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Robust quadratic stabilization of uncertain impulsive switched systems

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Abstract—This paper focuses on the quadratic stabilization problem for a class of uncertain impulsive switched systems. The uncertainty is assumed to be norm-bounded and enters both the state and the input matrices. Based on the Lyapunov methods, some results on robust stabilization and quadratic stabilization for the impulsive switched system are obtained. A stabilizing state feedback control law realizing the robust stabilization of the closed-loop system is constructed.

Keywords—Impulsive systems; Switched systems; Quadratic stabilization; Robust stabilization.

I. Introduction

SWITCHED systems are an important class of hybrid dynamical systems, which are composed of a family of continuous-time or discrete-time subsystems and a rule orchestrating the switch among them. These systems arise when modeling dynamical systems which exhibit switching among several subsystems due to jumping parameters or changing environmental factors. Some examples of switched control systems can be found (see,[1], [2]). Another category of hybrid systems is the system with impulse effects, namely, impulsive systems, which arose in scientific practice in 1950s in order to describe certain evolutionary processes and dynamical control systems that are subjected to sudden and sharp changes of states. Due to the existence of the states jump, these new class of hybrid systems cannot be well described by using pure continuous or pure discrete models [3], [4], [8].

In fact, many practical systems in physics, biology, engineering, and information science exhibit impulsive dynamical behaviors due to abrupt changes at certain instants during the dynamical process [15]. Although hybrid and switched system is an important model for dealing with many complex physical processes, it does not cover above dynamical process when the impulse effects appear at the switching points because state jump usually occurs under such circumstances. In recent years, the study of hybrid and switched control systems with impulse effects called impulsive switched systems provides many effective approaches for controlling highly non-linear complex dynamical systems and systems with large uncertainties and unknown parameters [15], [7].

Robust stability analysis and control of dynamic systems with parameter uncertainty are problems of considerable theoretical and practical significance that have been attracting the interest of a number of investigators for several decades. The main focus has been on stabilization for linear systems [17], nonlinear systems [11], and systems without delay as well as with delays [11], [17], [6] and so on. Recently, interest has been extended to the stabilization of impulsive switched systems with norm-bounded time-varying uncertainty. A feedback control law realizing the closed loop impulsive switched system is asymptotically stable and robustly stable with definite attenuance and H_∞ performance is given in [12]. Among various techniques for robust stabilization (e.g., [5]), the so-called quadratic stabilization theory (see, [18], [16]) seems to be most effective in dealing with time-varying parameter uncertainty see, for example, [14].

In this paper, we consider the quadratic stabilization for the case in which norm bounded uncertainty appears both the state and input matrices. The problem is to design a feedback control law such that the closed loop impulsive switched system is quadratically stable and robustly stable. Based on the Riccati equation approach, a new result on the robust quadratic stabilization is obtained.

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II. Preliminaries

Throughout, $\mathbb{R}$ denotes the set of all real numbers. $\mathbb{R}^n$ stands for the $n-$dimensional real vector space and $\mathbb{R}^{n\times m}$ is the space of $n \times m$ matrices with real entries. For matrix A in $\mathbb{R}^{n\times n}$, $A > 0$ (< 0) means that A is a symmetrical positive (negative) definite matrix and $A \geq 0$ (≤ 0) means that A is a symmetrical positive (negative) semi-definite matrix. We use $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ to denote the smallest and largest eigenvalue of A, respectively. $\mathbb{N}$ presents the set of all nonnegative integers and $\|\cdot\|$ denotes the Euclidean norm of vectors.

