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Robust Numerical Scheme for Pricing American Options under Jump Diffusion Models

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Abstract—The goal of option pricing theory is to help the investors to manage their money, enhance returns and control their financial future by theoretically valuing their options. However, most of the option pricing models have no analytical solution. Furthermore, not all the numerical methods are efficient to solve these models because they have nonsmoothing payoffs or discontinuous derivatives at the exercise price. In this paper, we solve the American option under jump diffusion models by using efficient time-dependent numerical methods. several techniques are integrated to reduced the overcome the computational complexity. Fast Fourier Transform (FFT) algorithm is used as a matrix-vector multiplication solver, which reduces the complexity from $\mathcal{O}(M^2)$ into $\mathcal{O}(M \log M)$. Partial fraction decomposition technique is applied to rational approximation schemes to overcome the complexity of inverting polynomial of matrices. The proposed method is easy to implement on serial or parallel versions. Numerical results are presented to prove the accuracy and efficiency of the proposed method.

Keywords—Integral differential equations, American options, jump–diffusion model, rational approximation.

I. Introduction

OPTION pricing models have been developed rapidly after the classical Black-Scholes model [1]. Just years later, empirical studies revealed that the Black-Scholes model is inconsistent with the market movements. Many studies have revealed to overcome these shortcomings, such as Lèvy models and jump-diffusion models (JDM) [2], [3]. Unlike in the European options [4], [5], American options do not have closed-form to calculate. Thus, extensive research in numerical methods has been conducted and applied in valuating American options [6], [7]. Different methods have been used to approximate the linear complementary problems (LCPs), such as the penalty method proposed by [8], [9]. operator splitting method [10], IMEX [11]–[14]. Khaliq et al. [15] developed numerical schemes based on Padè approximations of matrix exponential functions using exponential time differencing combined with Runge-Kutta (ETDRK) applied to the Black-Scholes model, Burgers equation, etc. (see [16], [17]). In this paper, we extend the work of Khaliq et al. [15] to apply it for American options with jump diffusion model. We use several methods to increase the computation speed, such as the FFT algorithm, partial fraction decomposition technique. This paper is organized as follows: the continuous problem is described in Section I. Discretization in space as well as in time are given in Sections III and IV respectively. The fast Fourier transform algorithm (FFA) is described in Section V. Several numerical examples are given in Section VI to show the efficiency and accuracy of our algorithms. We finalized the paper by a short conclusion in Section VII.

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II. Jump Diffusion Model

Consider that $v(x,\tau)$ is the value price of the asset x at time τ satisfying the integral differential equation

$$v_\tau - \frac{1}{2}\sigma v_{xx} - (r - \frac{1}{2}\sigma^2 - \kappa\lambda)v_x + (r+\lambda)v -$$

$$\lambda \int_{-\infty}^{\infty} v(z,\tau)\phi(z-x)dz = \tilde{p}(v,\tilde{v}) \qquad (1)$$

with the the following are the initial and boundary conditions:

- Call option

$$\begin{cases} v(x,0) = max(Ee^x - E), \\ v(X_{min},\tau) = 0, \\ v(X_{max},\tau) = Ee^{x_{max}} - E, \end{cases}$$

- Put option

$$\begin{cases} v(x,0) = max(E - Ee^x), \\ v(X_{min},\tau) = E, \\ v(X_{max},\tau) = 0, \end{cases}$$

where $v^{\sim}(x,\tau)$ is the payoff received at the exercising time $\tau \leq T$ and $p^{\sim}(V,\tilde{V})$ is the penalty term [18] given by

$$p(v,\tilde{v}) = \frac{1}{\epsilon} max\Big\{v(x,\tau) - \tilde{v}(x,\tau), 0\Big\}, \qquad 0 \leq \epsilon \leq 1$$

where E is the exercise price, σ is the volatility and r is the rate of interest, λ is the Poisson intensity and κ is the expectation of the the impulse function. For the boundary conditions, X_{min} and X_{max} are a boundaries of the truncated domain $\Omega \in (-\infty,\infty)$ and ϕ is the density function of the normal distribution function with mean μ, variance δ^2 and $\kappa = e^{(\mu - \frac{\delta^2}{2})} - 1$.

