International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:8, No:5, 2014

Sensitivity Analysis of Principal Stresses in Concrete Slab of Rigid Pavement Made From Recycled Materials

Aleš Florian, Lenka Ševelová

***Abstract*—**Complex sensitivity analysis of stresses in a concrete slab of the real type of rigid pavement made from recycled materials is performed. The computational model of the pavement is designed as a spatial (3D) model, is based on a nonlinear variant of the finite element method that respects the structural nonlinearity, enables to model different arrangements of joints, and the entire model can be loaded by the thermal load. Interaction of adjacent slabs in joints and contact of the slab and the subsequent layer are modeled with the help of special contact elements. Four concrete slabs separated by transverse and longitudinal joints and the additional structural layers and soil to the depth of about 3m are modeled. The thickness of individual layers, physical and mechanical properties of materials, characteristics of joints, and the temperature of the upper and lower surface of slabs are supposed to be random variables. The modern simulation technique Updated Latin Hypercube Sampling with 20 simulations is used. For sensitivity analysis the sensitivity coefficient based on the Spearman rank correlation coefficient is utilized. As a result, the estimates of influence of random variability of individual input variables on the random variability of principal stresses σ1 and σ3 in 53 points on the upper and lower surface of the concrete slabs are obtained.

***Keywords*—**Concrete, FEM, pavement, sensitivity, simulation.

I. Introduction

COMPLEX analysis of pavements is often very difficult for design practice. Rheological properties of materials, cracking, joints, contact of concrete slabs in joints, contact of slab and subsequent structural layer, temperature changes, non-homogeneity of pavement base, water regime in the subgrade, environmental changes, etc. influence serviceability of the structure in a decisive way. The problem is moreover complicated by the fact that the input data are generally random variables. Further source of uncertainties stems from vagueness of input data.

Taking into account specific properties of the particular type of structure, the combination of the proper analytical model with modern simulation techniques seems to be an effective tool for the solution of the problem [1]-[3]. The analysis of a pavement using these methods provides the designer with reliability limits of the structural response and enables the determination of possible critical development. The results of the analysis also enable finding out which input variables require special attention due to their random variability dominantly influencing the structural behavior. This type of analysis is called sensitivity analysis.

The behavior of the older type of rigid pavement is analyzed. This type of pavement is made from plain concrete, no dowels are used, and joints are made during laying of concrete. Dimensions of individual concrete slabs are 7.5 x 3.75m, see Fig. 1. The structure is loaded by the self-weight of concrete slabs, by the thermal loading due to the temperature difference between the upper and lower surface of the slab, and by the load of intensity 50 kN at a distance of 0.25m from the edge of slab - see point 26 in Fig. 1. Thus the total state of stress in the slab results from all three different sources of load acting together. Contrary to the traditional design, the base layer is supposed to be made from a recycled material instead of a natural one. It is made from recycled concrete of fractions 0–16mm.

The computational model is based on the nonlinear finite element method. Four concrete slabs, all other layers and longitudinal and transverse joints are modeled as 3D space. Joints, contact of slabs in joints, contact of slabs and subsequent layer, and the thermal loading are modeled in detail.

Total 17 basic random input variables describing layer thicknesses, mechanical properties of materials, characteristics of joints and temperature on both surfaces of concrete slabs are used in the study. They are described by the assumed cumulative distribution functions (generally three-parametric) and by the appropriate statistical parameters. The influence of uncertainties in input variables on the behavior of the pavement is respected in the analysis with help of numerical simulation techniques [4]. The modern simulation technique Updated Latin Hypercube Sampling with 20 simulations is used [5], [6].

