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Robust Nonlinear Control of a Miniature Autonomous Helicopter using Sliding Mode Control Structure

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Abstract—This paper presents an investigation into the design of a flight control system, using a robust sliding mode control structure, designed using the exact feedback linearization procedure of the dynamic of a small-size autonomous helicopter in hover. The robustness of the controller in the context of stabilization and trajectory tracking with respect to small body forces and air resistance on the main and tail rotor, is analytically proved using Lyapunov approach. Some simulation results are presented to illustrate the performance and robustness of such controller in the presence of small body forces and air resistance.

Keywords—Robust control, sliding mode, stability, Lyapunov approach.

I. Introduction

AMONG the many Unmanned Aerial Vehicle configurations available today, helicopters are one the most maneuverable and versatile platforms. They can take-off and landing without a runway and can hover in place. These capabilities have brought about the use of autonomous miniature helicopters. For these reasons, there is currently great interest in using these platforms in a wide range of civil and military applications that include traffic surveillance, search and rescue, air pollution monitoring, area mapping, agriculture applications, bridge and building construction inspection. For performing safely many types of these tasks, high maneuverability and robustness of the controllers with respect to disturbances and modeling errors are required. This has generated considerable interest in the robust flight control design. A number of recent works focused on autopilot for autonomous helicopters, [1] use the feedback linearization techniques to a good approximation to the helicopter dynamics and a trajectory planning is considered in [2], [3] based on differential flatness properties. In [4], [5] the Backstepping techniques have been applied to design a robust nonlinear control law of the approximate dynamic model of a scale model autonomous helicopter by ignoring the small body forces and later analyze the performance of the full system to ensure that for desired trajectories the unmodelled dynamics do not destroy the stability of the closed loop system. The dynamical variable structure controller scheme has been presented [6] for vertical regulation in helicopter which achieves robust asymptotic stabilization for altitude flight.

The paper addresses the design of a robust flight control for a small-size autonomous helicopter with six degrees of freedom model obtained using Newton's equations. The proposed controller based on sliding mode control approach combined with the exact feedback linearization procedure. Sliding mode controllers [6], [7] are known to be highly insensitive to external perturbation signals, modelling errors and parameter variations. The superiorities of this technique are its applicability on nonlinear systems, simplicity, high performance and robust character. To simplify the synthesis of the controller, we have neglected the small body forces (which couple torque inputs to translational dynamics) and we have considered that the air resistance on the main and tail rotor as an external perturbation, the simplified model is used in the control design which achieves a desired helicopter position and orientation in hover and achieves also tracking of a desired trajectory. To prove the robustness of the control law despite these perturbations a stability analysis based on the Lyapunov theory is presented for the complete helicopter dynamics.

The paper is arranged as follows: Section 2 introduces a general helicopter model and presents the simplification needed to control synthesis. Section 3 presents sliding mode controller design. Section 4 summarizes the stability analysis to prove the robustness of flight control. Section 5 presents the simulation results obtained with the full dynamic model. Finally, we present the conclusion of this paper.

II. Dynamic Model of Autonomous Helicopter

This section presents the dynamic model of a single main rotor and tail rotor autonomous helicopter. The dynamics of the helicopter [4] are described using a conventional six degree of freedom rigid body model driven by forces and moments that explicitly include the effects of main and tail rotor. Consider the helicopter depicted in Fig.1. Let $\xi = (x, y, z)$ denote the position vector of the center of mass of the helicopter relative to the inertial frame $\mathcal{I} = \{E_x, E_y, E_z\}$. The linear velocity of the center of mass expressed in the inertial frame is denoted $v = \dot{\xi}$. Let

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$\eta = (\psi, \theta, \phi)$ denote the vector of three Euler angles (yaw, pitch and roll angles). Let $R : \mathcal{B} \to \mathcal{I}$ be the rotation matrix representing the orientation of the body fixed frame $\mathcal{B} = \{E_1^b, E_2^b, E_2^b\}$ with respect to the inertial frame $\mathcal{I}$, where $R \in SO(3)$ is an orthogonal matrix. The vector $\Omega = (\Omega_1, \Omega_2, \Omega_3)$ denotes the angular velocity of the vehicle in the body fixed frame. Let m denotes the total mass and $\mathbf{I} = diag(\mathbf{I_1}, \mathbf{I_2}, \mathbf{I_3})$ is a diagonal matrix representing the inertia matrix of the helicopter expressed in the body fixed frame and g represents the gravity acceleration. e_1, e_2 and e_3 are standard basis in $\mathbb{R}^3$.