III. Spatial Discretization

The infinite space is truncated to $\Omega = [X_{min}, X_{max}]$ [19]. Then we descritize the differentiation term by the 2nd order central finite difference and fourth order Chebyshev Spectral Method [20] whereas the integral term is approximated the 2nd order composite Trapezoidal Rule and a fourth order quadrature rule at each subinterval in space. Therefore, the PIDE(1) can be written as a semi-linear system of ODEs given by

$$v'(\tau) + Av = \lambda F(v, \tau) \tag{2}$$

where A is given by

$$A = \frac{1}{2}\sigma^2 D_M^2 + \left(r - \frac{1}{2}\sigma^2 - \kappa\lambda\right) D_M - (r+\lambda)I$$

in case of Chebyshev spectral methods, where D_M and D_M^2 are the first and second Chebyshev spectral differentiation matrices respectively whereas, in case of the FDM, A is given by

$$A = \text{tridiag}\left(\frac{c_1}{2h^2} - \frac{c_2}{2h}\ , -\frac{c_1}{h^2} - r - \lambda\ , \frac{c_1}{2h^2} + \frac{c_2}{2h}\right)$$

where

$$c_1 = \sigma^2 \qquad \text{and} \qquad c_2 = r - \lambda\kappa - \frac{\sigma^2}{2}$$

Approximating the integral term by either composite Trapezoidal Rule, which is less accurate or Clenshaw-Curtis Quadrature, which is higher accurate, will end up with either tridiagonal Toeplitz matrix which is computationally cheaper or a dense matrix which is computationally exhaustive respectively. The semi-linear function F is the approximated integral term and the penalty term of the PIDE(1).

IV. Full Discretization

For the semi-discretized system of the ODEs given in (2), define $k \geq 0$ by the time step size, $\tau_n = nk$, $0 \leq n \leq N$. Thus,using Duhamel principle [15], the semi-discretized system of the ODEs has exact solution given by

$$v(\tau) = e^{-kA}v(\tau) + \int_0^\tau e^{-kA(\tau-\nu)}F(v(\nu), \nu)d\nu, \tag{3}$$

by changing variable, $\nu - \tau = ks$ and putting $v_n = v(\tau_n)$, we get a simpler form of the exact solution

$$v_{n+1} = e^{-kA}v_n + k\int_0^1 e^{-kA(1-s)}F(v(\tau_n + ks), \tau_n + ks)ds, \tag{4}$$

Several time stepping schemes were developed (see [21], [22]) to approximate the recurrence formula (4). However, their schemes require to invert matrix higher order polynomials which is computationally expensive. Later, [23] used partial fraction form of rational approximations to approximate the matrix exponential functions arising in the exact solutions.

A. Padé Approximation

Padè approximation, type of rational approximation, is an important tool to approximate matrix exponential functions. It is named after the French mathematician Henri Padé (1863-1953). Padè schemes are the approximants derived by a ratio of two power series approximations.Due to the rational form of Padè approximations, it is better than Taylor expansions when approximating functions containing poles. Following [24], Padé approximation of order $(n+m+1)$ of the exponential function e^{-x} is given by

$$R_m^n(x) = \frac{P_m^n(x)}{Q_m^n(x)} + O(x^{n+m+1})$$

where $P_m^n(x)$ and $Q_m^n(x)$ are polynomials of order n and m respectively given by

$$P_m^n(x) = \sum_{i=0}^{n} \frac{(m+n+i)n!}{(m+n)!i!(n-i)!}(-x)^i$$

$$Q_m^n(x) = \sum_{j=0}^{m} \frac{(m+n+j)n!}{(m+n)!j!(m-i)!}(x)^j$$

with the property $R_m^n(x) = e^{-x} + O(x^{n+m+1})$ when $x \to 0$.
Illustrative examples: The following functions are different lower and higher orders Padé approximations of e^{-x} :

$R_1^0(x) = (1+x)^{-1}$ (Backward Euler)
$R_1^1(x) = (1 - \frac{1}{2}x)(1+\frac{1}{2}x)^{-1}$ (Crank-Nicolson)
$R_2^1(x) = (1 - \frac{1}{3}x)(1 + \frac{2}{3}x + \frac{1}{6}x^2)^{-1}$
$R_4^0(x) = 24(24 + 24x + 12x^2 + 4x^3 + x^4)^{-1}$

B. Partial Fraction form Padè Approximation

The ETDRK schemes mentioned in the previous section contain lower and higher order polynomials of matrices which cause computational difficulties. In this regard, [25] and [26] have made important contributions to address this issue. They used the partial fraction technique to implement the Padè schemes. Not only this, but also they implemented efficient serial and parallel algorithms. Yousuf et al. [27] developed algorithms to implement diagonal and damping subdiagonal schemes and obtained the following version of schemes. Although we are interest only on two particular types of Padè approximations, for the sake of generalization, we shall give schemes of (n,m)- Padè in general for any positive integer numbers n, m.