The sensitivity analysis of principal stresses σ1 and σ3 in concrete slabs is performed to show possibilities of reliability methods in analysis of real pavement structures. The stresses are evaluated in 53 points on the upper and lower surface (totally 106 points) of the concrete slabs, see Fig. 1. Sign convention is chosen so the positive stresses are tensile, while the negative stresses are compressive. Principal stress σ1 represents an extreme value of tensile stress in the given point of the structure, while σ3 is an extreme value of compressive stress, which arises due to the spatial stress state. Although the calculation and measurement of deflections on the pavement

A. Florian is with the Faculty of Civil Engineering, Brno University of Technology, Czech Republic (phone: +420-541147378; e-mail: florian.a@fce.vutbr.cz).

L. Ševelová is with the Department of Landscape Formation and Protection, Mendel University of Brno, Czech Republic (e-mail: lenka.sevelova@mendelu.cz).

plays the most important role in today's engineering practice, the calculation of stresses seems to be in fact much more important. Principal stress represents the extreme normal stress at a given point of the structure and thus it is the crucial characteristic which would be used in dimensioning process. If the principal stress exceeds the tensile strength of the material, a local tensile crack is created. If it exceeds the compressive strength of the material (only hypothetically for pavements), the material is locally crushed.

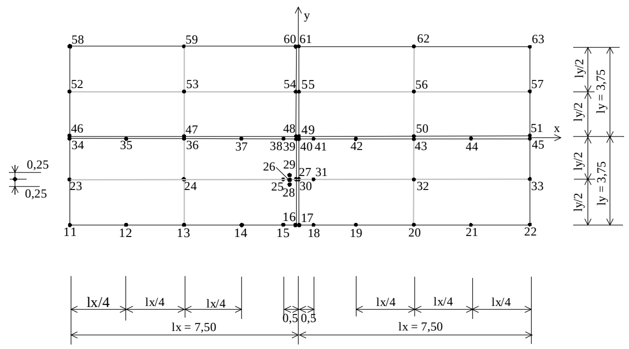

Fig. 1 Points for evaluation of stresses

The result example of sensitivity analysis of principal stresses is provided. The aim is to identify those input variables which random variability influences random variability of principal stresses the most and thus to demonstrate the practical applicability of reliability methods in analyzing the behavior of pavements, their possibilities to supplement or even replace experimental methods, the nature of the information obtained, and finally, the possibility to quantify the reliability of pavements.

II. Analytical Model

The computational model of the pavement is developed in *ANSYS* system. Its main features are:

- a parametric model, open and flexible,
- a nonlinear model that respects the structural nonlinearity,
- designed as a spatial (3D) model that models the four adjacent concrete slabs with subsequent layers and the surrounding soil,
- it allows to model different arrangement of joints,
- the interaction of adjacent slabs is realized with help of special contact elements which prevent the transmission of tensile stress,
- contact between a slab and a subsequent layer is also realized with the contact elements, which allow realistic modeling of lifting of corners and the center of the slab,
- each layer can be of any thickness and of any material,
- the model can be loaded by thermal loading,
- external load can be applied at any point of the slab,
- displacements, rotations, strains, normal or tangential stresses etc. can be evaluated at any point in any layer of the pavement structure.

The geometry of the structure is modeled by elements BRICK45. It is the eight-mode element with three degrees of freedom (UX, UY, UZ) in each node.

Attention is paid to modeling of concrete slab contact with subsequent layer and interaction of adjacent slabs in joints. In these areas the so-called contact problem occurs, in which tensile stresses cannot be transmitted. This is the case of so called structural nonlinearity and the pavement modeling therefore becomes nonlinear. Thus the solution is divided into individual iteration steps and the Newton-Raphson method is used in each step. The actual contacts are modeled with the help of special contact elements. The contact element connects appropriate nodes of finite element mesh and operates in compression as the imaginary spring with given stiffness, but if it is pulled, the nodes behave independently.

The proposed model of joints tries to simulate at least some of the complex phenomena that occur here. The adjacent slabs interact due to their mutual contact, due to the material in joints, and also due to stresses transmitted through the other layers of the structure.

