix} 1 & 120 & 1191 & 2416 & 1191 & 120 & 1 & \ldots & 0 \\ 7 & 392 & 1715 & 0 & -1715 & -392 & -7 & \ldots & 0 \\ 42 & 1008 & 630 & -3360 & 630 & 1008 & 42 & \ldots & 0 \\ \acute{a}_i & \acute{b}_i & \acute{m}_i & \acute{d}_i & \acute{e}_i & \acute{f}_i & \acute{g}_i & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & \acute{a}_i & \acute{b}_i & \acute{m}_i & \acute{d}_i & \acute{e}_i & \acute{f}_i & \acute{g}_i \\ 0 & \ldots & 1 & 120 & 1191 & 2416 & 1191 & 120 & 1 \\ 0 & \ldots & 7 & 392 & 1715 & 0 & -1715 & -392 & -7 \\ 0 & \ldots & 42 & 1008 & 630 & -3360 & 630 & 1008 & 42 \end{pmatrix}, \qquad (23)$$

$$C = (c_{-3}^{n+1}, c_{-2}^{n+1}, \ldots, c_{N+2}^{n+1}, c_{N+3}^{n+1})^T, \qquad (24)$$

$$Q = \begin{cases} \Big(g_0(\delta t), 0, 0, u(x_0, \delta t) - \frac{\delta t}{2}(\alpha\frac{\partial^2 u}{\partial x^2}(x_0, \delta t) + \upsilon\frac{\partial^4 u}{\partial x^4}(x_0, \delta t)), \ldots, \\ u(x_N, \delta t) - \frac{\delta t}{2}(\alpha\frac{\partial^2 u}{\partial x^2}(x_N, \delta t) + \upsilon\frac{\partial^4 u}{\partial x^4}(x_N, \delta t)), g_1(\delta t), 0, 0\Big)^T, if \ \ t = \delta t, \\ \\ \Big(g_0(t), 0, 0, \Psi_0^n, \ldots, \Psi_N^n, g_1(t), 0, 0\Big)^T, \qquad if \ \ t > \delta t. \end{cases} \qquad (25)$$

IV. Stability analysis

In this section, we present the stability of the septic B-spline approximation (17) using the Von Numann method [9],[10]. According to the Von-Neumann method, we have

$$c_i^n = \xi^n exp(\lambda khi), \qquad \lambda^2 = -1, \tag{26}$$

where k is the mode number and h is the element size. We obtain the equation:

$$\xi^{n+1}\Big(\acute{a}_i exp((i-3)\lambda kh) + \acute{b}_i exp((i-2)\lambda kh) + \acute{m}_i exp((i-1)\lambda kh) + \acute{d}_i exp((i)\lambda kh) + \acute{e}_i exp((i+1)\lambda kh) + \acute{f}_i exp((i+2)\lambda kh) + \acute{g}_i exp((i+3)\lambda kh)\Big) =$$
$$\xi^{n}\Big(\breve{a}_i exp((i-3)\lambda kh) + \breve{b}_i exp((i-2)\lambda kh) + \breve{m}_i exp((i-1)\lambda kh) + \breve{d}_i exp((i)\lambda kh) + \breve{e}_i exp((i+1)\lambda kh) + \breve{f}_i exp((i+2)\lambda kh) + \breve{g}_i exp((i+3)\lambda kh)\Big). \tag{27}$$

Dividing both sides of (27) by exp($i\lambda$kh), we can be written as:

$$\xi^{n+1}\Big(\acute{a}_i exp((-3)\lambda kh) + \acute{b}_i exp((-2)\lambda kh) + \acute{m}_i exp((-1)\lambda kh) + \acute{d}_i + \acute{e}_i exp((1)\lambda kh) + \acute{f}_i exp((2)\lambda kh) + \acute{g}_i exp(3)\lambda kh)\Big) = \xi^{n}\Big(\breve{a}_i exp((-3)\lambda kh) + \breve{b}_i exp((-2)\lambda kh) + \breve{m}_i exp((-1)\lambda kh) + \breve{d}_i + \breve{e}_i exp((1)\lambda kh) + \breve{f}_i exp((2)\lambda kh) + \breve{g}_i exp((3)\lambda kh)\Big). \tag{28}$$

Eq. (28) can be rewritten in a simple form as:

$$\xi = \frac{(\acute{X} + \lambda\acute{Y})}{(X + \lambda Y)}, \tag{29}$$

where

$X = (2 + 2x + 84z + 1680w)cos(3kh) + (240 + 240x + 2016z)cos(2kh) + (2382 + 2382x + 1260z - 15120w)cos(kh) + (2416 + 2416x - 3360z + 13400w)$,

$Y = (14y)sin(3kh) + (784y)sin(2kh) + (3430y)sin(kh)$,

$\acute{X} = 2(1 - 42z - 840w)cos(3kh) + 2(120 - 1008z)cos(2kh) + 2(1191 - 63z + 750w)cos(kh) + (2416 + 3360z - 13440w)$,

$\acute{Y} = 0$,

here x, y, z, w have their predefined definition given in section (III). The Eq. (29) and above equations imply $\mid \xi \mid \leq 1$. Therefore, the linearized numerical scheme for the Kuramoto-Sivashinsky equation is unconditionally stable.