Case1: $n < m$,

$$R_m^n(x) = \sum_{j=1}^{q_1} \frac{\omega_j}{x - c_j} + 2\sum_{j=q_1+1}^{q_1+q_2} \Re\left(\frac{\omega_j}{x - c_j}\right)$$

Case2: $n = m$,

$$R_m^m(x) = (-1)^m + \sum_{j=1}^{q_1} \frac{\omega_j}{x - c_j} + 2\sum_{j=q_1+1}^{q_1+q_2} \Re\left(\frac{\omega_j}{x - c_j}\right)$$

where q_1 and q_2 are the number of real and non-real poles $\bar{c}_j$ of $\bar{R}_m^n$.

Therefore, our fully discretized scheme is given by

$$v_{n+1} = R_4^0(kA)v_n + \mathcal{P}_1(kA)F(v_n,t_n) + \mathcal{P}_2(kA)\big(F(\alpha_n, t_n + k/2 + F(\beta_n, t_n + k/2)\big) + \mathcal{P}_3(kA)F(\gamma_n, t_n), \quad (5)$$

where

$$\alpha_n = \bar{R}_4^0(kA)v_n + \bar{\mathcal{P}}(kA)F(v_n,t_n),$$

$$\beta_n = \bar{R}_4^0(kA)v_n + \bar{\mathcal{P}}(kA)F(\alpha_n,t_n + k/2),$$

$$\gamma_n = \bar{R}_4^0(kA)v_n + \bar{\mathcal{P}}(kA)\Big(2F(\beta_n,t_n + k/2) - F(v_n,t_n)\Big),$$

and

$$R_4^0(x) = 2\Re\left(\frac{\omega_1}{x - c_1}\right) + 2\Re\left(\frac{\omega_2}{x - c_2}\right),$$

$$\mathcal{P}_i = 2\Re\left(\frac{\omega_{i1}}{x - c_1}\right) + 2\Re\left(\frac{\omega_{i2}}{x - c_2}\right), \quad i = 1, 2, 3.$$

$$\bar{R}_4^0(x) = 2\Re\left(\frac{\bar{\omega}_1}{x - \bar{c}_1}\right) + 2\Re\left(\frac{\bar{\omega}_2}{x - \bar{c}_2}\right),$$

and

$$\mathcal{P}(x) = 2\Re\left(\frac{\Omega_1}{x - \bar{c}_1} + 2\Re\left(\frac{\Omega_2}{x - \bar{c}_2}\right)\right),$$

where the poles c_i and $\bar{c_i}$ and the corresponding weights ω_i, $\bar{\omega_i}$, ω_{ji} and Ω_i, $i = 1, 2$ and $j = 1, 2, 3$ are given by

$$\begin{aligned}
c_1 &= -1.72944423106769 + i0.888974376121862.\\
c_2 &= -0.2705557689322 - i2.50477590436244.\\
\omega_1 &= 0.541413348429182 - i1.58885918222330.\\
\omega_2 &= -0.541413348429154 - i0.248562520866115.\\
\omega_{11} &= 0.244153693956274 - i0.0497524711964030.\\
\omega_{12} &= -0.244153693956268 - i0.0750708534900480.\\
\omega_{21} &= -0.0240066687966667 - i0.210771761184790.\\
\omega_{22} &= 0.0240066687966698 + i0.110830774318527.\\
\omega_{31} &= 0.473042583717175 + i0.293424221840328.\\
\omega_{32} &= 0.0269574162828241 - i0.165188084403066\\
\bar{c}_1 &= -3.45888846213543 - i1.77794875224371.\\
\bar{c}_2 &= -0.541111537864595 - i5.00955180872487.\\
\bar{\omega}_1 &= 1.08282669685827 + i3.17771836444659.\\
\bar{\omega}_2 &= -1.08282669685831 - i0.497125041732246.\\
\Omega_1 &= -0.621169602486758 - i0.599415294095229.\\
\Omega_2 &= 0.121169602486770 - i0.203064159380992.
\end{aligned}$$

C. Stability Analysis

Consider the nonlinear ODE,

$$v_t = cv + F(v) \quad (6)$$

where $F(v)$ is the nonlinear term. We assume that there exist a fixed point $v_0 = v(t_0)$, such that $cv_0 + F(v_0) = 0$. We linearize about the fixed point to lead to

$$v_t = cv + \lambda v. \quad (7)$$

where v becomes the perturbation to v_0, whereas, $\lambda = F'(v_0)$. If $\Re(c + \lambda) < 0$, then the fixed point v_0 is stable [21].