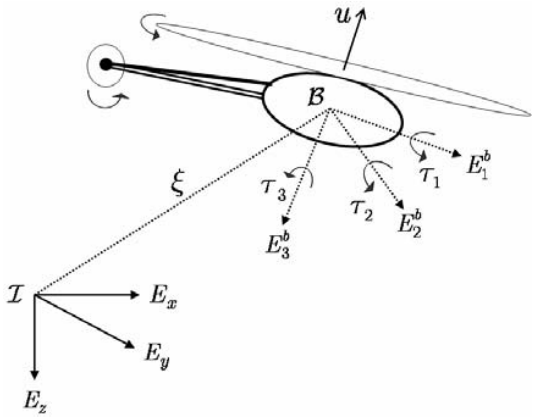

Fig. 1 Reference frames for an autonomous helicopter with force and torques control

The equations of motion for the autonomous helicopter are as the following:

$$\begin{cases} \dot{\xi} = v \\ m\dot{v} = mge_3 - Rue_3 + R\mathcal{K}\tau \\ \dot{R} = Rs(\Omega) \\ \mathbf{I}\dot{\Omega} = -\Omega \times \mathbf{I}\Omega + |Q_M|e_3 - |Q_T|e_2 + \tau \end{cases} \quad (1)$$

where $u \in \mathbb{R}$ is the main rotor thrust input and $\tau = (\tau_1, \tau_2, \tau_3) \in \mathcal{B}$ is the torque input applied to the helicopter at the center of mass. The term $R\mathcal{K}\tau$ represents the small body force perturbations due to mechanism used to obtain torque control and $\mathcal{K}$ is a constant matrix depending on the geometric parameters of the helicopter, $|Q_M|$ and $|Q_T|$ represent the air resistance (anti-torque) on the main and tail rotor. The notation $s(\Omega)$ denotes the skew symmetric matrix such that $s(\Omega)q = \Omega \times q$ for the vector cross product $\times$ and any vector $q \in \mathbb{R}^3$.

Notice that the model (1) is highly nonlinear and presents the small body forces which couple the torque inputs to translational dynamics, therefore control theoretical issues such as stability and tracking becomes much more complicated. To reduce the computational complexity on the control law, we will start by designing a controller for the approximate model in the absence of small body forces, it can be shown that the resulting approximate system is differentially flat, and hence feedback linearizable [1]. For this reason, we design a robust controller using sliding mode control combined with exact feedback linearization for the simplified model, and we analyze the robustness of the closed loop system for the full dynamic helicopter model.

III. Sliding Mode Controller Design

This section focuses on the design of a control flight based on the sliding mode control combined with the exact feedback linearization procedure for the simplified helicopter model, where the small body force and air resistance are neglected.

Consider the approximate helicopter model

$$\begin{cases} \dot{\xi} = v \\ m\dot{v} = mge_3 - Rue_3 \\ \dot{R} = Rs(\Omega) \\ \mathbf{I}\dot{\Omega} = -\Omega \times \mathbf{I}\Omega + \tau \end{cases} \quad (2)$$

The objective of the control flight is to design an autopilot $(u, \tau_1, \tau_2, \tau_2)$ for the miniature helicopter to let the vertical, lateral, longitudinal and yaw attitude dynamics to track a desired smooth trajectories $\xi_d = (x_d, y_d, z_d)$ and ψ_d, for which the tracking errors $e_\xi = \xi - \xi_d$ and $e_\psi = \psi - \psi_d$ converge asymptotically to zero.

In order to render the approximate model (2) completely linearizable, we will use a dynamic extension procedure. This is done by two integrators of the thrust control input u, we will thus consider as the control inputs the vector $(\ddot{u}, \tau_1, \tau_2, \tau_2)$.

To simplify the following analysis, consider a linearizing control input transformation given by

$$\tilde{\tau} = -\mathbf{I}^{-1}\Omega \times \mathbf{I}\Omega + \mathbf{I}^{-1}\tau \quad (3)$$

With this choice, we have $\dot{\Omega} = \tilde{\tau}$, where $\tilde{\tau} = (\tilde{\tau}_1, \tilde{\tau}_2, \tilde{\tau}_3)$ is the new control input.