III. Simulation Technique

The modern simulation technique Updated Latin Hypercube Sampling [5], [6] with 20 simulations is used for statistical analysis. It is an improved variant of Latin Hypercube Sampling [4]. The method keeps the methodology of Latin Hypercube Sampling, but uses the improved strategy of generating input samples based on specially modified tables of random permutations of rank numbers. The modified tables consist of random permutations that are mutually statistically independent. Using of Updated Latin Hypercube Sampling generally results to the further increase of accuracy, quality and reliability of the results obtained from reliability analysis. The detailed description of Updated Latin Hypercube Sampling can be found in [5], [6].

To measure the relative influence of random variability of each input variable on the random variability of the output (principal stresses in 106 points in concrete slabs), the sensitivity coefficient based on the Spearman rank correlation coefficient is proposed, see [7]. It is not limited to the linear relationship like Pearson correlation coefficient. The sensitivity coefficient is defined as

$$r_s = 1 - \frac{6\sum_i d_i^2}{N(N-1)(N+1)} \tag{1}$$

where r_s is the sensitivity coefficient among the k-th input variable and the output, d_i is the difference between the rank numbers of the k-th input variable and the rank numbers of the output, and N is the number of simulation.

The sensitivity coefficient ranges within interval from –1 to +1. The higher is the coefficient (in absolute value), the higher is the sensitivity of the output to the appropriate input variable. The sign of the coefficient indicates positive or negative influence. Sensitivity coefficient lower than 0.30 (in absolute value) can be explained as practically no influence, higher than 0.30 as a low influence, higher than 0.50 as a moderate influence, higher than 0.70 as a high influence, and sensitivity coefficient higher than 0.90 as a dominant influence. In the illustrative figures, the input variables with sensitivity coefficient higher than 0.30 are shown.

TABLE I
DERIVED STATISTICAL PARAMETERS OF INPUT VARIABLES

No.	Layer	Input variable	mean	COV	skewness
X1	concrete slab	thickness	220	0,09	-0,6
X2		Young modulus	37500	0,12	0,0
X3		Poisson's coefficient	0,20	0,02	0,0
X4	base	thickness	200	0,15	0,0
X5		Young modulus	150	0,40	0,0
X6		Poisson's coefficient	0,30	0,08	0,0
X7	sub-base	thickness	250	0,24	1,0
X8		Young modulus	120	0,18	0,9
X9		Poisson's coefficient	0,30	0,07	0,0
X10	subgrade	Young modulus	80	0,32	0,5
X11		Poisson's coefficient	0,35	0,075	0,9
X12	joints	width transversal	20	0,15	0,0
X13		width longitudinal	1,5	0,75	2*
X14		Young modulus	150	0,39	1,0
X15		coefficient of friction	0,5	0,34	0,0
X16	temperature	upper surface	11	0,95	0,5
X17		lower surface	10	0,25	0,9

* - truncation parameter

IV. INPUT RANDOM VARIABLES

Total of 17 variables are considered as random input variables, see Table I (units in MPa, mm, ^{0}C). Their statistical parameters are carefully evaluated taking into account the data obtained from the in-situ measurements, experimental tests, data from technological handbooks and scientific publications and the corresponding standards. Information about random properties of recycled concrete used in base layer is taken from [8]. To derive appropriate statistical parameters of input variables, the following procedure is utilized. At first, the limits are specified (minimum, maximum and mean value) in which input variables will occur with a high probability. Then, based on the assumption that values smaller than the minimum value and higher than the maximum value can occur only with a low probability, and choosing the appropriate cumulative distribution function (CDF) (normal, three-parametric lognormal, truncated normal), the other required statistical parameters are determined – coefficient of variation (COV) and skewness, see Table I. The normal and truncated normal CDF are used for symmetrically distributed variables, the three-parametric lognormal CDF for the other case. For simplicity, the mutual statistical independence of input variables is considered with the following exception - the temperatures of the upper and lower surface of the slabs are supposed to be fully statistically dependent.