Consider the uncertain impulsive switched system given by

$$\begin{aligned} &\dot{x}(t) = A_{\sigma(t)}(t)x(t) + \bar{A}_{\sigma(t)}(t)u(t),\ t \neq t_k, \\ &\Delta x(t) = D_k x(t) + g_k(t, x(t)),\ t = t_k, \\ &x(t_0) = x_0, \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $\sigma(t) : [0, \infty) \to \mathbb{M}$ is the switching signal mapping time to some finite index set $\mathbb{M}$ and a piecewise-constant continuous-from-the-left function taking values in $\mathbb{M}$, $\sigma(t) = i_k \in \mathbb{M}$ for $t \in [t_k, t_{k+1})$, $\mathbb{M} = \{1, 2, \cdots, m\}$, $k \in \mathbb{N}$. Under the control of a switching signal σ, coupling with the impulsive effects, system (1) enters from the i_{k-1} subsystem to the i_k subsystem at the point $t = t_k$, t_k is impulsive switching time point satisfying $0 = t_0 < t_1 < \cdots < t_k < \cdots < t_\infty = \infty$. $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^+) = \lim_{h\to 0^+} x(t_k + h)$ and $x(t_k) = x(t_k^-) = \lim_{h\to 0^+} x(t_k - h)$ mean that the solution of the system (1) is left continuous.

For each k, $D_k \in \mathbb{R}^{n\times n}$ is known matrix. $A_{i_k}(t)$ and $\bar{A}_{i_k}(t)$ are assumed to be uncertain and satisfy

$$\begin{bmatrix} A_{i_k}(t) & \bar{A}_{i_k}(t) \end{bmatrix} = \begin{bmatrix} A_{i_k} & \bar{A}_{i_k} \end{bmatrix} + E_{i_k}F_{i_k}(t)\begin{bmatrix} H_{i_k} & \bar{H}_{i_k} \end{bmatrix} \quad (2)$$

with A_{i_k}, $\bar{A}_{i_k}$, E_{i_k}, H_{i_k}, $\bar{H}_{i_k} \in \mathbb{R}^{n\times n}$ are known real matrices and $F_{i_k}(t) \in \mathbb{R}^{p\times q}$ is unknown time-varying matrix satisfying $\|F_{i_k}(t)\| \leq 1$. Besides, the elements of $F_{i_k}(t)$ are lebesgue measurable. Impulsive perturbation $g_k(t, x(t)) : [t_0, \infty) \times \mathbb{R}^n \to \mathbb{R}^n$ satisfies

$$g_k^T(t, x(t))g_k(t, x(t)) \leq x^T(t)(I + D_k)^T(I + D_k)x(t). \quad (3)$$

Also, $g_k(t, 0) \equiv 0$ for all $t \in [t_0, \infty)$.

Throughout this paper we shall use the following concept of quadratic stability and quadratic stabilization for system (1).

Definition 1: ([14]) The system (1)with $u(t) = 0$ is said to be quadratically stable if there exists a positive definite matrix $P_{i_k} \in \mathbb{R}^{n\times n}$ such that, for any admissible uncertainty satisfying (2), there exists a Lyapunov function $V(t) = x^T(t)P_{i_k}x(t)$ such that $V(t)$ decreases along every nonzero trajectory of system (1).

Definition 2: The system (1) is said to be quadratically stabilizable via linear state feedback if there exists a state feedback control $u(t) = -K_{i_k}x(t)$ such that the closed-loop system is quadratically stable.

Next, we present some lemmas and assumptions that are useful in deriving the principal contribution of this paper.

Lemma 1: ([9]) Let E, F and H be real matrices of appropriate dimensions with $\|F\| \leq 1$. Then for any scalars $\varepsilon > 0$,

$$EFH + H^TF^TE^T \leq \varepsilon^{-1}EE^T + \varepsilon H^TH.$$

Lemma 2: ([10]) If the algebraic Riccati equation $G^TS + SG - SWS + Q = 0$, $W > 0$, has a positive definite solution S, then for any $0 < W_1 \leq W$ and $Q_1 \geq Q$, the equation $G^TS_1 + S_1G - S_1W_1S_1 + Q_1 = 0$ has a positive definite solution $S_1 \geq S$.