Using the input-output feedback linearization procedure of the position ξ which has relative degree of 4 with respect to the control input, we have:

The third time derivative of ξ which any input appears in its expression yields

$$\xi^{(3)} = -\frac{1}{m}Rus(\Omega)e_3 - \frac{1}{m}R\dot{u}e_3 \quad (4)$$

Taking the fourth time derivative of the position ξ, it follows that

$$\begin{aligned} \xi^{(4)} &= -\frac{2}{m}R\dot{u}s(\Omega)e_3 - \frac{1}{m}Rus(\dot{\Omega})e_3 - \frac{1}{m}R\ddot{u}e_3 \quad (5) \\ &= -\frac{2}{m}R\dot{u}s(\Omega)e_3 - \frac{1}{m}Ru(\dot{\Omega} \times e_3) - \frac{1}{m}R\ddot{u}e_3 \end{aligned}$$

Combining equation (3) and recalling that $s(\dot{\Omega})e_3 = \dot{\Omega} \times e_3 = \tilde{\tau} \times e_3 = (\tilde{\tau}_2, -\tilde{\tau}_1, 0)^T$, equation (5) can be written as

$$\xi^{(4)} = -\frac{2}{m} R\dot{u}s(\Omega)e_3 - \frac{1}{m} RA(u)(\tilde{\tau}_1, \tilde{\tau}_2, \ddot{u})^T \quad (6)$$

where

$A(u) = \begin{pmatrix} 0 & u & 0 \\ -u & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $\dot{u}s(\Omega)e_3 = \begin{pmatrix} \dot{u}\Omega_2 \\ -\dot{u}\Omega_1 \\ 0 \end{pmatrix}$, since the thrust input u is never to be zero while the helicopter is in hover then the matrix $A(u)$ is nonsingular and the control signals $\ddot{u}$, $\tilde{\tau}_1$ and $\tilde{\tau}_2$ can be determined using sliding mode control design. To obtain the control input $\tilde{\tau}_3$ that stabilizes the tracking error of the yaw attitude, it is necessary to use the kinematic relationship between the Euler angles and the angular velocity in the body frame. The generalized velocities $\dot{\eta} = (\dot{\psi}, \dot{\theta}, \dot{\phi})$ are related to the angular velocity Ω by the relation

$$\dot{\eta} = \mathcal{W}(\eta)\Omega = \frac{1}{\cos\theta} \begin{pmatrix} 0 & \sin\phi & \cos\phi \\ 1 & \cos\phi & -\sin\phi \\ 1 & \sin\phi\sin\theta & \cos\phi\sin\theta \end{pmatrix} \Omega \quad (7)$$

The first time derivative of the yaw attitude is given by

$$\dot{\psi} = e_1^T \mathcal{W}(\eta)\Omega = \frac{\sin\phi}{\cos\theta}\Omega_2 + \frac{\cos\phi}{\cos\theta}\Omega_3 \quad (8)$$

Compute the second derivative of ψ and using equation (3), we have

$$\ddot{\psi} = \alpha(\eta, \dot{\eta}, \Omega) + \frac{\sin\phi}{\cos\theta}\tilde{\tau}_2 + \frac{\cos\phi}{\cos\theta}\tilde{\tau}_3 \quad (9)$$

where $\alpha(\eta, \dot{\eta}, \Omega) = e_1^T \dot{\mathcal{W}}(\eta)\Omega$ and $\theta, \phi \in]-\pi/2, \pi/2[$

Define now a nonlinear change of coordinates $\zeta_1 = \xi$, $\zeta_2 = \dot{\xi}$, $\zeta_3 = \ddot{\xi}$, $\zeta_4 = \xi^{(3)}$, $\zeta_5 = \psi$ and $\zeta_6 = \dot{\psi}$. In the new system of coordinates, we can put the approximate model (2) into the canonical form: $\dot{\zeta}_1 = \zeta_2$, $\dot{\zeta}_2 = \zeta_3$, $\dot{\zeta}_3 = \zeta_4$, $\dot{\zeta}_4 = \xi^{(4)}$, $\dot{\zeta}_5 = \zeta_6$ and $\dot{\zeta}_6 = \ddot{\psi}$.

Let us $e_\xi^{(n)} = \zeta_{n+1} - \xi_d^{(n)}$, $n = 0,1,2,3$, $e_\xi^{(4)} = \dot{\zeta}_4 - \xi_d^{(4)}$, $e_\psi = \zeta_5 - \psi_d$, $\dot{e}_\psi = \zeta_6 - \dot{\psi}_d$ and $\ddot{e}_\psi = \dot{\zeta}_6 - \ddot{\psi}_d$.

