

# Seismic Response of Reinforced Concrete Buildings: Field Challenges and Simplified Code Formulas

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## I. INTRODUCTION

**Abstract**—Building code-related literature provides recommendations on normalizing approaches to the calculation of the dynamic properties of structures. Most building codes make a distinction among types of structural systems, construction material, and configuration through a numerical coefficient in the expression for the fundamental period. The period is then used in normalized response spectra to compute base shear. The typical parameter used in simplified code formulas for the fundamental period is overall building height raised to a power determined from analytical and experimental results. However, reinforced concrete buildings which constitute the majority of built space in less developed countries pose additional challenges to the ones built with homogeneous material such as steel, or with concrete under stricter quality control. In the present paper, the particularities of reinforced concrete buildings are explored and related to current methods of equivalent static analysis. A comparative study is presented between the Uniform Building Code, commonly used for buildings within and outside the USA, and data from the Middle East used to model 151 reinforced concrete buildings of varying number of bays, number of floors, overall building height, and individual story height. The fundamental period was calculated using eigenvalue matrix computation. The results were also used in a separate regression analysis where the computed period serves as dependent variable, while five building properties serve as independent variables. The statistical analysis shed light on important parameters that simplified code formulas need to account for including individual story height, overall building height, floor plan, number of bays, and concrete properties. Such inclusions are important for reinforced concrete buildings of special conditions due to the level of concrete damage, aging, or materials quality control during construction.

Overall results of the present analysis show that simplified code formulas for fundamental period and base shear may be applied but they require revisions to account for multiple parameters. The conclusion above is confirmed by the analytical model where fundamental periods were computed using numerical techniques and eigenvalue solutions. This recommendation is particularly relevant to code upgrades in less developed countries where it is customary to adopt, and mildly adapt international codes.

We also note the necessity of further research using empirical data from buildings in Lebanon that were subjected to severe damage due to impulse loading or accelerated aging. However, we excluded this study from the present paper and left it for future research as it has its own peculiarities and requires a different type of analysis.

**Keywords**—Seismic behavior, reinforced concrete, simplified code formulas, equivalent static analysis, base shear, response spectra.

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EARTHQUAKE response of buildings is affected by their fundamental period more so than higher order modes due to the nature and frequency range of ground motion [1], [2]. On that basis, most codes and design guidelines provide simplified formulas that help estimate the fundamental period of a building as long as its structure meets regularity conditions [3], [4]. Structures with complex geometries, irregularities, or discontinuities whether in mass distribution or stiffness along their heights, are prescribed a dynamic analysis and are excluded from the use of simplified period formulas [3], [5].

Our series of analyses on models that simulate reinforced concrete buildings of various properties showed that there is a need for improvement over commonly used simplified code formulas. Most currently used simplified formulas condense the fundamental period expression to a constant that multiplies total building height raised to a certain power determined from mathematical derivations and experimental results [6]. Such expressions, however, do not capture other important features of the building that pertain to their layout and architectural configuration such as number of bays, story height, and plan dimensions.

Research in literature addressed the stiffness of reinforced concrete elements, which is especially important for moment resisting frames where columns are expected to resist both vertical and lateral forces [7], [8]. Serviceability and story drift requirements need to be investigated as well to distinguish between sway and non-sway conditions in column design [9]. Other research focused on interior partitioning and the effect of masonry infill panels. Although infill panels are not assigned structural loads during the design phase, it was found that they have a net effect on building period, and therefore on its seismic response [10]. Recent research explored the inclusion of devices within the structural system to enhance damping rather than relying on hysteresis behavior that often entails excessive deformations in the main structural system [11], [12].

## II. THEORETICAL MODEL

### A. Development and Assumptions

This paper uses a mathematical derivation while emphasizing direct practical applications rather than focusing on the details of the derivation. Consider the most basic structural configuration of a single bay in a single story. Denote by  $m_s$  the concrete slab contribution to the story mass and  $m'_c$  the column contribution. Denote by  $k_c$  the lateral

stiffness provided by one typical column in the structural system. Assume that the portal acts as a shear frame whereby column end nodes do not rotate significantly in the portal's deformed shape. We are interested in the ratio  $m^*/k^*$  where  $m^*$  and  $k^*$  represent the generalized mass and stiffness, respectively. For the simplest case, we have:

$$\frac{m^*}{k^*} = \frac{m_s + 2m_c}{2k_c} \quad (1)$$

Denote by  $N$  the total number of bays in the structure. If we consider the general case where  $N$  can take the value of any positive integer, we can write:

$$\frac{m^*}{k^*} = \frac{N m_s + \frac{N+1}{2} m_c}{(N+1) k_c} \quad (2)$$

Expression (2) uses the assumptions of a lumped mass model at slab level only, a degree of freedom (DOF) condensation to one horizontal DOF at each floor according to a rigid diaphragm action. Column mass contribution in middle stories away from first floor and roof is  $m_c = 2m'_c$ , but is much smaller than slab contribution as will be discussed in the following sections. This is often referred to as a simple shear building model.

#### B. Proposed Simplified Formulas

In typical design cases we investigate an empirical relationship between  $m_c$  and  $m_s$  that we denote by the ratio  $r = m_c/m_s$ . Introducing  $r$  in (2) and rearranging, we can express the generalized period of the system,  $T^*$ , in terms of  $N$ ,  $r$ , and a basic period that we define as the fundamental period of a simple structural unit that represents the mass of one slab panel in a typical tributary area, and the stiffness of one column,  $T_o = 2\pi (m_s/k_c)^{1/2}$  that represent the basic properties of the building at hand. This leads to:

$$T^* = \left[ \frac{N}{N+1} + \frac{r}{2} \right]^{1/2} T_o \quad (3)$$

In most practical cases, and particularly for reinforced concrete buildings,  $m_c$  is small compared to  $m_s$ . Empirical data that we collected from a sample of 151 reinforced concrete buildings in Lebanon show that  $r$  ranges between 0.1 and 0.2. This can be readily shown by referring, for example, to a typical residential building that uses a 20cm thick slab supported by 30cm x 60cm rectangular section columns laid out at a typical spacing of 4.5m x 5m with a 3m story height. The spacing represents the tributary area supported by a column. The ratio  $r = m_c/m_s$  in this numerical example is less than 0.14.

Further simplification to (3) could be performed by comparing the value of the generalized period for a range of number of bays,  $N$ . The error in neglecting the term  $r/2$  in (3) diminishes as  $N$  increases. For one bay or  $N = 1$  neglecting  $r/2$  corresponds to a 7% error while for a 6 bay building, for example, the error is less than 3%. On that basis, we could adopt a relationship between the generalized period of the

building and the basic unit period as follows:

$$T^* = \left[ \frac{N}{N+1} \right]^{1/2} T_o \quad (4)$$

Equation (4) can be applied to  $n$  stories by taking a unit displacement at the top of the building which would mobilize a tributary mass of  $n m_s$  and an equivalent stiffness of  $k_c/n$ . Substituting in (4), an expression for the generalized period for a building that has  $n$  floors can be written as:

$$T_{(n)}^* = \left[ \frac{n^2 N}{N+1} \right]^{1/2} T_o \quad (5)$$

Expression (5) slightly overestimates the period obtained from modal analysis because it takes the mass contribution from all floors in a uniform distribution. In modal analysis, the contribution of higher floors is larger than lower floors based on the predominance of the first mode shape in the displacement response [13]. Nevertheless, as will be shown in subsequent sections, (5) offers an improved estimate for preliminary design compared to code formulas that are widely used in literature.

Regression analysis was performed on our data set of 151 buildings to establish a detailed relationship between the fundamental period and properties of the structure. The relationship explicitly shows the effect of story plan dimensions, story and building height, number of bays, and concrete properties. Positive and statistically significant correlation was found between the fundamental period and story height, building height, and floor plan dimension and a statistically significant negative correlation was found with concrete properties and number of bays. The detailed statistical relationship was left outside the scope of this paper as we intend to focus here on the proposed simplified formula.

### III. CODE FORMULAS

#### A. Code-Based Equivalent Static Analysis

An equivalent static analysis allows the designer to perform a preliminary seismic design based on simplified formulas used to determine the fundamental period, building base shear, and story shear distribution applied statically. This approach is supposed to be equivalent to a dynamic analysis as long as the structure meets certain requirements [3]-[5]. The equivalent static analysis, which was initially developed for traditionally supported buildings, was extended in 1990 to base isolated buildings whereby the fundamental period is shifted outward and away from the damaging frequency range of common ground motion records [14].