Lemma 3: ([12]) Let $P \in \mathbb{R}^{n\times n}$ be a given positive definite matrix and $U \in \mathbb{R}^{n\times n}$ be a given symmetric matrix. Then

$$\lambda_{\min}(P^{-1}U)\Phi(t) \leq x^T(t)Ux(t) \leq \lambda_{\max}(P^{-1}U)\Phi(t),$$

where $\Phi(t) = x^T(t)Px(t)$, $x(t) \in \mathbb{R}^n$.

Lemma 4: ([9]) Given any $x \in \mathbb{R}^n$, $\max\{(x^TPEFHx)^2 : F^TF \leq I\} = x^TPEE^TPxx^TH^THx$.

Lemma 5: ([13]) Given $n \times n$ matrices $X \geq 0$, $Y < 0$, and $Z \geq 0$ such that $(\xi^TY\xi)^2 - 4(\xi^TX\xi\xi^TZ\xi) > 0$ for all $\xi \in \mathbb{R}^n$ with $\xi \neq 0$. Then there exists a constant $\delta > 0$ such that $\delta^2X + \delta Y + Z < 0$.

Lemma 6: (Finsler's Theorem) Let X be a given symmetric matrix and Z be a matrix such that $\xi^TX\xi < 0$ for all nonzero vectors ξ such that $Z\xi = 0$. Then there exists a constant $\vartheta > 0$ such that $X - \vartheta Z^TZ < 0$.

Assumption 1: Suppose $\mathrm{rank}(\bar{H}_{i_k}) = j \leq q$. Define $\Sigma_{i_k} \in \mathbb{R}^{j\times m}$ such that $\mathrm{rank}(\Sigma_{i_k}) = j$ and $\bar{H}_{i_k}^T\bar{H}_{i_k} = \Sigma_{i_k}^T\Sigma_{i_k}$. Let $\Phi_{i_k} \in \mathbb{R}^{(m-j)\times m}$ be chosen such that $\Phi_{i_k}\Sigma_{i_k}^T = 0$ and $\Phi_{i_k}\Phi_{i_k}^T = I$ ($\Phi_{i_k} = 0$ if $j = m$). Let $\Xi_{i_k} = \Sigma_{i_k}^T(\Sigma_{i_k}\Sigma_{i_k}^T)^{-2}\Sigma_{i_k}$. (Clearly, $\Xi_{i_k} = \Xi_{i_k}^T$).

In fact, if $\bar{H}_{i_k} = 0$, then $\Sigma_{i_k} = 0$, $\Phi_{i_k} = I$, and $\Xi_{i_k} = 0$. Also, if $\mathrm{rank}(\bar{H}_{i_k}) = m$, then Σ_{i_k} is square and nonsingular, $\Xi_{i_k} = (\Sigma_{i_k}^T\Sigma_{i_k})^{-1} = (\bar{H}_{i_k}^T\bar{H}_{i_k})^{-1}$ and $\Phi_{i_k} = 0$. Besides, since $\Phi_{i_k}\Sigma_{i_k}^T = 0$, then $\Phi_{i_k}\bar{H}_{i_k}^T = 0$.

III. Main results

In the sequel we shall present a sufficient condition for the quadratic stabilization of the uncertain impulsive switched system (1) via linear non-dynamic state feedback.

Theorem 1: The system (1) is quadratically stabilizable via linear control if there exists positive scalars ε and λ_k, positive definite matrix P_{i_k} such that the following conditions are satisfied: (i)

(i) $0 < \lambda_k < 1$, $\lambda_k = (2 + 2\frac{\lambda_{\max}(P_{i_k})}{\lambda_{\min}(P_{i_k})})\lambda_{\max}[P_{i_{k-1}}^{-1}(I + D_k)^TP_{i_k}(I + D_k)]$.

