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Robust Conversion of Chaos into an Arbitrary Periodic Motion

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***Abstract*—**One of the most attractive and important field of chaos theory is control of chaos. In this paper, we try to present a simple framework for chaotic motion control using the feedback linearization method. Using this approach, we derive a strategy, which can be easily applied to the other chaotic systems. This task presents two novel results: the desired periodic orbit need not be a solution of the original dynamics and the other is the robustness of response against parameter variations. The illustrated simulations show the ability of these. In addition, by a comparison between a conventional state feedback and our proposed method it is demonstrated that the introduced technique is more efficient.

***Keywords*—**chaos, feedback linearization, robust control, periodic motion.

I. Introduction

NONLINEAR deterministic dynamic systems manifesting "*chaos*" do exist and are not exceptional [1]. It also turned out that the methods describing chaotic behavior occur in many areas of science and technology [2] and sometimes are more suitable for describing non-regular oscillations and uncertainties than the stochastic, probabilistic methods [3].

Until only recently, the field of nonlinear dynamics has remained within the confines of academia, and has found limited practical application to engineering problems. However, this situation is now undergoing a revolution of sorts, given (1) the several paradigm-shifting discoveries that have taken place in the closely related fields of chaos, fractals, and wavelets; (2) the advent of powerful computing tools that make the complex numerical simulation of nonlinear phenomena possible; as well as (3) the ever more pressing need to account for, and deal with nonlinear effects that can no longer be adequately handled by mere linear approaches.

Perhaps the most important problem in this context is "control of chaos" which has set up a great challenge [4]. After Yorke and his collaborators' work [5] this problem has found its position between the other open problems. In this present task, we are going to present a strategy for control of a chaotic motion. Obviously, we can classify the control of chaos problems in three sets [6]:*stabilization*, *chaotization,* and *synchronization*.

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Each problem has its specific techniques and tricks. However here we will concentrate on the first problem: stabilization.

So far, the methods proposed for solving "control of chaos" is much more than we could list them here [7]. However, they approximately have employed the conventional control methods. In addition, they have tried to drive the chaos exhibited in an artificial or industrial system into a fixed equilibrium point or into one of the periodic solutions of the original system [8]. We will show that using the powerful feedback linearization method [9], it is easy to change the dynamic of chaos and then employ the conventional state feedback method [10] to replace the chaotic motion with an arbitrary periodic motion.

This paper is organized as follows: in section 2 a survey of chaos theory is presented. Section 3 is devoted to problem statement. Simulations are brought in section 4. Finally some concluding remarks close the paper.

II. A Survey on Chaos Theory

The field of dynamics concerns the study of systems whose internal parameters (states) obey a set of temporal rules, essentially encompassing all observable phenomena.
This endeavor divides into three sub disciplines, namely:
- *applied dynamics* [11], which concerns the modeling process that transforms actual system observations into an idealized mathematical dynamical system.
-*mathematical dynamics* [12], which primarily focuses on the qualitative analysis of the model dynamical system; and
-*experimental dynamics* [13], which ranges from controlled laboratory experiments to the numerical simulation of state equations on computers.

The state temporal behavior is either viewed as a traditional time series or, more usually, in a phase space perspective wherein the system states are plotted against each other in an n dimensional space with time as an implicit parameter.

The latter framework affords a more natural geometrical setting that possesses an arsenal of analysis tools. A dynamical system is said to be linear or nonlinear depending on whether the superposition rule holds.

The latter is nonlinear dynamics; since it leads to a virtual universe of effects (chaos is but one) with potential practical import that is just beginning to be realized.

One of the most well known and potentially useful nonlinear dynamical effects is the bounded, random-like behavior called chaos-in essence, "deterministic noise"[7]. Chaos has been found to occur in a whole myriad of dynamical systems modeling phenomena from astronomy to zoology [14], and in frequency ranges from baseband to optical. This phenomena, and its closely related fractal cousin have been put forth as a new paradigm for understanding and modeling the world around us [2]. This stems from their underlying principle of self-similarity at different scales that matches closely with what is observed in nature.
There are three fundamental characteristics of chaos:

1. an essentially continuous and possibly banded frequency spectrum that resembles random noise;
2. *sensitivity to initial conditions,* that is, nearby orbits diverge very rapidly;
3. an *ergodicity and mixing* of the dynamical orbits which in essence implies the wholesale visit of the entire phase space by the chaotic behavior and a loss of information.