To obtain the stability region of the numerical methods, we first denote $\xi = \lambda k$ and $\eta = ck$, where k is the time step-size,

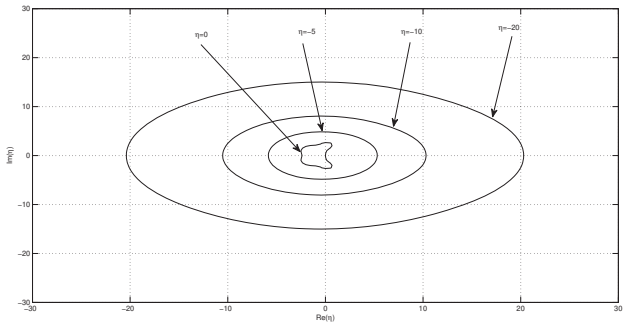

Fig. 1 Stability regions of (0,4)-Pad`e scheme in the complex ξ-plane

then we apply (5) to the ODE (6) leading to a recurrence relation involving v_n and v_{n+1}.

$$\frac{v_{n+1}}{v_n} = r(\xi,\eta) = \frac{c_0 + c_1\xi + c_2\xi^2 + c_3\xi^3 + c_4\xi^4}{D} \quad (8)$$

where

$$D = (24 - 24\eta + 12\eta^2 - 4\eta^3 + \eta^4)(384 - 192\eta + 48\eta^2 - 8\eta^3 + \eta^4)^3$$

where c, λ, ξ and η are complex numbers. Following [21], for a better and useful method, the stability regions grow as $|\eta|$ becomes larger. Therefore, we fix η with several negative real values , $\eta = 0$, $\eta = -5$, $\eta = -10$ and $\eta = -20$, in the complex ξ-plane.

In Fig. 1, we show the stability region in the complex ξ-plane for different negative real η, $\eta = 0$, $\eta = -5$, $\eta = -10$ and $\eta = -20$. When $\eta \to 0$ the stability region tends to the fourth-order Runge-Kutta scheme; and, as η degreases from -10 to -20 the region grows. This region gives an indication of the stability of the (0,4)-Padè.

V. The FFT Algorithm

Thanks to the approximation of the integral part is a Toeplitz matrix, we can reduce the cost of vector-matrix multiplication using what is so-called Fast Fourier Transform (FFT) algorithm [28]

Definition 1. The Discrete Fourier Transform (DFT) is a Transform on $\mathbf{C^m}$ given as

$$[D\{b\}]_j = \frac{1}{\sqrt{m}} \sum_{k=0}^{m-1} b_k e^{-i2\pi jk/m}, \qquad j = 0, 1, ..., m-1 \quad (9)$$

where b is a vector and D is a Fourier matrix $\in \mathbf{C^{m\times m}}$

Whereas the inverse DFT of a vector b is given by

$$[D^{-1}\{b\}]_j = \frac{1}{\sqrt{m}} \sum_{k=0}^{m-1} b_k e^{i2\pi jk/m}, \qquad j = 0, 1, ..., m-1 \quad (10)$$

The use FFT algorithm is to evaluate the The Discrete Fourier Transform (DFT) of a vector b of length M whose operations cost is $O(M^2)$ with a cost of $O(M\,logM)$ operation [29].

Definition 2. A square matrix $[A]_{M\times M}$ is called a Toeplitze Matrix if it has the form

$$A=\begin{bmatrix} a_0 & a_{-1} & \dots & a_{2-M} & a_{1-M} \\ a_1 & a_0 & a_{-1} & \ddots & a_{1-M} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ a_{M-2} & \ddots & a_1 & a_0 & a_{-1} \\ a_{M-1} & a_{M-2} & \dots & a_1 & a_0 \end{bmatrix}$$

We may generate the toeplitze matrix using the mathematical software Matlab by calling $toeplitz(a)$, where $a=(a_{M-1},a_{M-2},\dots,a_0,a_{-1},a_{-2}\dots,a_{1-M})$