The first step in designing a sliding mode control for the input-output linearized dynamics is to design the sliding surface. Let the sliding surfaces $\sigma = (\sigma_\xi, \sigma_\psi)$ where $\sigma_\xi = (\sigma_1, \sigma_2, \sigma_3) \in \mathbb{R}^3$ and $\sigma_\psi = \sigma_4 \in \mathbb{R}$ associate respectively to the position ξ and yaw attitude ψ. The sliding surfaces are chosen as functions of the tracking error such that

$$\sigma_\xi = e_\xi^{(3)} + \Lambda_3\ddot{e}_\xi + \Lambda_2\dot{e}_\xi + \Lambda_1 e_\xi \quad (10)$$

$$\sigma_\psi = \dot{e}_\psi + \lambda_4 e_\psi \quad (11)$$

where $\Lambda_j = diag(\lambda_{j1}, \lambda_{j2}, \lambda_{j3}), j = 1,2,3$ are positive definite diagonal matrices which the diagonal elements are chosen such that the polynomials $P_k(s) = s^3 + \lambda_{3k}s^2 + \lambda_{2k}s + \lambda_{1k}$, $k = 1,2,3$ are Hurwitz polynomials and λ_4 is a positive parameter.

The sliding surfaces are designed so that the state trajectory, restricted to $\sigma = 0$ at every $t \geq t_1$, for some $t_1 > 0$ show some desired behavior such as stability or tracking. After this step the objective is to determine the control inputs which drive the state trajectory along the surfaces (10) and (11).

The following proposition gives the first result of the paper.

Proposition:

The following discontinuous control signals when applied to the approximate helicopter model (2) via the control input transformation (3)

$$(\tilde{\tau}_1, \tilde{\tau}_2, \ddot{u})^T = A^{-1}(u)[-2\dot{u}s(\Omega)e_3 - mR^T(\xi_d^{(4)} - \Lambda_3 e_\xi^{(3)} - \Lambda_2\ddot{e}_\xi - \Lambda_1\dot{e}_\xi - Gsign(\sigma_\xi))] \quad (12)$$

$$\tilde{\tau}_3 = \frac{\cos\theta}{\cos\phi}[-\frac{\sin\phi}{\cos\theta}\tilde{\tau}_2 - \alpha(\eta, \dot{\eta}, \Omega) + \ddot{\psi}_d - \lambda_4\dot{e}_\psi - g_4 sign(\sigma_\psi)] \quad (13)$$

asymptotically stabilize the tracking errors $e_\xi = \xi - \xi_d$ and $e_\psi = \psi - \psi_d$ to zero. With $G = diag(g_1, g_2, g_3)$ being a strictly positive definite diagonal matrix and g_4 is a positive parameter, '$sign$' denotes the signum function.

Proof:

Differentiating the sliding surfaces (10) and (11) with respect to time, we have

$$\dot{\sigma}_\xi = \xi^{(4)} - \xi_d^{(4)} + \Lambda_3 e_\xi^{(3)} + \Lambda_2\ddot{e}_\xi + \Lambda_1\dot{e}_\xi \quad (14)$$

$$\dot{\sigma}_\psi = \ddot{\psi} - \ddot{\psi}_d + \lambda_4\dot{e}_\psi \quad (15)$$

Combining (6) and (12) into (14), it follows that

$$\dot{\sigma}_\xi = -Gsign(\sigma_\xi) \quad (16)$$

On the other hand, introducing (9) and (13) into (15), we have

$$\dot{\sigma}_\psi = -g_4 sign(\sigma_\psi) \quad (17)$$

The dynamics in (16) and (17) guarantees the finite time reachability of the sliding surfaces to zero from any given initial conditions $\sigma_\xi(0)$ and $\sigma_\psi(0)$ provided that the gains G and g_4 are strictly positive. Moreover, the dynamics in (16) and (17) guarantees that $\sigma_\xi^T\dot{\sigma}_\xi < 0$ and $\sigma_\psi\dot{\sigma}_\psi < 0$ (the condition needed to guarantees sliding phase). Hence, σ_ξ and σ_ψ are driven to zero in finite time, the tracking errors e_ξ and e_ψ are governed respectively after such a finite time by the third order dynamics $e_\xi^{(3)} + \Lambda_3\ddot{e}_\xi + \Lambda_2\dot{e}_\xi + \Lambda_1 e_\xi = 0$ and the first order differential equation $\dot{e}_\psi + \lambda_4 e_\psi = 0$. Thus the tracking errors will converge asymptotically to zero as $t \to +\infty$ because Λ_1, Λ_2 and Λ_3 are positive definite diagonal matrices and λ_4 is a positive parameter.