III. Problem Statement

In the presence of external perturbation, the non-stationary model:

$$\dot{x} = F(t, x(t), u) \tag{1}$$

is a general model for a complex dynamic system [15]. We implicitly assume that the system is full observable and for initial modeling, the parameters assumed are deterministic. The problems of suppressing the chaotic oscillations by reducing them to the regular oscillations [16] or suppressing them completely [7] can be formalized as follows:

Let us consider a free (uncontrolled, $u(t) \equiv 0$) motion $x_*(t)$ of system (1) with the initial condition $x_*(0) = x_{*0}$. Let this motion be T-periodic, that is, $x_*(t+T) = x_*(t)$ be satisfied for all $t \geq 0$. We need to stabilize it, that is, reduce the solutions $x(t)$ of system (1) to $x_*(t)$:

$$\lim_{t\to\infty}(x(t) - x_*(t)) = 0 \tag{2}$$

under the initial state $x(0) = x_0 \in \Omega$ where Ω is the given set of initial conditions.

If the control law would have been choosing that be able to vanish the nonlinearity of the given system, based on Lie algebra[17] and under some weak conditions the system could be transformed into a new linear system.

Based upon Anosov lemma [18] there exist a periodic orbit as close as possible to a chaotic orbit which using a proper control law one can drive that chaotic orbit to that periodic motion [4]. However following the methods introduced in [19] here a new strategy is introduced as follows. Before starting let's state the important key theorem in chaos study:

Theorem [4]: *Any trajectory belonging to a compact minimum invariant set is recurrent. Any compact invariant minimum set is the closure of some recurrent trajectory.*

This is called Birkhoff theorem. Upon this theorem, we will be able to establish the following proposed facts. Suppose an arbitrary periodic function $\eta(t)$ (periodicity on time) that is not necessarily a solution of (1). Our general objective of control is:

$$\lim_{t\to\infty} Q(x(t), t) = 0 \tag{3}$$

where Q is a proper cost function. A suitable Q for converting chaos into an arbitrary periodic motion, e.g. $\eta(t)$ is:

$$Q(x, t) = (x - x_*(t))^T \Gamma (x - x_*(t)) \tag{4}$$

where Γ is a positive definite symmetric matrix [20]. In fact, generally for any control of chaos problems choosing a suitable Q can be considered [21].

To simplify the analyses we have taken an affine-control scheme for an arbitrary system:

$$\dot{x} = f_1(t, x) + g_1(x, t)u \tag{5}$$

f_1, g_1 are two nonlinear functions that play the main role in chaos generation. Actually we exert the control to this key variable according to f_1, g_1.

Let the control law be:

$$u = u_f + u_t \tag{6}$$

such that the feedback control u_f is chosen by the conventional feedback linearization methods and feedforward control u_t can be chosen following [22].

$$\begin{aligned}\dot{x} &= f_1(t, x) + g_1(t, x)(u_f + u_t) \\ &= f_1(t, x) + g_1(t, x)\left(\frac{-f_1(t, x)}{g_1(t, x)} + u_t\right) \\ &= g_1(t, x)u_t\end{aligned} \tag{7}$$

Indeed, in the linear time-varying case with periodic A-matrix, upon the well-known Floquet theorem [9], the given system can be transformed to a system in which the A-matrix is any constant matrix. Additionally based on the fundamental theorems in the first course of topology and differential geometry [23], which is applied in nonlinear control theory, one can easily see that the transformed complex system into the new linear version of that is gratefully robust against the parameter variations. The simulations show these assertions obviously.

IV. Simulation

A good example to simulate this technique is the famous *Duffing's oscillator* [9] exhibiting chaos. The Duffing equation that has been simulated here is:

$$\dot{x}_1 = x$$
$$\dot{x}_2 = x_1 - .5x_1^{\,3} - .168x_2 + a\cos(\omega t) + u \qquad (8)$$

where $a = .9, \omega = 1\&10\, {rad}/{sec}$.

in this example we have:

$$f_1(t,x) = \begin{bmatrix} x_1 \\ x_1 - .5x_1^{\,3} - .168x_2 + a\cos(\omega t) \end{bmatrix}$$
$$g_1(t,x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (9)$$

As previously stated we can choose the controller as:

$$u = (-x_1 + .5x_1^3 + .168x_2 - a\cos(\overline{\omega} t)) + u_t \qquad (10)$$

In which $\overline{\omega}$ is the midpoint in the interval of the frequency variation $[\omega_{\min}, \omega_{\max}]$. Also u_t is the feedforward control signal.