(ii)
$$\Omega_{i_k} = \begin{bmatrix} \Omega_{11i_k} & P_{i_k} \\ * & -\Omega_{22i_k} \end{bmatrix} < 0,$$
where
$\Omega_{11i_k} = (A_{i_k} - \bar{A}_{i_k}\Xi_{i_k}\bar{H}_{i_k}^TH_{i_k})^TP_{i_k} + P_{i_k}(A_{i_k} - \bar{A}_{i_k}\Xi_{i_k}\bar{H}_{i_k}^TH_{i_k}) + H_{i_k}^T(I - \bar{H}_{i_k}\Xi\bar{H}_{i_k}^T)H_{i_k}$, $\Omega_{22i_k}^{-1} = E_{i_k}E_{i_k}^T - \bar{A}_{i_k}\Xi_{i_k}\bar{A}_{i_k}^T - \frac{1}{\varepsilon}\bar{A}_{i_k}\Phi_{i_k}^T\Phi_{i_k}\bar{A}_{i_k}$.

(iii) A stabilizing state feedback control law is given by
$$u(t) = -\left[\left(\frac{1}{2\varepsilon}\Phi_{i_k}^T\Phi_{i_k} + \Xi_{i_k}\right)\bar{A}_{i_k}^TP_{i_k} + \Xi_{i_k}\bar{H}_{i_k}^TH_{i_k}\right]x(t).$$

Proof: Let the control law $u(t)$ be defined as in ((iii). We now show that system (1) with control law in ((iii) is quadratically stabilizale with the Lyapunov function

$$V(t) = x^T(t)P_{i_k}x(t).$$

Firstly, at the impulsive switching time point $t = t_k$, noticing that $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, we have

$$\begin{aligned}
&V(t_k^+)\\
=&x^T(t_k^+)P_{i_k}x(t_k^+)\\
=&(\Delta x(t_k) + x(t_k^-))^TP_{i_k}(\Delta x(t_k) + x(t_k^-))\\
=&[(I + D_k)x(t_k) + g_k(t_k, x(t_k))]^TP_{i_k}\\
&\times [(I + D_k)x(t_k) + g_k(t_k, x(t_k))].
\end{aligned}$$

By Lemma 1 and Lemma 3, we further derive that

$$\begin{aligned}
&V(t_k^+)\\
\leq&2x^T(I + D_k)^TP_{i_k}(I + D_k)x(t_k)\\
&+ 2g_k(t_k, x(t_k))^TP_{i_k}g_k(t_k, x(t_k))\\
\leq&(2 + 2\frac{\lambda_{\max}(P_{i_k})}{\lambda_{\min}(P_{i_k})})x^T(t_k)(I + D_k)^TP_{i_k}(I + D_k)x(t_k)\\
\leq&(2 + 2\frac{\lambda_{\max}(P_{i_k})}{\lambda_{\min}(P_{i_k})})\lambda_{\max}[P_{i_{k-1}}^{-1}(I + D_k)^TP_{i_k}(I + D_k)]\\
&\times x^T(t_k)P_{i_{k-1}}x(t_k)\\
=&\lambda_kV(t_k^-).
\end{aligned} \quad (4)$$

Since $0 < \lambda_k < 1$, then $V(t)$ is non-increasing at all impulsive switching time points t_k, $k \in \mathbb{N}$.

Secondly, we claim that the Lyapunov functional $V(t)$ is decreasing on each $(t_k, t_{k+1}]$.

In fact, for $t \in (t_k, t_{k+1})$, consider the upper right-hand derivative of $V(t)$ along the trajectory of system (1). According to the (2) and ((iii), it follows that

$$\begin{aligned}D^+V(t) =&x^T(t)\left[(A_{i_k}+E_{i_k}F_{i_k}(t)H_{i_k})^T P_{i_k}\right.\\&+P_{i_k}(A_{i_k}+E_{i_k}F_{i_k}(t)H_{i_k})]x(t)-\\&2x^T(t)P_{i_k}(\bar{A}_{i_k}+E_{i_k}F_{i_k}(t)\bar{H}_{i_k})\\&\times\left[\left(\frac{1}{2\varepsilon}\Phi_{i_k}^T\Phi_{i_k}+\Xi_{i_k}\right)\bar{A}_{i_k}^T P_{i_k}+\Xi_{i_k}\bar{H}_{i_k}^T H_{i_k}\right]x(t)\end{aligned}$$