V. Numerical examples

In order to illustrate the performance of the septic B-spline collocation method in solving the Kuramoto-Sivashinsky equation and justify the accuracy and efficiency of the present method, we consider the following examples. To show the efficiency of the present methd for our problem in comparsion with the exact solution, we report the global relative error and L_∞ using formulae

$$GRE = \frac{\sum_i \mid U(x_i,t) - u(x_i,t) \mid}{\sum_i \mid u(x_i,t) \mid}, \tag{30}$$

$$L_\infty = \max_i \mid U(x_i,t) - u(x_i,t) \mid, \tag{31}$$

where U is numerical solution and u denotes analytical solution.

Example 1. Consider the Kuramoto-Sivashinsky equation with $\alpha = 1$ and $\beta = 1$ in the interval $[-30, 30]$, with the exact solution

$$u(x,t) = \ddot{b} + \tfrac{15}{19}(\tfrac{11}{19})^{\frac{1}{2}}\Big(-9\tanh(k(x - \ddot{b}t - x_0)) + 11\tanh^3(k(x - \ddot{b}t - x_0))\Big).$$

The boundary conditions and the initial conditions is taken from the exact solution. We have taken $\ddot{b} = 5, k = \frac{1}{2}(\frac{11}{19})^{\frac{1}{2}}$ and $x_0 = -12$. In order to compare the solutions with [11], we have taken $\delta t = 0.01$ and $N = 150$. In Table II give a comparison between the global relative error found by our method and method in [11] for N=150.

Table II:COMPARISON OF GRE FOR EXAMPLE 1 AT DIFFERENT TIME WITH N =150 AND $\delta t = 0.01$.

Time	1	2	3	4
Present method	9.14572e-06	1.71198e-05	2.47542e-05	3.17003e-05
Method in [11]	6.7923e-04	1.1503e-03	1.5941e-03	2.0075e-03

Table III: COMPARISON OF GRE AND L_∞ FOR EXAMPLE 1 AT DIFFERENT TIME WITH N =300 AND $\delta t = 0.01$.

Time	1	2	3	4
GRE	4.65545e-06	8.62414e-06	1.24180e-05	1.5897e-05
L_∞	1.66067e-03	2.46738e-03	2.83154e-03	2.78573e-03

Table IV: COMPARISON OF GRE AND L_∞ FOR EXAMPLE 1 AT DIFFERENT TIME AND DIFFERENT PARTITIONS.

partitions	$\delta t = 0.01, N = 50$		$\delta t = 0.001, N = 100$	
Time	GRE	L_∞	GRE	L_∞
0.5	3.40634e-04	1.63837e-02	7.16415e-06	1.03619e-03
1	4.42380e-04	1.95299e-02	1.38513e-05	1.63762e-03
1.5	5.21724e-04	2.20100e-02	2.02052e-05	2.07273e-03
2	6.07455e-04	2.37631e-02	2.60686e-05	2.48375e-03
2.5	6.84635e-04	2.55042e-02	3.22695e-05	2.79434e-03
3	8.51473e-04	2.75543e-02	3.86268e-05	3.00439e-03
3.5	1.01856e-03	2.85097e-02	4.47998e-05	3.16038e-03
4	1.20163e-03	2.85479e-02	4.99723e-05	3.43704e-03

Figure 1, shows that the solution obtained by our method is close to the exact solution and Figure 3, shows absolute error for different values of time. Also from Table III we see the GRE and L_∞ error decrease as δt decreases.

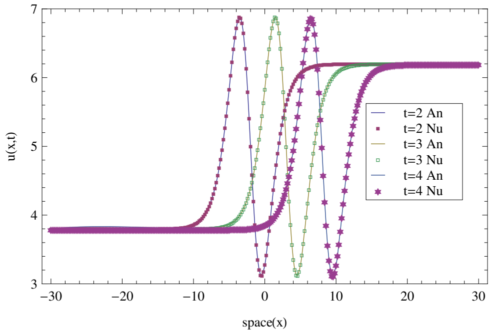

Fig. 1. Analytical-estimated graphs of Example 1 for $x \in [-30, 30]$ and $t = 2, 3, 4$ with $\delta t = 0.01$ and $N = 300$.