IV. Stability Analysis

In this section, we present the analysis of the closed loop performance of the controllers given by equations (12), (13) and (3) designed above for the approximate model (2) when applied to the full dynamic helicopter model (1) by considering the small body forces and air resistance.

Consider the complete helicopter dynamics (1) and compute the time derivative of the new system of coordinates, we have

$\dot{\zeta}_1 = \zeta_2$, $\dot{\zeta}_2 = \zeta_3 + R\mathcal{K}\tau/m$, $\dot{\zeta}_3 = \zeta_4$,

$\dot{\zeta}_4 = -\frac{2}{m}R\dot{u}s(\Omega)e_3 - \frac{1}{m}RA(u)(\tilde{\tau}_1, \tilde{\tau}_2, \ddot{u})^T + \frac{|Q_T|}{m\mathbf{I}_2}Rue_1$,

$\dot{\zeta}_5 = \zeta_6$ and $\dot{\zeta}_6 = \alpha(\eta, \dot{\eta}, \Omega) + \frac{\sin\phi}{\cos\theta}\tilde{\tau}_2 + \frac{\cos\phi}{\cos\theta}\tilde{\tau}_3 + \frac{\cos\phi}{\mathbf{I}_3\cos\theta}|Q_M| - \frac{\sin\phi}{\mathbf{I}_2\cos\theta}|Q_T|$.

The following theorem gives the second result of the paper.

Theorem:

Consider the full dynamic (1) and given that the original control inputs u and τ are bounded. There exist $\beta_1 > 0$, $\beta_2 > 0$, there exist a positive definite diagonal matrix $G = diag(g_1, g_2, g_3)$ satisfy $g_{\min} = \beta_1 + |Q_T| \sup_{t\geq 0}|u|/m\mathbf{I}_2 + \|\Lambda_2\|\|\mathcal{K}\| \sup_{t\geq 0}\|\tau\|/m$ and a positive design parameter g_4 satisfy $g_4 = \beta_2 + |Q_T|/\mathbf{I}_2 + Q_M/\mathbf{I}_3$, then the discontinuous control signals given by equations (12) and (13) when applied to the full helicopter model (1) via the control input transformation (3), stabilize asymptotically the tracking errors to zero.

Proof:

Consider the quadratic Lyapunov function

$$V(\sigma) = V_\xi + V_\psi = \frac{1}{2}\sigma_\xi^T\sigma_\xi + \frac{1}{2}\sigma_\psi^2 \tag{18}$$

with σ_ξ and σ_ψ are the sliding surfaces defined in (10) and (11). The time derivative of V_ξ along the trajectories of the full dynamic (1) gives

$$\dot{V}_\xi = \sigma_\xi^T\dot{\sigma}_\xi = \sigma_\xi^T(\dot{\zeta}_4 - \xi_d^{(4)} + \Lambda_3 e_\xi^{(3)} + \Lambda_2\ddot{e}_\xi + \Lambda_1\dot{e}_\xi) \tag{19}$$

Substituting $\dot{\zeta}_4$ by its expression and introducing into (19) the control signals $(\tilde{\tau}_1, \tilde{\tau}_2, \ddot{u})$ from (12), we obtain

$$\begin{aligned}\dot{V}_\xi &= \sigma_\xi^T(-Gsign(\sigma_\xi) + |Q_T|Rue_1/m\mathbf{I}_2 + \Lambda_2 R\mathcal{K}\tau/m) \\ &= -\sigma_\xi^T Gsign(\sigma_\xi) + |Q_T|\sigma_\xi^T Rue_1/m\mathbf{I}_2 + \sigma_\xi^T\Lambda_2 R\mathcal{K}\tau/m\end{aligned} \tag{20}$$

Then it follows that

$$\dot{V}_\xi \leq -\|G\|\|\sigma_\xi\| + |Q_T||u|\|\sigma_\xi\|/m\mathbf{I}_2 + \|\Lambda_2\|\|\mathcal{K}\|\|\tau\|\|\sigma_\xi\|/m \tag{21}$$