The derived statistical parameters of input variables used in the presented study take into account uncertainties due to their random nature and also the uncertainties due to our incomplete knowledge of the structure, insufficient experimental research, modeling errors, vagueness of input data etc.

V. RESULTS

A. Principal Stress $\sigma 1$

Sensitivity analysis of principal stress $\sigma 1$ (maximal tensile stress) is performed in all points of the upper and lower surface of concrete slabs. Stresses on the both surfaces are generally affected by random variability of the input variables differently. Illustrative results are presented for some important points on the lower surface only, see Fig. 2.

Nominal values of stress obtained from deterministic analysis (based on mean values of input variables) on the lower surface have the character of tensile stress at all points, while on the upper surface there are both tensile and compressive stresses as well. In both cases, the nominal values are very close to the minimum values obtained from the statistical analysis taking into account uncertainties in input variables. Mean values of stress on both surfaces in all points have the character of tensile stress and are generally always greater than the nominal values. The nominal as well as mean values reach their maximum at points near the external load application (points 25 – 29), see Fig. 1.

Some points on the lower surface are influenced only by the minimum number of input variables. The stress at points near the external load application, the centers of slabs and the longitudinal edge of slabs (points 12-14, 20, 24-29, 32, 37, 38, 53, 56, 58, 63) are influenced only by two or three variables. The largest number of input variables - eight - has an influence in the outer corners of the loaded slab as well as the adjacent slab (points 11, 16, 22).

Most points on the lower surface are influenced by the temperature field acting on the upper (X16) and lower (X17) surface of the concrete slabs. Influence of these variables can be considered as dominant. Influence of the modulus of elasticity of joints material (X14), Poisson coefficient for sub-base layer (X9), and the width of transverse joints (X12) can be generally considered as moderate, in the case of the modulus of elasticity and the joint width at some points as high. Low influence show the modulus of elasticity of subgrade layer (X10); the thickness, the modulus of elasticity and Poisson coefficient of concrete slab (X1, X2, X3); the thickness, the modulus of elasticity and Poisson coefficient of base layer (X4, X5, X6); the thickness and the modulus of elasticity of sub-base layer (X7, X8); the coefficient of friction in joints (X15), and the width of the longitudinal joint (X13).

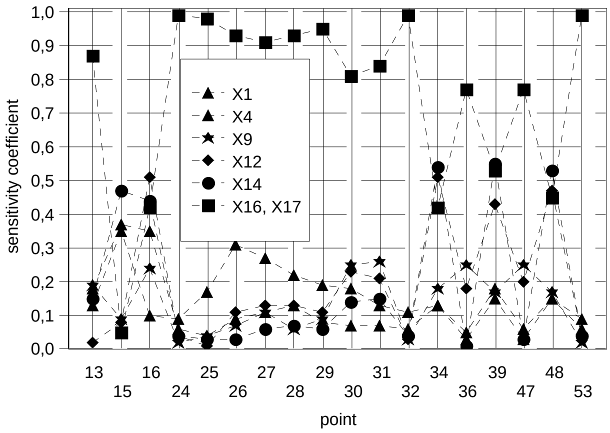

Fig. 2 Sensitivity coefficients of principal stress σ1, lower surface

B. Principal Stress σ3

Sensitivity analysis of principal stress σ3 (maximal compressive stress) is performed in all points of the upper and lower surface of concrete slabs. Stresses on the both surfaces are generally affected by random variability of the input variables differently. Illustrative results are presented for some important points on the lower surface only, see Fig. 3.

Nominal values of stress obtained from deterministic analysis (based on mean values of input variables) on the upper surface have the character of compressive stress at all points, while on the lower surface there are also minimal tensile stresses in points near the external load application (points 28, 30). In both cases, the nominal values are very close to the minimum values obtained from statistical analysis. Mean values of stress on both surfaces in all points have the same character as nominal values and are generally always greater than the nominal values.