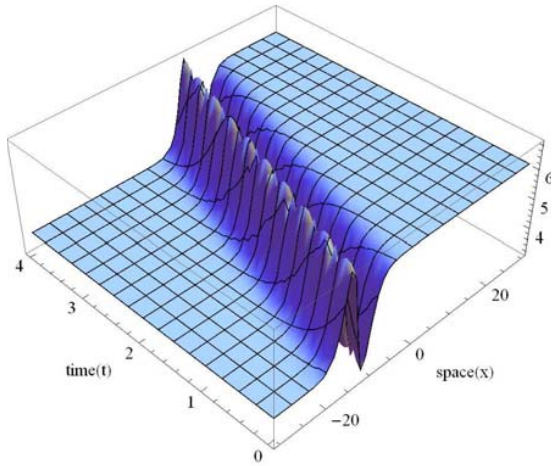

Fig. 2. Three-dimensional plot for Example 1.

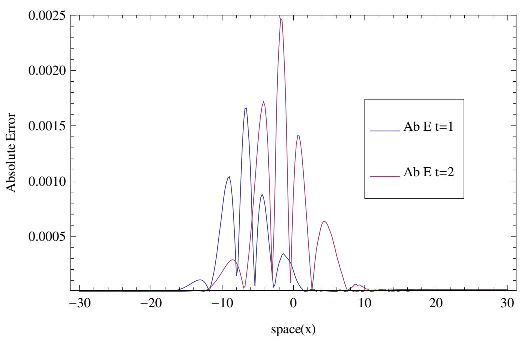

Fig. 3. Absolute error graphs of Example 1 for $x \in [-30, 30]$ and $t = 1, 2$ with $\delta t = 0.01$ and $N = 300$.

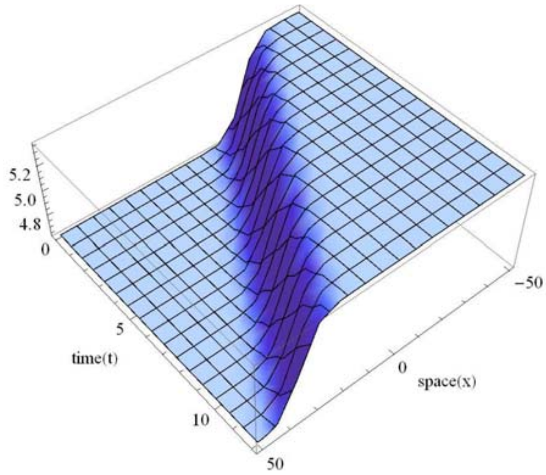

Fig. 4. Three-dimensional plot for Example 2.

Example 2. Consider the Kuramoto-Sivashinsky equation with $\alpha = -1$ and $\beta = 1$ in the interval $[-50, 50]$, with the exact solution
$u(x,t) = \ddot{b} + \frac{15}{19}(\frac{1}{19})^{\frac{1}{2}}\Big(-3\tanh(k(x-\ddot{b}t-x_0)) + \tanh^3(k(x-\ddot{b}t-x_0))\Big)$.
The boundary conditions and the initial conditions is taken from the exact solution. We have taken $\ddot{b} = 5, k = \frac{1}{2}(\frac{1}{19})^{\frac{1}{2}}$, $x_0 = -25$. The comparison of the global relative error by our method and method in [11] given in Table V for N=200.

Table V: COMPARISON OF GRE FOR EXAMPLE 2 AT DIFFERENT TIME WITH N =200 AND $\delta t = 0.01$.

Time	6	8	10	12
N=200	1.62464e-07	1.94032e-07	2.22878e-07	5.31428e-07
Method in [11]	7.8808e-06	9.5324e-06	1.0891e-05	1.1793e-05

Table VI: COMPARISON OF GRE AND L_∞ FOR EXAMPLE 2 AT DIFFERENT TIME WITH N =300 and $\delta t = 0.01$.

Time	1	2	3	4	5
GRE	2.95431e-08	4.96862e-08	6.70086e-08	8.23450e-08	9.60820e-08
L_∞	3.17905e-06	5.90193e-06	8.28073e-06	1.03852e-05	1.22640e-05

Table VII: COMPARISON OF GRE AND L_∞ FOR EXAMPLE 2 AT DIFFERENT TIME AND DIFFERENT PARTITIONS.