In this paper we consider the American Uniform Building Code (UBC) as the basis for comparison with (5) because its consecutive editions provide a thorough historic development of seismic requirements. In particular, we use UBC'88 for illustrative purposes [15]. The International Conference of Building Officials (ICBO) typically adopts the seismic section of the Blue Book developed by the Structural Engineers Association of California (SEAOC) into UBC. Most of the

existing concrete buildings that we surveyed were built over eight years ago, but the present comparison applies to new construction as well because IBC has adopted, to a large extent, provisions from UBC, especially in its seismic design sections. Within that scope, simplified code formulas in an equivalent static analysis are used in lieu of full dynamic analyses. They help during the preliminary design phases to avoid large discrepancies between the physical behavior of the building, once in operation, and the estimates computed prior to construction.

### B. Conditions for Simplified Formulas

Most design codes require specific conditions for the use of simplified formulas, as they define a range for their applicability based on physical building properties. These are conditions of regularity and continuity.

UBC'88 allows the equivalent static method for regular structures under 240 feet (73 m) in height with a lateral force resisting system described in UCB'88 Table 23-O. A structure that has a stiffness, weight, or geometric irregularity is prescribed a dynamic analysis rather than equivalent static analysis. According to UBC'88, there are five types of irregularities labeled A through E [15].

Type A corresponds to a soft story defined as having 70% less stiffness than the story above or less than 80% of the average of the three stories above. Practically, this may happen if there is a considerable difference in story height, as the stiffness of a vertical element is inversely proportional to the cube of its length. Buildings with large, high ceiling, lobbies or buildings with a ground floor parking characterized by large open spaces in between columns, fall in that category.

Type B has to do with irregularity in weight (or mass) distribution. This applies to any story that has 1.5 times or more the mass of the adjacent story, i.e. either above it or below it. Practically, this may occur in stories where there is a significant change in dead load due to slab thickness, mezzanines, partitions, elevated gardens within the building and the like.

Type C corresponds to geometric irregularity whereby a story has a lateral force resisting member that has a horizontal dimension that it 1.3 times or more than that on the adjacent story. A sudden change in the length of a shear wall for architectural reasons would be an example of such irregularity.

Type D applies when there is an in-plane discontinuity in the vertical lateral force resisting element, where the discontinuity or offset is greater than the length of that element. A sudden interruption in an elevator shaft or an offset to a lesser number of shafts such as in buildings where lower stories are serviced by a larger number of elevators may fall in that category.

Type E has to do with discontinuity in story capacity. This is typically described from a strength perspective; shear or combined shear and flexural strength of the lateral force resisting system. If the strength on a given story is 80 percent or less of that of adjacent stories in terms of resisting story shear, then we term it a Type E discontinuity.

### C. Code-Based Fundamental Period

Consider the UBC '88 formula commonly used for the computation of the fundamental period:

$$T = C_t (h_n)^{3/4} \quad (6)$$

where  $C_t$  equals 0.03 for reinforced concrete moment-resisting frame buildings, 0.035 for steel moment-resisting frames, and 0.02 for all other buildings, and  $h_n$  is the building height in feet.

Equation (6) represents the empirical formula for the fundamental vibration period of buildings as specified in several U.S. building codes such as UBC 1997, Applied Technology Council 3-06 (ATC 3-06) [16], Structural Engineers Association of California 1996 (SEAOC 1996) Blue Book, and the National Earthquake Hazards Reduction Program (NEHRP-94) [17], which consists of Technical Briefs published by the National Institute of Standards and Technology (NIST). An even simpler formula applicable to buildings of twelve stories or less in height was adopted by NEHRP-94 before (6):

$$T = \frac{n}{10} \quad (7)$$

where  $n$  is the number of stories. Equation (7) over-simplifies the calculation of  $T$  as it does not require the substitution of any features related to the aspect ratio of the building, number of bays, or the basic properties related to a typical slab thickness or typical column dimensions.

Based on the variety of simplified formulas, the following sections provide a comparison with the proposed simplified formula in this paper, and the eigenvalue solution. The designer may therefore find a balance between simplicity of calculations and accuracy of results.

## IV. COMPARATIVE PERIOD CALCULATION

### A. Proposed Formula vs. Code Formulas

Both (5) and (7) have the number of stories  $n$  in their numerators indicating that total height is the main parameter as expressed in (6). However, the main difference between the two expressions is that the role of a typical slab and column, represented by the basic unit period  $T_o$  as explicitly shown in (5), and the role of the number of structural bays serve as correction factors over the code formula in (7). While (7) multiplies the number of stories  $n$  by 0.1, one would multiply  $n$  by the factor  $[N / (N+1)]^{1/2} T_o$  as computed by (5).