Thus, one can easily verify that

$$\dot{V}_\xi \leq (-g_{\min} + |Q_T||u|/m\mathbf{I}_2 + \|\Lambda_2\|\|\mathcal{K}\|\|\tau\|/m)\|\sigma_\xi\| \tag{22}$$

where $g_{\min} = \min\{g_1, g_2, g_3\}$, then there exists a sufficiently large positive value of the minimum element diagonals $g_{\min}$ of the gain matrix G such that

$$\dot{V}_\xi \leq -\beta_1\|\sigma_\xi\|,\ \beta_1 > 0 \tag{23}$$

We can choose $g_{\min}$ as follows

$$g_{\min} = \beta_1 + |Q_T| \sup_{t\geq 0}|u|/m\mathbf{I}_2 + \|\Lambda_2\|\|\mathcal{K}\| \sup_{t\geq 0}\|\tau\|/m$$

Similarly, taking the derivative of V_ψ along the trajectories of the closed loop dynamics (1) gives

$$\dot{V}_\psi = \sigma_\psi\dot{\sigma}_\psi = \sigma_\psi(\dot{\zeta}_6 - \ddot{\psi}_d + \lambda_4\dot{e}_\psi) \tag{24}$$

Substituting $\dot{\zeta}_6$ by its expression and using the control input given by equation (13), it yields

$$\dot{V}_\psi = \sigma_\psi(-g_4 sign(\sigma_\psi) - \frac{\sin\phi}{\mathbf{I}_2\cos\theta}|Q_T| + \frac{\cos\phi}{\mathbf{I}_3\cos\theta}|Q_M|) \tag{25}$$

Therefore, equation (25) become

$$\dot{V}_\psi \leq (-g_4 + |Q_T|/\mathbf{I}_2 + |Q_M|/\mathbf{I}_3)|\sigma_\psi| \tag{26}$$

Thus, for a sufficiently large positive value of the gain parameter g_4, the derivative of the Lyapunov function V_ψ can always be made negative, with the choice $g_4 = \beta_2 + |Q_T|/\mathbf{I}_2 + |Q_M|/\mathbf{I}_3$, $\beta_2 > 0$, we get

$$\dot{V}_\psi \leq -\beta_2\|\sigma_\psi\| \tag{27}$$

Finally, the dynamic (23) and (27) guarantees the finite time reachability of the sliding surfaces to zero from any given initial conditions $\sigma_\xi(0)$ and $\sigma_\psi(0)$. Hence the tracking errors of the full helicopter dynamics (1) will converge asymptotically to zero as $t \to +\infty$.

Remark:

The implementation of the proposed controller that involves the $sign$ function is not very practical because it involves infinitely switching when the sliding surface $\sigma = 0$, this leads to chattering phenomena and excite high frequency and unmodelled dynamics. To avoid the problem with chattering, a boundary layer around the sliding surface can be used. For this reason, the $sign$ function can be replaced by a saturation function $sat(\sigma/B)$, where B is the boundary layer thickness.

V. Simulation Results

In this section, simulations are presented to illustrate the performance and robustness of proposed control law when applied to the full helicopter model with the small body forces and air resistance in the case of stabilization and trajectory tracking. The parameters values used for the dynamic model are as follows [4], $m = 9.6$, $\mathbf{I} = diag(04,056, 0.22)$ and $g = 9.8$. The air resistances are estimated to be the following constants ($|Q_M| = 0.02$, $|Q_T| = 0.002$). The matrix $\mathcal{K}$ which couple the torques input τ and the matrix R is $\mathcal{K} = \begin{pmatrix} 0 & -2.2 & 0 \\ 2.2 & 0 & 0.7 \\ 0 & 0 & 0 \end{pmatrix}$. The initial conditions are $\xi = \dot{\xi} = \eta = \Omega = 0$, $R_0 = I_3$, $u_0 = mg$, $\dot{u}_0 = 0$. The

parameters of the controller Λ_j, $j=1,2,3$ are chosen such that the corresponding poles of $(-1,-1,-1)$ and $\lambda_4=1$. The control gains selected are $G=diag(5,5,5)$ and $g_4=5$. These gains satisfy the conditions of theorem. The simulation results presented in Figures 2-5 were obtained in the case of stabilization of the helicopter to a set point in hover. The desired position and yaw attitude are chosen to be $\xi_d=(2,2,-3)$ and $\psi_d=\pi/4$. The simulation results presented in figures 6-9 consider the case of trajectory tracking. The desired trajectory was chosen as a vertical helix ascending given by $\xi_d=(3\cos\psi_d, 3\sin\psi_d, -0.2t-0.5)$ and $\psi_d=\pi t/10+0.8$.