Noticing that $\Phi_{i_k}\bar{H}_{i_k}^T=0$ and applying Lemma 1, one can obtain

$$\begin{aligned}&D^+V(t)\\&=x^T(t)(A_{i_k}^T P_{i_k}+P_{i_k}A_{i_k}-\frac{1}{\varepsilon}P_{i_k}\bar{A}_{i_k}\Phi_{i_k}^T\Phi_{i_k}\bar{A}_{i_k}^T P_{i_k}\\&\quad-2P_{i_k}\bar{A}_{i_k}\Xi_{i_k}\bar{A}_{i_k}^T P_{i_k}-P_{i_k}\bar{A}_{i_k}\Xi_{i_k}\bar{H}_{i_k}^T H_{i_k}\\&\quad-H_{i_k}^T\bar{H}_{i_k}\Xi_{i_k}\bar{A}_{i_k}^T P_{i_k})x(t)+2x^T(t)P_{i_k}E_{i_k}F_{i_k}(t)\\&\quad\times[H_{i_k}-\bar{H}_{i_k}\Xi_{i_k}(\bar{A}_{i_k}^T P_{i_k}+\bar{H}_{i_k}^T H_{i_k})]x(t)\\&\le x^T(t)(A_{i_k}^T P_{i_k}+P_{i_k}A_{i_k}-\frac{1}{\varepsilon}P_{i_k}\bar{A}_{i_k}\Phi_{i_k}^T\Phi_{i_k}\bar{A}_{i_k}^T P_{i_k}\\&\quad-2P_{i_k}\bar{A}_{i_k}\Xi_{i_k}\bar{A}_{i_k}^T P_{i_k}-P_{i_k}\bar{A}_{i_k}\Xi_{i_k}\bar{H}_{i_k}^T H_{i_k}\\&\quad-H_{i_k}^T\bar{H}_{i_k}\Xi_{i_k}\bar{A}_{i_k}^T P_{i_k})x(t)+x^T(t)P_{i_k}E_{i_k}E_{i_k}^T P_{i_k}x(t)\\&\quad+x^T(t)[H_{i_k}-\bar{H}_{i_k}\Xi_{i_k}(\bar{A}_{i_k}^T P_{i_k}+\bar{H}_{i_k}^T H_{i_k})]^T\\&\quad\times[H_{i_k}-\bar{H}_{i_k}\Xi_{i_k}(\bar{A}_{i_k}^T P_{i_k}+\bar{H}_{i_k}^T H_{i_k})]x(t).\end{aligned}\tag{5}$$

Substituting $\Xi_{i_k}=\Sigma_{i_k}^T(\Sigma_{i_k}\Sigma_{i_k}^T)^{-2}\Sigma_{i_k}$ and $\bar{H}_{i_k}^T\bar{H}_{i_k}=\Sigma_{i_k}^T\Sigma_{i_k}$ defined in Assumption 1 into (5), we get

$$\begin{aligned}&D^+V(t)\\&\le x^T(t)(A_{i_k}^T P_{i_k}+P_{i_k}A_{i_k}-\frac{1}{\varepsilon}P_{i_k}\bar{A}_{i_k}\Phi_{i_k}^T\Phi_{i_k}\bar{A}_{i_k}^T P_{i_k}\\&\quad-P_{i_k}\bar{A}_{i_k}\Xi_{i_k}\bar{A}_{i_k}^T P_{i_k}-P_{i_k}\bar{A}_{i_k}\Xi_{i_k}\bar{H}_{i_k}^T H_{i_k}\\&\quad-H_{i_k}^T\bar{H}_{i_k}\Xi_{i_k}\bar{A}_{i_k}^T P_{i_k}+P_{i_k}E_{i_k}E_{i_k}^T P_{i_k}+H_{i_k}^T H_{i_k}\\&\quad-H_{i_k}^T\bar{H}_{i_k}\Xi_{i_k}\bar{H}_{i_k}^T H_{i_k})x(t).\end{aligned}$$

For all $x(t)\neq 0$, $t\in(t_k,t_{k+1})$, taking ((ii) into account, yields

$$D^+V(t)<0.\tag{6}$$

Moreover, since $x(t_{k+1})=x(t_{k+1}^-)$, then $V(t)$ is decreasing on $(t_k,t_{k+1}]$ as the claim.