In the points on the upper surface the influence of temperature field acting on the upper (X16) and lower (X17) surface of the concrete slabs are dominant. Influence of other variables can be considered as minimal or negligible.

On the lower surface the influence of temperature field (X16, X17) is still dominant, but the influence of other variables is no longer negligible. It may be mentioned the influence of the width of the transverse joint (X12), as well as the mechanical properties of the concrete slab (X2, X3), the thickness of base layer (X4), and the modulus of elasticity of sub-base layer (X8).

VI. Conclusion

The presented sensitivity analysis of principal stresses in concrete slabs of rigid pavement shows that the principal stresses in the slabs are influenced by random variability of input variables in a different way. That is, random variability influences differently principal stress σ1 and principal stress σ3, differently principal stresses on the upper and lower surface of concrete slabs and differently its influence also varies in individual points in which sensitivity coefficients are calculated. The dominant influence shows the random variability of temperature field acting on the upper and lower surface of the concrete slabs.

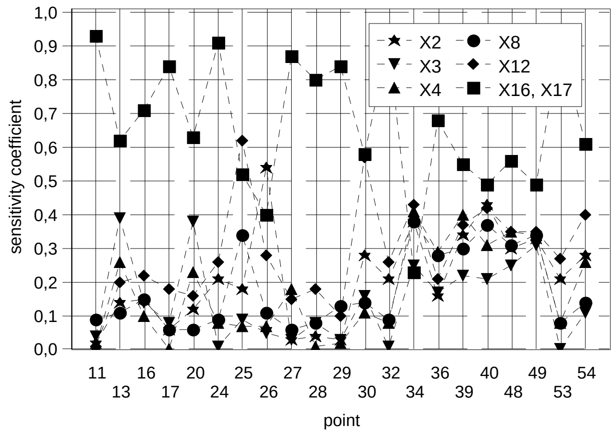

Fig. 3 Sensitivity coefficients of principal stress σ3, lower surface

Acknowledgment

The research was supported by the project TA01020326 "Optimization of design and realization of low capacity road pavements" of the Technology Agency of Czech Republic.

References

[1] S. F. Wojtkiewicz, L. Khazanovich, G. Gaurav, and R. Velasquez, "Probabilistic Numerical Simulation of Pavement Performance using MEPDG," *Road Materials and Pavement Design*, vol. 11, no. 2, 2010, pp. 291-306.

[2] S. W. Lee, J. H. Jeong, and B. J. Chon, "Probabilistic Modelling of Pavement Joint Opening," *Proc. of the Institution of Civ. Eng. Transport*, vol. 163, no. 1, 2010, pp. 9-17.

[3] A. Florian, L. Ševelová, and R. Hela, "Reliability Analysis of Deflections of Concrete Pavement," *Proc. of the 2nd Int. Conference on Best Practices for Concrete Pavements IBRACON 2011*, Florianopolis, Brazil, 2011, pp. 1-10.

[4] M. McKay, R. J. Beckman, and W. J. Conover, "A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code," *Technometrics*, no. 2, 1979, pp. 239-245.

[5] A. Florian, "An Efficient Sampling Scheme: Updated Latin Hypercube Sampling," *J. Probabilistic Engineering Mechanics*, vol. 7, no. 2, 1992, pp. 123-130.

[6] A. Florian, "Optimization of parametric studies using Updated Latin Hypercube Sampling," *Proc. of the ICOSSAR 2005*, Rome, Italy, 2005, pp. 2319-2323.

[7] A. Florian, J. Navrátil, "Reliability Analysis of the Cable Stayed Bridge in Construction and Service Stages," *Proc. of the ICOSSAR'93*, Innsbruck, Austria, 1993, pp. 869-877.

[8] A. Florian, and L. Ševelová, "Computer Simulation of Low Volume Roads Made From Recycled Materials", *International Journal of Civil, Architectural Science and Engineering*, vol. 7, no. 10, 2013, pp. 805-809.