Another comparison is made between (5) and (6) where the effect of basic system properties is missing. A comparison is provided in Fig. 1 for a range of buildings that belong to our sample of 151 buildings, as well as results from our mathematical model using an exact matrix eigenvalue solution. Note how the results from (5) have a relatively small error in reference to the eigenvalue solution, while (6) and (7) are closer to each other but show a large discrepancy with both (5) and the eigenvalue results. This discrepancy is clearly illustrated for reinforced concrete buildings up to eight stories.

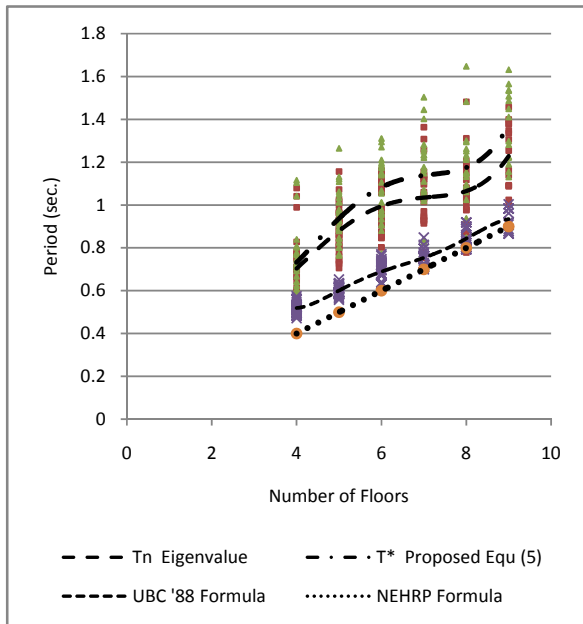


Fig. 1 Comparative period values from Eigenvalue solution, (5), UBC '88, and NEHRP

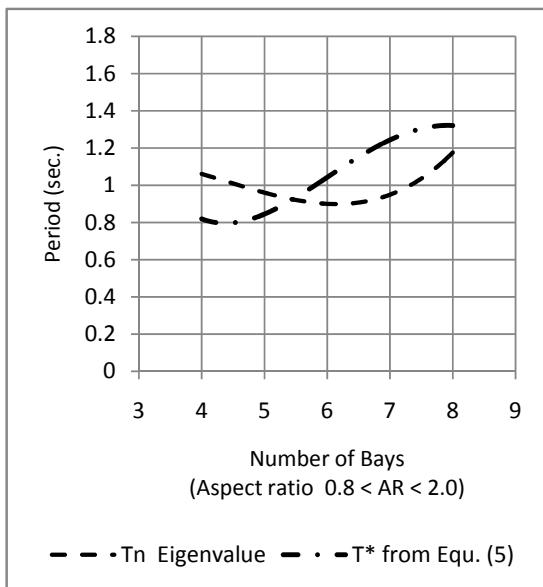


Fig. 2 Period values from Eigenvalue solution and (5), versus N for building aspect ratios between 0.8 and 2.0

Another comparison is made between (5) and the eigenvalue analysis results showing period as a function of the number of bays in the building. Although this graph on its own has no direct practical application because the number of bays by itself is not indicative of the structural configuration, it still shows that for buildings with aspect ratios between 0.8 and 2.0, which was the case for most of our data sample, the two solutions yield close results (Fig. 2).

A main advantage of including the basic unit period in the simplified formula is that it shows explicitly the mechanical

properties of a typical column in a concrete moment-resisting frame. While code formulas account for the structural system through the constant  $C_t$ , the difference between the multiplier for a moment-resisting frame and *all other buildings* as stated in UBC '88 is 50%. Therefore if the reinforced concrete building includes a shear wall, irrespective of its aspect ratio, the period computed by the code formula is expected to be significantly lower than the one of a moment-resisting frame. It is our view that such differences require a more accurate evaluation rather than simply assigning shear wall buildings to a general category.

The differentiation between moment-resisting frames and shear wall configurations is essential as it greatly influences the design procedure of reinforced concrete elements. The American Concrete Institute provisions emphasize this point by distinguishing between sway and non-sway frames in terms of magnification factors for slender column design when the column is expected to contribute to lateral force resistance, which is certainly the case in moment-resisting frames [18].