From (4) and (6), we conclude that system (1) via the linear control law ((iii) is quadratically stabilizable.

Next, we suppose (1) is quadratically stabilizable via linear state feedback $u(t)=-K_{i_k}x(t)$, $K_{i_k}\in\mathbb{R}^{m\times n}$. It follows from Definition 1 that there exists a positive definite symmetric matrix S_{i_k} such that

$$\begin{aligned}&[A_{i_k}+E_{i_k}F_{i_k}(t)H_{i_k}-(\bar{A}_{i_k}+E_{i_k}F_{i_k}(t)\bar{H}_{i_k})K_{i_k}]^T S_{i_k}\\&+S_{i_k}[A_{i_k}+E_{i_k}F_{i_k}(t)H_{i_k}-(\bar{A}_{i_k}+E_{i_k}F_{i_k}(t)\bar{H}_{i_k})K_{i_k}]<0.\end{aligned}$$

The above inequality can be rewritten as

$$\begin{aligned}&(A_{i_k}-\bar{A}_{i_k}K_{i_k})^T S_{i_k}+S_{i_k}(A_{i_k}-\bar{A}_{i_k}K_{i_k})\\&<S_{i_k}E_{i_k}F_{i_k}(t)(\bar{H}_{i_k}K_{i_k}-H_{i_k})-(H_{i_k}-\bar{H}_{i_k}K_{i_k})^T F_{i_k}^T E_{i_k}^T S_{i_k}.\end{aligned}$$

Thus, for all $x(t)\in\mathbb{R}^n,\ x\neq 0$,

$$\begin{aligned}&x^T(t)[(A_{i_k}-\bar{A}_{i_k}K_{i_k})^T S_{i_k}+S_{i_k}(A_{i_k}-\bar{A}_{i_k}K_{i_k})]x(t)\\&<-2x^T(t)S_{i_k}E_{i_k}F_{i_k}(t)(H_{i_k}-\bar{H}_{i_k}K_{i_k})x(t).\end{aligned}$$

Clearly,

$$\begin{aligned}&x^T(t)[(A_{i_k}-\bar{A}_{i_k}K_{i_k})^T S_{i_k}+S_{i_k}(A_{i_k}-\bar{A}_{i_k}K_{i_k})]x(t)\\&<-2\max[x^T(t)S_{i_k}E_{i_k}F_{i_k}(t)(H_{i_k}-\bar{H}_{i_k}K_{i_k})x(t)]\le 0.\end{aligned}$$

Using Lemma 4, we see that

$$\begin{aligned}&\{x^T(t)[(A_{i_k}-\bar{A}_{i_k}K_{i_k})^T S_{i_k}+S_{i_k}(A_{i_k}-\bar{A}_{i_k}K_{i_k})]x(t)\}^2\\&\quad>4x^T(t)S_{i_k}E_{i_k}E_{i_k}^T S_{i_k}x(t)x^T(t)(H_{i_k}-\bar{H}_{i_k}K_{i_k})^T\\&\qquad\times(H_{i_k}-\bar{H}_{i_k}K_{i_k})x(t).\end{aligned}$$