partitions	$\delta t = 0.01, N = 50$		$\delta t = 0.1, N = 100$	
Time	GRE	L_∞	GRE	L_∞
1	2.71992e-07	5.06881e-06	6.63279e-06	3.12842e-04
2	3.92102e-07	9.93310e-06	1.24271e-05	5.82322e-04
3	4.58610e-07	1.04124e-05	1.75327e-05	8.18514e-04
4	5.56721e-07	1.47454e-05	2.20976e-05	1.02792e-03
5	6.09751e-07	1.44221e-05	2.61869e-05	1.21516e-03
6	7.05997e-07	1.81599e-05	2.99008e-05	1.38367e-03
7	7.43306e-07	1.76446e-05	3.32457e-05	1.53615e-03
8	8.30062e-07	2.08271e-05	3.62610e-05	1.67472e-03
9	8.54613e-07	2.02786e-05	3.89815e-05	1.80114e-03
10	9.50095e-07	2.29967e-05	4.14260e-05	1.91684e-03
11	1.12428e-06	2.26008e-05	4.35000e-05	2.02302e-03
12	3.09898e-06	1.46694e-04	4.44660e-05	2.12072e-03

Figure 5 shows that the solution obtained by our method is close to the exact solution.

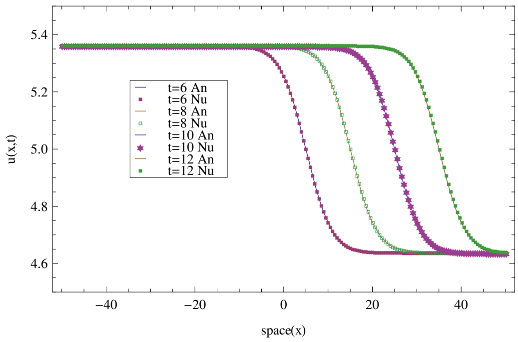

Fig. 5. Analytical-estimated graphs of Example 2 for $x \in [-50, 50]$ and $t = 6, 8, 10, 12$ with $\delta t = 0.01$ and $N = 300$.

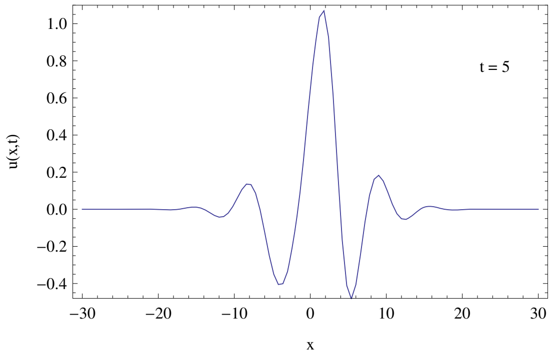

Fig. 6. Numerical solutions at t=5 with $\delta t = 0.01$ and $N = 100$ for Example 3.

Example 3. As a last study we consider here a numerical solution of the Kuramoto-Sivashinsky equation with $\alpha = 1$, $\beta = 1$ and with the initial condition u(x,t)=exp(-x^2) in the interval $[-30, 30]$. We can see clearly that the result shows same behavior as in [7]. The numerical solution at different time is presented in figs 6 ,7 ,8.

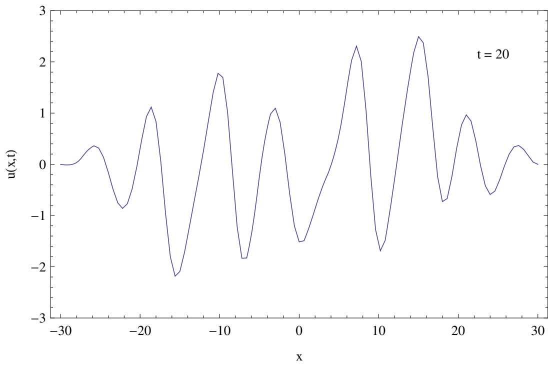

Fig. 7. Numerical solutions at t=20 with $\delta t = 0.01$ and $N = 100$ for Example 3.

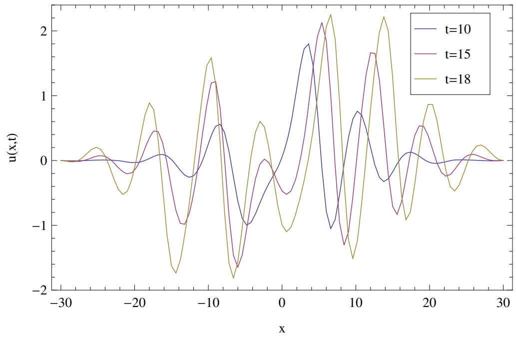

Fig. 8. Numerical solutions at different times with $\delta t = 0.01$ and $N = 100$ for Example 3.

CONCLUSION

The septic B-spline collocation method is used to solve the Kuramoto-Sivashinsky equation with initial and boundary conditions. The stability analysis of the method is shown to be unconditionally stable. The numerical results given in the previous section demonstrate the good accuracy stable of the scheme proposed in this research.

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