#### B. Discussion of Simplified Code Formulas for the Fundamental Period

As seen in previous sections, most simplified seismic code formulas express fundamental periods of buildings as a function of overall building height only, denoted by  $H$  in meters (m) or feet (ft) raised to a certain power and multiplied by a constant. It is our view that architectural codes need to be considered as well, side by side with engineering codes. Although the use of  $H$  as a single parameter is convenient and provides a simple determination of fundamental period, it would remain useful to include architectural features of the building to allow for revisions and coordination between the architect and the engineer.

Buildings that have a uniform stiffness and mass distribution along their heights lend themselves to an equivalent static analysis. ATC3-06 uses (6) in a slightly different format,  $T = C_t H^{0.75}$  ( $H$ , ft) with  $C_t = 0.03$  for reinforced concrete moment-resisting frames, and  $H$  equals total building height. This issue becomes even more critical when considering architectural and engineering code requirements side by side. Architectural codes address clearance and overhead requirements of a typical single story, which establishes a direct relationship between number of stories and overall building height. For instance, codes in several countries require apartment buildings to have a minimum floor to ceiling clearance of 2.7m. Designers may pack as many floors as the architectural code allows within the overall height limit, while other designers may favor a spacious 3.6m floor clearance, with a lesser number of stories, as it is the case in older buildings. The simplified Code formula would compute the same fundamental period for these two buildings because they would have the same total height, while in reality they have significantly different periods.

Alternatively, suppose that you are within the overall building height (architectural) restriction for both buildings, and you wish to build nine stories with a 2.7 m and 3.6 m floor clearance, respectively. Further, consider a slab thickness that ranges between 20 cm for a flat plate and 36 cm for a hordis system. A ten-story building could therefore have a total

height of 29 m; while another ten-story building could have a total height over 39 m. Expression (6) would then result in a period of 0.91 sec for the first building, and a period of 1.15 sec for the second building. The difference between these two periods, of about 26%, results in discrepancies in design spectra acceleration ratios, while (7) would compute a period of 1 sec for both buildings.

## V. IMPLICATIONS ON RESPONSE SPECTRA ANALYSIS

### A. Normalized Response Spectra Curves

The ratio of spectral acceleration to the effective peak ground acceleration is commonly referred to as a normalized response, also known as ratio of response spectra (RRS), and is given as a function of the fundamental period of the system under consideration [15], [16]. Spectral shapes for various soil conditions were developed following a statistical study of over one hundred records from twenty earthquakes most of which were in California [19], [20]. For all soil types, the shapes start at a single point of 1.0, which is the normalized acceleration for  $T = 0$  sec. as those shapes are developed on the basis of a peak ground acceleration adopted from hazard maps for rock conditions. For longer periods, the curves are different with great dependence on soil conditions. Based on that logic, simplified spectral shapes were developed by the Applied Technology Council and adopted in the Uniform Building Code [15], [16]. Those shapes included at the time three soil types S1, S2, and S3, which represented a range starting from stiff soils and rock (S1), deep cohesionless or stiff clay soils (S2), to soft to medium clays and sands (S3). At a later stage, and following the experience of the Mexico City earthquake which exhibited very low frequency ground motion, an S4 soil was added to account for deep soft clayish local site conditions [21].

The signal processing of similar records showed that the three soil conditions can be expressed in a digitized form for the purpose of response spectrum analysis as shown in (8a), (8b), and (8c), where the dimensionless value  $S_i$ -RRS corresponds to soil type  $S_i$  for  $i=1, 2$ , and 3, respectively, and  $T$  is the system period in seconds [22].

$$S_1\text{-RRS} = \begin{cases} 10T + 1.0 & 0 \leq T < 0.15 \\ 2.5 & 0.15 \leq T \leq 0.4 \\ 1.0193 T^{-1.013} & 0.4 < T \leq 3.0 \end{cases} \quad (8a)$$

$$S_2\text{-RRS} = \begin{cases} 10T + 1.0 & 0 \leq T < 0.15 \\ 2.5 & 0.15 \leq T \leq 0.575 \\ 1.4494 T^{-0.967} & 0.575 < T \leq 1.5 \\ 1.4302 T^{-0.936} & 1.5 < T \leq 3.0 \end{cases} \quad (8b)$$

$$S_3\text{-RRS} = \begin{cases} 7.5T + 1.0 & 0 \leq T < 0.2 \\ 2.5 & 0.2 \leq T \leq 0.9 \\ 2.284 T^{-0.992} & 0.9 < T \leq 3.0 \end{cases} \quad (8c)$$