In both simulations, from Fig. 3 and Fig. 7, it can be seen that the actual position and yaw attitude of the helicopter converge to their desired values. Hence the feedback controller is robust with respect to the small body forces and air resistance. Fig.4 and Fig.8 show the thrust control input u and the torque control input τ. It can be seen that the chattering in the control signals is eliminated by introducing the boundary layer around the sliding surface. Fig.5 and Fig.9 show the sliding surfaces σ_ξ and σ_ψ. It can be seen from these figures that the sliding surfaces converge to zero.

VI. Conclusion

In this paper, a robust flight control has been presented for a small-size autonomous helicopter using sliding mode control combined with the exact feedback linearization procedure. The robustness of the proposed controller with respect to small body forces and air resistance was analytically analyzed that achieves robust tracking and stabilization. To avoid the chattering problem and guarantees the smoothed control inputs, a boundary layer around the sliding surface was introduced. Simulation results show the effectiveness of the proposed controller.

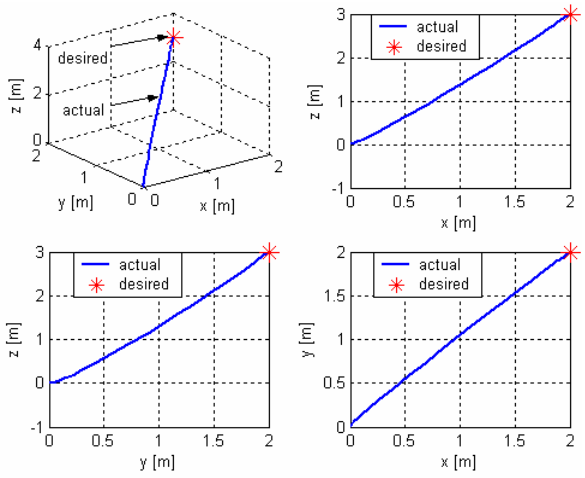

Fig. 2 3D and 2D motion of the helicopter

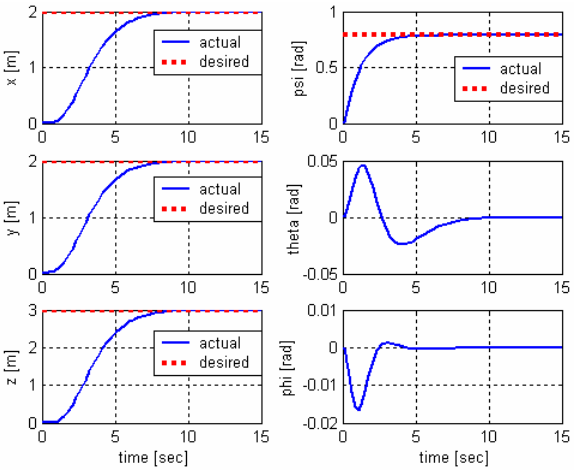

Fig. 3 Position (x,y,z) and orientation (ψ,θ,ϕ)