Fig. 1 shows the uncontrolled system for two cases $\omega = 1\&10$ rad/sec.

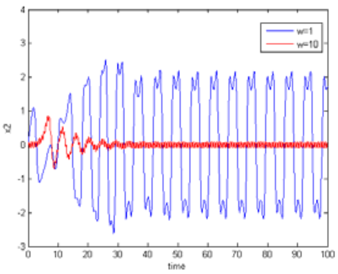

Fig. 1 uncontrolled Duffing's oscillator- x_2 .

Fig. 2 shows the uncontrolled system phase portrait where there exists a parameter variation in the system. It is easy to see the presence of chaos in both cases of frequencies.

Fig. 3 shows simultaneously the controlled system phase portrait in presence of parameter variations. As can be seen the difference is very small and clearly depicts the robustness of control system. Since a quasi-circle in the phase portrait is a periodic motion in the time domain we see that the proposed method is able to drive the chaotic behavior into an arbitrary periodic motion which is not necessarily a solution of the original chaotic system. We can name this methodology a targeting technique which is an important tactic in open-loop control of chaos [24].

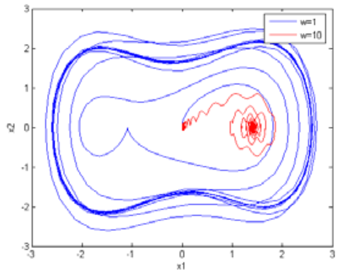

Fig. 2 uncontrolled Duffing's oscillator phase portrait in presence of parameter variations.

Fig. 4 shows the time series of the states of the original chaotic system in the presence of control signal. Also note that in spite of the variations of the frequencies in the Duffing's oscillator, the period of the states in both cases are similar. This can be considered as the robustness ability of the proposed technique.

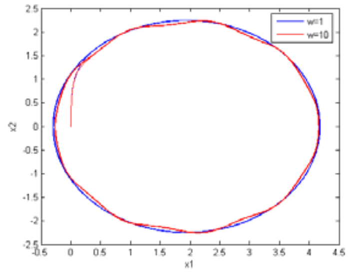

Fig. 3 controlled Duffing's oscillator using proposed method in the phase portrait format.

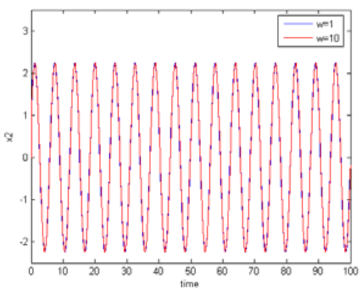

Fig. 4 controlled Duffing's oscillator- x_2 .

To prove the capability of the introduced technique, we compare the results with the response of the system which is controlled by state feedback controller [10]. In computing the state feedback gains it is easy to utilize the optimal control methods.

As can be seen from Figures (5) and (6) the rate of convergence of the chaotic motion to the desired periodic motion in state feedback method is considerably lesser than that of our presented technique.

V. Conclusion

The apparently random phenomenon of chaos has become increasingly observed in the behavior of myriad nonlinear deterministic systems, that is, those described accurately by partial or ordinary differential equations or difference equations. These observations are being made not only experimentally, but also by computer simulations. Examples abound in a wide gamut of disciplines ranging from solid state physics to cosmology, from electrical engineering to biology. Chaos is found in systems that are forced or unforced (also known as non-autonomous or autonomous, respectively), lossless or dissipative, discrete in time and of any dimension, or continuous in time and of dimension three or higher.

Since “control of chaos” today is an attractive research field, the published papers which have tried to solve this problem partially is so numerous. In this task, we have introduced a new strategy based on feedback linearization methods and employing the essential concepts from differential geometry. The two important advantages of this technique are its robustness against parameter variations and the second is that the desired periodic motion does not need to be a solution of the original dynamic system. These two advantages have been proved by comparison with the conventional state feedback method. As a result, we can state that among other published articles [4] this technique has brought a novel robust chaos control method.