By Lemma 5 then yields that there exists a constant $\delta>0$ such that

$$\begin{aligned}&\delta^2 S_{i_k}E_{i_k}E_{i_k}^T S_{i_k}+\delta[(A_{i_k}-\bar{A}_{i_k}K_{i_k})^T S_{i_k}\\&\quad+S_{i_k}(A_{i_k}-\bar{A}_{i_k}K_{i_k})]\\&\quad+(H_{i_k}-\bar{H}_{i_k}K_{i_k})^T(H_{i_k}-\bar{H}_{i_k}K_{i_k})<0.\end{aligned}\tag{7}$$

We now define $P_{i_k}=\delta S_{i_k}$, then (7) becomes

$$\begin{aligned}&A_{i_k}^T P_{i_k}+P_{i_k}A_{i_k}+H_{i_k}^T H_{i_k}+P_{i_k}E_{i_k}E_{i_k}^T P_{i_k}\\&-K_{i_k}^T(\bar{H}_{i_k}^T H_{i_k}+\bar{A}_{i_k}^T P_{i_k})-(\bar{H}_{i_k}^T H_{i_k}+\bar{A}_{i_k}^T P_{i_k})^T K_{i_k}\\&+K_{i_k}^T\bar{H}_{i_k}^T\bar{H}_{i_k}K_{i_k}<0.\end{aligned}\tag{8}$$

Set $T_{i_k}=[\Sigma_{i_k}^T\ \Phi_{i_k}^T]\in\mathbb{R}^{m\times m}$ and $L=[L_1^T\ L_2^T]^T=T_{i_k}^{-1}K_{i_k}$, then T is nonsingular and $K_{i_k}=T_{i_k}L$. Besides, if $\bar{H}_{i_k}=0$, then $T_{i_k}=\Phi_{i_k}$. It follows that

$$\begin{aligned}&K_{i_k}^T\bar{H}_{i_k}^T\bar{H}_{i_k}K_{i_k}-K_{i_k}^T(\bar{H}_{i_k}^T H_{i_k}+\bar{A}_{i_k}^T P_{i_k})\\&\quad-(\bar{H}_{i_k}^T H_{i_k}+\bar{A}_{i_k}^T P_{i_k})^T K_{i_k}\\&=L_1^T(\Sigma_{i_k}\Sigma_{i_k}^T)^2 L_1-L_1^T\Sigma_{i_k}(\bar{H}_{i_k}^T H_{i_k}+\bar{A}_{i_k}^T P_{i_k})\\&\quad-(H_{i_k}^T\bar{H}_{i_k}+P_{i_k}\bar{A}_{i_k})\Sigma_{i_k}^T L_1\\&\quad-L_2^T\Phi_{i_k}\bar{A}_{i_k}^T P_{i_k}-P_{i_k}\bar{A}_{i_k}\Phi_{i_k}^T L_2\\&=[(\Sigma_{i_k}\Sigma_{i_k}^T)^{-1}\Sigma_{i_k}(\bar{H}_{i_k}^T H_{i_k}+\bar{A}_{i_k}^T P_{i_k})\\&\quad-\Sigma_{i_k}\Sigma_{i_k}^T L_1]^T[(\Sigma_{i_k}\Sigma_{i_k}^T)^{-1}\Sigma_{i_k}(\bar{H}_{i_k}^T H_{i_k}\\&\quad+\bar{A}_{i_k}^T P_{i_k})-\Sigma_{i_k}\Sigma_{i_k}^T L_1]-(H_{i_k}^T\bar{H}_{i_k}+P_{i_k}\bar{A}_{i_k})\\&\quad\times\Xi_{i_k}(\bar{H}_{i_k}^T H_{i_k}+\bar{A}_{i_k}^T P_{i_k})\\&\quad-L_2^T\Phi_{i_k}\bar{A}_{i_k}^T P_{i_k}-P_{i_k}\bar{A}_{i_k}\Phi_{i_k}^T L_2.\end{aligned}\tag{9}$$

