

# Robust Control Design and Analysis Using SCILAB for a Mass-Spring-Damper System

Yoonsoo Kim

**Abstract**—This paper introduces an open-source software package SCILAB, an alternative of MATLAB, which can be used for robust control design and analysis of a typical mass-spring-damper (MSD) system. Using the previously published ideas, this popular mechanical system is considered to provide another example of usefulness of SCILAB for advanced control design.

**Keywords**—Robust Control, SCILAB, Mass-Spring-Damper (MSD).

## I. INTRODUCTION

IN [1], [2], it is shown that SCILAB [3] is an enabling design and analysis tool for robust control of real-world systems. Unlike MATLAB [4], SCILAB is an open-source software package (no need to pay even a ‘penny’), and one can find many success stories of SCILAB in many engineering fields, e.g. optimization algorithm development [5], vehicle stability control [6] and aircraft flutter test [7]. In this paper, one more success story of SCILAB shall be added to the list; SCILAB-based robust control design and analysis is also possible for a typical mass-spring-damper (MSD) system. One challenge in robust control design for MSD is to take care of the inverse multiplicative uncertainty appearing in the mass parameter. This paper clearly shows that SCILAB is capable of handling such uncertainties with little effort. It should be noted that this paper is written with the intention of disseminate the ideas of how to use SCILAB for teaching robust control theory using a standard textbook such as [8] or performing related research works. Therefore, any technical detail on robust control theory is simply referred to the textbook.

## II. ROBUST CONTROL DESIGN AND ANALYSIS USING ROBUST CONTROL TOOLBOX: MIXED-SENSITIVITY $H_\infty$ CONTROL DESIGN AND ANALYSIS FOR A MASS-SPRING-DAMPER SYSTEM

In order to perform robust control design and analysis using SCILAB, one needs to obtain a robust control toolbox available on the website [9]. Although incomprehensive, some useful information on functions built in the toolbox can also be found on the website. In what follows, these functions shall be used to design and analyze a robust controller for a MSD system found in [10]. All the SCILAB programs in this paper were tested with the SCILAB version 5.2.0-beta-1 on a personal computer (operating system: Windows XP).

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Consider a familiar mechanical system described by the differential equation  $m \ddot{x}(t) + b \dot{x}(t) + kx(t) = u(t)$  or the block diagram in Fig. 1. Suppose one wants to design a control input such that (normalized) disturbances with frequencies less than 10 rad/sec can be rejected while the control input magnitude is less than 100 N.

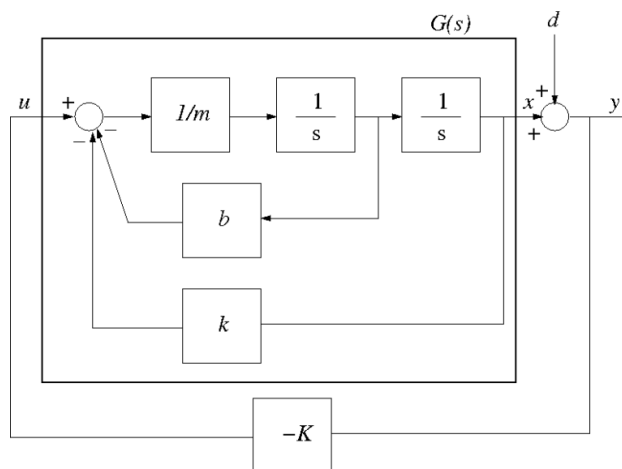


Fig. 1 A MSD system

For this purpose, an  $H_\infty$  controller is desired which minimizes  $\| [w_p S \ w_u K S]^T \|$  or  $\gamma$  such that  $\| [w_p S \ w_u K S]^T \| < \gamma$  with suitable  $w_p$  and  $w_u$ . In order to address the said control objectives, the following weights are chosen:  $w_u = \frac{1}{100}$  and  $w_p = \frac{s/M + w_B}{s + w_B A}$ , where  $w_B = 10$ ,  $M = 2$