References

[1] R.M.May, “Simple Mathematical Models With Very Complicated Dynamics”, Nature 261, 459. 1976.

[2] R.M.Crownover, "Introduction to Fractals and chaos", Jones and Bartlett Publishers, 1995.

[3] A.J.Lichtenber, and M.A.Liebermann,"Regular and Stochastic Motion. Springer, Heidelberg-New York.1982.

[4] B.R. Andrievskii, and A.L.Fradkov, " Control of chaos: Methods and applications. I. Methods:, Avtom.Telemkh, no. 5, pp.3-45. 2003.

[5] Li, T. and Yorke, J.A., Period Three Implies Chaos, Am. Math. Monthly.,1975, vol. 82, pp. 985-992..

[6] A.Razminia, and M.A.Sadrnia, “Chua’s circuit regulation using a nonlinear adaptive feedback technique”, International journal of electronics, circuits and systems, Vol 2. No1. 2008.

[7] Yongai Zheng," Controlling chaos based on an adaptive adjustment mechanism", Chaos, Solitons and Fractals,2005.

[8] .Battle, E.Fossas, " Stabilization of periodic orbits of the buck converter by time delayed feedback", Int. J. Circ. Theory Appl., vol. 27, pp. 617-631. 1999.

[9] H.Khalil, “ Nonlinear Systems”, Prentice hall, 2006..

[10] J Chaohong Cai , Zhenyuan Xu, Wenbo Xu, Converting chaos into periodic motion by state feedback control, Automatica 38 (2002),pp. 1927 – 1933.

[11] R. Ghidossi, D. Veyret, P. Moulin, “Computational fluid dynamics applied to membranes: State of the art and opportunities”, Chemical Engineering and Processing, Volume 45, Issue 6, June 2006, Pages 437-454.

[12] P. Magni, M. Simeoni, I. Poggesi, M. Rocchetti, G. De Nicolao, “A mathematical model to study the effects of drugs administration on tumor growth dynamics”, Mathematical Biosciences, Volume 200, Issue 2, April 2006, Pages 127-151.

[13] Shu Chen, Guo-Hua Hu, Chen Guo, Hui-Zhou Liu, “Experimental study and dissipative particle dynamics simulation of the formation and stabilization of gold nanoparticles in PEO–PPO–PEO block copolymer micelles” Chemical Engineering Science, Volume 62, Issues 18-20, September-October 2007, Pages 5251-5256.

[14] Franz S. Hover, Michael S. Triantafyllou, “Application of polynomial chaos in stability and control” Automatica, Volume 42, Issue 5, May 2006, Pages 789-795.

[15] R.L.Devaney, "A First Course in Chaotic Dynamical Systems: Theory and Experiment" Addison-Wesley, 1997.

[16] J.Guckenheimer and Holmes, "Nonlinear oscillation, Dynamical Systems, and Bifurcation of Vector Fields", SpringerVerlag, New York, 1990.

[17] David A. Towers, Vicente R. Varea, “Elementary Lie algebras and Lie A-algebras” Journal of Algebra, Volume 312, Issue 2, 15 June 2007, Pages 891-901.

[18] Jorge Lauret, Cynthia E. Will, “On Anosov automorphisms of manifolds” Journal of Pure and Applied Algebra, Volume 212, Issue 7, July 2008, Pages 1747-1755.

[19] Chen, G., 1996, "Controlling Chaotic Trajectories to Unstable Limit Cycles", Proceedings of the American Control Conference, San Diego, California, pp. 2413-2414.

[20] C.D.Meyer, “Matrix analysis and applied linear algebra”, SIAM 2000J.

[21] .Wang, and J.Zhang, “Chaotic secure communication based on nonlinear autoregressive filter with changeable parameters”, Phy.Lett.A 357, pp.323-329, 2006.

[22] Sinha, S.C., and Joseph, P., "Control of General Dynamical Systems with Periodically Varying Parameters Via Lyapunov-Floquet Transformation", ASME Journal of Dynamical Systems, Measurement, and Control, Vol. 116, pp. 650-658, 1994.

[23] R.L.Devaney, " Topological Bifurcation" Topology Proceedings 28 pp.99-112, 2004.

[24] Alexander L. Fradkov, Robin J. Evans, “Control of chaos: Methods and applications in engineering,”, Annual Reviews in Control, Volume 29, Issue 1, 2005, Pages 33-56.