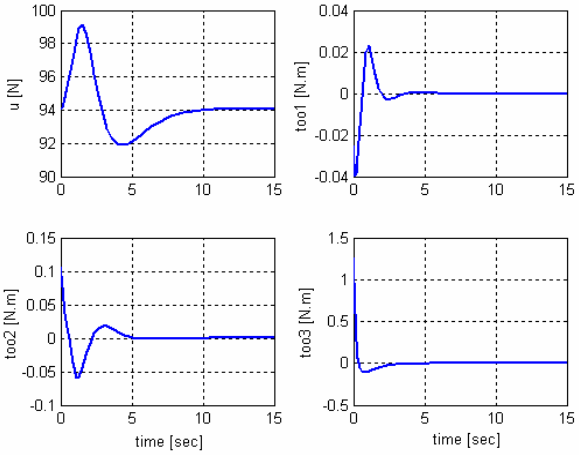

Fig. 4 Control inputs u and $\tau=(\tau_1,\tau_2,\tau_3)$

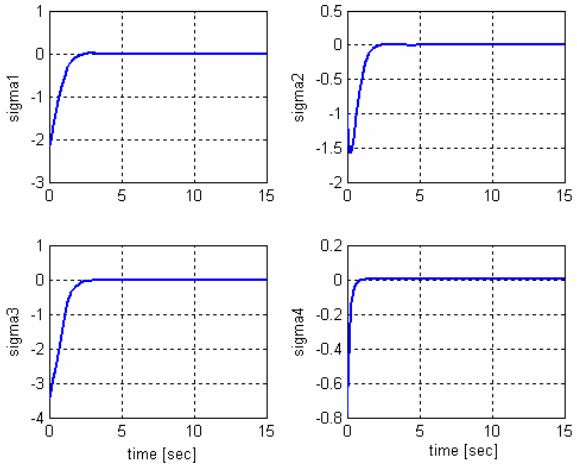

Fig. 5 Sliding surfaces $(\sigma_1,\sigma_1,\sigma_3)$ and σ_4

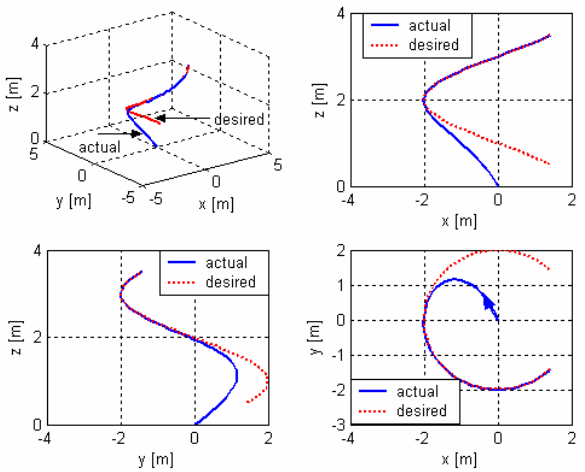

Fig. 6 3D and 2D motion of the helicopter

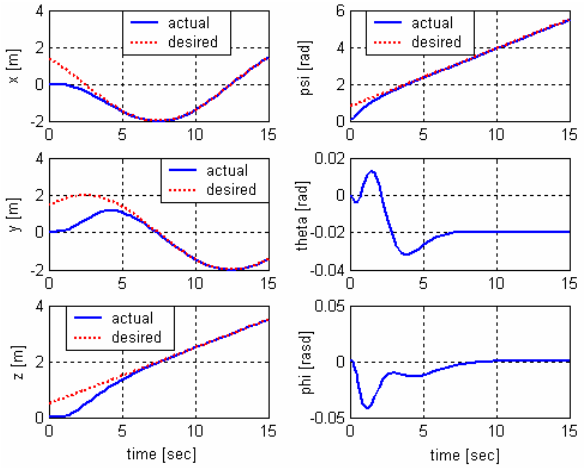

Fig. 7 Position (x, y, z) and orientation (ψ, θ, ϕ)

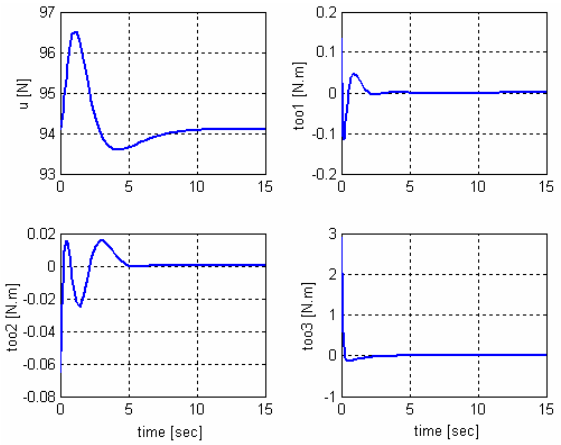

Fig. 8 Control inputs u and $\tau = (\tau_1, \tau_2, \tau_3)$

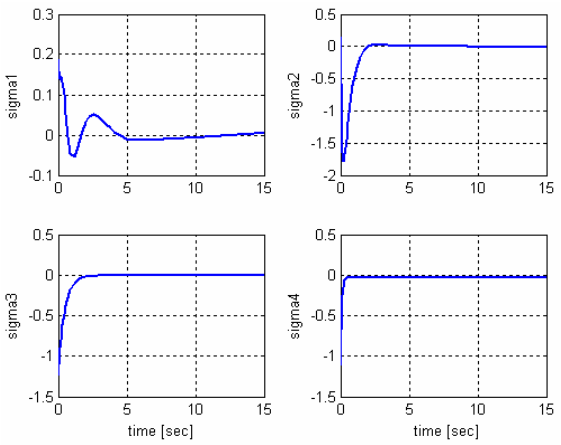

Fig. 9 Sliding surfaces $(\sigma_1, \sigma_1, \sigma_3)$ and σ_4

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