Definition 3. A Circulant matrix $[C]_{M\times M}$ is special case of a Toeplitz matrix when for each of its rows is a right cyclic shift of the row preceding it, $(i.e)$

$$C=\begin{bmatrix} a_0 & a_{M-1} & \dots & a_2 & a_1 \\ a_1 & a_0 & a_{M-1} & \ddots & a_2 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ a_{M-2} & \ddots & a_1 & a_0 & a_{M-1} \\ a_{M-1} & a_{M-2} & \dots & a_1 & a_0 \end{bmatrix}$$

To evaluate the vector-Matrix multiplication $[A]_{M\times M}\ [b]_{M\times 1}$, we embed [28] the matrix A into a circulant matrix $[C]_{(2M-1)\times(2M-1)}$, let $\hat{b}=[b,0,\cdots,0]$ where $\hat{b}$ is of size $2M-1$. Next step is to apply the DFT transform to the vector $\hat{b}$ and vector c where c is the first row and column of the circulant matrix C, then apply the inverse DFT transform to the product of transformed two vectors. $(i.e)$

$$\mathbf{v}=\mathbf{ifft}\big(\mathbf{fft}(\mathbf{c}).\mathbf{fft}(\hat{\mathbf{b}})\big) \tag{11}$$

Finally we extract the first M elements of the vector $\mathbf{v}$ to be the desired result.

VI. Numerical Experiments

We test performance of our scheme by showing its efficiency and accuracy. Our concern is not only the convergence, but also on the computational cost.

A. Convergence Test

To compare convergence of finite difference method and the spectral method, we shall use the second order central finite difference method and the Chebychev spectral method to discretize the domain Ω in space. Regarding to the integral term (jump part), we use composite trapezoidal rule when using finite difference method. Whereas, the Clenshaw-Curtis Quadrature is used to approximate the integral term when using the spectral method. We consider the truncated domain $\Omega=[-2.2,2.2]$, with $E=100$, $\sigma=0.15$, $\delta=0.25$ $r=0.04$, $\lambda=1$,$\mu=0$,$T=1$ and $\epsilon=0.01$.

TABLE I
Order of Convergence in Space of FD and Spectral Methods at a Fixed Number of Time-Steps $N=100$

	Central FD Method			Spectral method		
M	Value	Error	Order	Value	Error	Order
40	12.49678	–	–	12.71511	–	–
80	12.66399	1.67208E-01	–	12.71794	2.83265E-03	–
160	12.70438	4.03889E-02	2.04960	12.71811	1.74943E-04	4.01719
320	12.71462	1.02421E-02	1.97944	12.71813	1.09423E-05	3.99889
640	12.71723	2.60203E-03	1.97680	12.71813	6.84221E-07	3.99931

From Table I, it is noticed that the second and fourth order convergence are achieved when using the second order central finite difference method and spectral method respectively in the spatial discretizations.

B. Efficiency Test

In this experiment we compare the the efficiency our scheme when using two different spatial discretization methods as well as two different integral approximation methods.

TABLE II
Computational Costs (Time in Seconds) Required When Using Finite Difference Method vs Chebychev Spectral Method

Steps	Central FD Method		Spectral Method	
M=N	Value	CPU	Value	CPU
40	14.84558872	0.040	15.03228280	0.041
80	14.98433811	0.100	15.03467073	0.121
160	15.02087991	0.120	15.03467073	0.184
320	15.03080592	0.418	15.03497510	1.241
640	15.03362563	1.659	15.03498484	9.471
1280	15.03449027	7.661	15.03498720	73.595

It can be observed from Table II that the required computational time is almost the same in the two different methods. However, when the discretisation becomes finer, which makes the matrix is larger, the first method (FDM) requires much less time compared with the spectral method. The reason behind that FFT method is used as a matrix-vector multiplication in the first method because we have a Toeplitz-tridiagonal matrix. However,when applying the spectral method, we will have to dense large matrix-multiplication in each time iteration.

VII. Conclusion

We have developed a robust and stable time-dependent method for pricing American options with Morton's jump diffusion model. We use different useful techniques to reduce the computational complexity. We have used the rational Padè approximation to approximate the exponential function of matrices, then used its partial fraction decomposition technique to overcome inverting polynomials of matrices. We also applied the FFT methods to as a matrix-vector multiplication solver. We finally gave numerical examples to test our algorithm.

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