### B. Application to Typical Buildings

In our examples above, we make reference to older codes that were in effect at the time the existing buildings that we surveyed were designed to simulate the provisions that the designer had to meet ten to thirty years ago. In UBC 1988, the design spectrum given in chapter 23, Fig. 3, distinguishes

three types of soil, type 1 (S1) for rock and stiff soils, type 2 (S2) for deep cohesionless or stiff clay soil, and type 3 (S3) for soft to medium clays [15].

To illustrate how the use of total height as the sole parameter for period calculation can throw off the design by a sizeable percentage, we considered two buildings from our data set that have practically equal heights. The first building is 13 m high with a clear floor space of 3m, and has a period of 0.56 sec from eigenvalue analysis. The second building is 13.8 m high with clear floor space of 3.25 m, and has a period of 0.83 sec from eigenvalue analysis. Using these two fundamental periods, and for a site type  $S_2$  for example, the ratio of the spectral acceleration to effective peak ground acceleration is 2.5 for the first building, and 1.73 for the second. This difference in the acceleration ratio causes a difference in base shear, which would have a significant impact on the lateral force distribution, and therefore on member sizing. But if we were to apply (6) to these two buildings we would have periods of 0.5 sec and 0.52 sec respectively, yielding practically the same spectral acceleration ratio for any soil type and exhibiting no difference in the lateral load requirements, or in the design of structural members. Many similar examples exit in the data set whereby the computed periods have significant differences with periods computed using the simplified code formula in (6) or (7), and hence in the acceleration ratios.

## VI. COMMENTS ON REINFORCED CONCRETE PARTICULARITIES

### A. Effect of Bond and Changes in Member Stiffness

Reinforced concrete has its own particularities as it is not an isotropic material and poses challenges in the calculation of flexural and shear strength and stiffness [23]-[25]. Combined beam-column theory applied to concrete design makes assumptions of strain compatibility and bond whereby the reinforcing steel and the surrounding concrete deform in the same rate [26], [27].

The flexural stiffness of the concrete section changes significantly when bond is reduced due to cracking, aging, or fatigue. The ACI recommends a reduction coefficient applied to the stiffness of the gross concrete section of 0.35 and 0.70 for the cracked and un-cracked section respectively [27]. Under these conditions, the effect of cracking on the fundamental period of the structure cannot be ignored and such considerations become very important in the retrofit of damaged concrete buildings [28], [29]. The basic unit period figuring in the proposed formula in (5) is affected by a factor of 1.19 for un-cracked sections and 1.6 for cracked sections [27].

The reductions in gross area moment of inertia that we used were in accordance with ACI provisions. The modified properties were included in the eigenvalue analysis of each building surveyed depending on the shape of the building. Comparative results showed a reasonable concurrence between the shape of the building and the calculated fundamental period.

### B. Effect of Concrete Aging and Long Term Volumetric Change

Another set of challenges presented by reinforced concrete buildings is related to aging. Creep, shrinkage, and volumetric changes due to mixing and placement affect the performance of the concrete member and in particular its stiffness [30].

Buildings that exhibited excessive damage due to impulse loading, and that were part of our data collection, were left outside this present treatment because they pose a different set of challenges. For all other buildings in the data set, the effect of aging was taken into consideration per ACI requirements.

## VII. CONCLUSIONS

Simplified code formulas for the determination of the fundamental period of reinforced concrete buildings offer a rapid way to estimate spectral response and seismic forces but require careful reconsideration because they lack important structural features. A theoretical model is developed to account for basic features of the building that translate into mass and stiffness parameters. It is found that individual story height, floor dimensions, and number of bays, when included in the period evaluation, yield results that are closer to the exact matrix eigenvalue solution than do simplified code formulas. The simplified code formulas, whether pertaining to total building height, or total number of stories, exhibited large discrepancies with the eigenvalue solution.

Particularities of reinforced concrete were taken into account in our calculations according to ACI's reduction factors applied to the moment of inertia of the gross section. However, more detailed treatment of the effects of loss of bond in damaged concrete structures requires further study on aging, excessive loading or impulse excitations, and was left for future research.

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