Combining (8) and (9), we obtain

$$\begin{aligned}&[(\Sigma_{i_k}\Sigma_{i_k}^T)^{-1}\Sigma_{i_k}(\bar{H}_{i_k}^T H_{i_k}+\bar{A}_{i_k}^T P_{i_k})-\Sigma_{i_k}\Sigma_{i_k}^T L_1]^T\\&\times[(\Sigma_{i_k}\Sigma_{i_k}^T)^{-1}\Sigma_{i_k}(\bar{H}_{i_k}^T H_{i_k}+\bar{A}_{i_k}^T P_{i_k})-\Sigma_{i_k}\Sigma_{i_k}^T L_1]\\&-(H_{i_k}^T\bar{H}_{i_k}+P_{i_k}\bar{A}_{i_k})\Xi_{i_k}(\bar{H}_{i_k}^T H_{i_k}+\bar{A}_{i_k}^T P_{i_k})-L_2^T\Phi_{i_k}\bar{A}_{i_k}^T P_{i_k}\\&-P_{i_k}\bar{A}_{i_k}\Phi_{i_k}^T L_2+A_{i_k}^T P_{i_k}+P_{i_k}A_{i_k}+H_{i_k}^T H_{i_k}\\&+P_{i_k}E_{i_k}E_{i_k}^T P_{i_k}<0.\end{aligned}$$

For all $x(t)\neq 0$ such that $\Phi_{i_k}\bar{A}_{i_k}^T P_{i_k}x(t)=0$. Then

$$\begin{aligned}&x^T(t)[A_{i_k}^T P_{i_k}+P_{i_k}A_{i_k}+H_{i_k}^T H_{i_k}+P_{i_k}E_{i_k}E_{i_k}^T P_{i_k}\\&-(H_{i_k}^T\bar{H}_{i_k}+P_{i_k}\bar{A}_{i_k})\Xi_{i_k}(\bar{H}_{i_k}^T H_{i_k}+\bar{A}_{i_k}^T P_{i_k})]x(t)<0.\end{aligned}$$

Applying Lemma 6, one can derive that there exists a positive scalar ε such that

$$\begin{aligned}&A_{i_k}^T P_{i_k} + P_{i_k} A_{i_k} + H_{i_k}^T H_{i_k} + P_{i_k} E_{i_k} E_{i_k}^T P_{i_k}\\ &- (H_{i_k}^T \bar{H}_{i_k} + P_{i_k} \bar{A}_{i_k}) \Xi_{i_k} (\bar{H}_{i_k}^T H_{i_k}\\ &+ \bar{A}_{i_k}^T P_{i_k}) - \frac{1}{\varepsilon} P_{i_k} \bar{A}_{i_k} \Phi_{i_k}^T \Phi_{i_k} \bar{A}_{i_k}^T P_{i_k} < 0.\end{aligned}$$

That is,

$$\begin{aligned}&(A_{i_k} - \bar{A}_{i_k} \Xi_{i_k} \bar{H}_{i_k}^T H_{i_k})^T P_{i_k} + P_{i_k} (A_{i_k} - \bar{A}_{i_k} \Xi_{i_k} \bar{H}_{i_k}^T H_{i_k})\\ &+ H_{i_k}^T (I - \bar{H}_{i_k} \Xi \bar{H}_{i_k}^T) H_{i_k} + P_{i_k} E_{i_k} E_{i_k}^T P_{i_k}\\ &- P_{i_k} \bar{A}_{i_k} \Xi_{i_k} \bar{A}_{i_k}^T P_{i_k} - \frac{1}{\varepsilon} P_{i_k} \bar{A}_{i_k} \Phi_{i_k}^T \Phi_{i_k} \bar{A}_{i_k} P_{i_k} < 0.\end{aligned} \tag{10}$$

By Schur complement, (10) is equivalent to the condition given by ((ii). This completes the proof of the theorem. ■

IV. Conclusions

In this paper, we considered a class of uncertain impulsive switched systems. By applying Lyapunov function method, some sufficient conditions of robust stabilization and quadratic stabilization for such systems were established. These conditions can guarantee the design of the stabilizing state feedback control law.

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