and  $A = 10^{-4}$ . Assuming  $m = 3$ ,  $b = 1$  and  $k = 2$  with appropriate units, the  $H_\infty$  controller and the associated closed-loop responses can be obtained using the program as follows:

```
s=poly(0,"s"); G=200/(10*s+1)/(0.05*s+1)^2;
M=1.5; wB=10; A=10^(-4);
wp=(s/M^(.5)+wB)^2/(s+wB*A^(.5))^2;
wu=1; r=[1 1]; P=[0 wu; wp*G; -1 -G];
[K,ro]=h_inf(P,r,0,1,30); gamma=1/sqrt(ro);
S=syslin('c',1/(1+G*K));
Bode(S); wp_inverse=syslin('c',1/wp);
Bode(wp_inverse); gamma=h_norm(wp*S);
gamma=h_norm(wp*S); S=syslin('c',1/(1+G*K));
t=0:0.05:5; step=ones(1,length(t));
plot(t,csim(step,t,S)); plot(t,csim(step,t,-K*S));
```

The obtained  $H_\infty$  controller is

$$K(s) = \frac{192.5s^2 + 64.15s + 128.3}{s^2 + 12.76s + 0.01276}$$

and the corresponding value of  $\gamma$  is 1.9883. Fig. 2 shows the unit step disturbance response and the control signal, respectively. As expected from the value of  $\gamma$ , the control signal in Fig. 2 (b) does not meet the requirement (the maximum magnitude is clearly greater than 100). However, one could reduce the magnitude of the control signal by increasing the value of  $w_u$  at the expense of increasing the value of  $\gamma$ . Note that the SCILAB command *csim* is used to plot the graphs and its third parameter comes from the fact that

$$Y(s) = \frac{1}{1+GK} D(s) = SD(s)$$

and

$$U(s) = -\frac{K}{1+GK} D(s) = -KSD(s),$$

where  $Y(s)$ ,  $D(s)$  and  $U(s)$  are the Laplace transforms of  $y(t)$ ,  $d(t)$  and  $u(t)$ .

Suppose now that the three physical parameters  $m$ ,  $b$  and  $k$  are not known exactly, but are believed to lie in known intervals. In particular, the actual mass is within 10% of its nominal mass  $\bar{m} = 3$ , the actual damping value is within 20% of its nominal value of  $\bar{b} = 1$ , and the spring stiffness is within 30% of its nominal value  $\bar{k} = 2$ , i.e.

$$m = \bar{m}(1 + w_m \delta_m), \quad b = \bar{b}(1 + w_b \delta_b) \quad \text{and} \quad k = \bar{k}(1 + w_k \delta_k),$$

where  $w_m = 0.1$ ,  $w_b = 0.2$ ,  $w_k = 0.3$ , and the uncertainty parameters  $\delta_m$ ,  $\delta_b$  and  $\delta_k$  are assumed to lie in the interval  $[-1, 1]$ . Then, it is not difficult to obtain the following generalized plant  $P$  using the standard block diagram algebra:

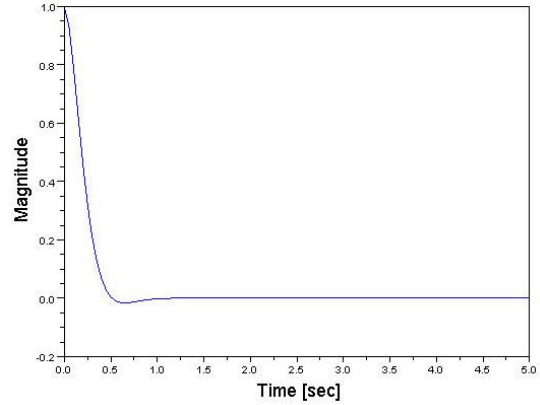
$$P = [P_{ij}] / (\bar{m}s^2 + \bar{b}s + \bar{k}) \quad (i, j = 1, 2, 3), \quad \text{where}$$

$$P_{11} = -\begin{bmatrix} \bar{m}w_m s^2 & \bar{m}w_b s^2 & \bar{m}w_k s^2 \\ \bar{b}w_m s & \bar{b}w_b s & \bar{b}w_k s \\ \bar{k}w_m & \bar{k}w_b & \bar{k}w_k \end{bmatrix}, \quad P_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad P_{13} = \begin{bmatrix} \bar{m}s^2 \\ \bar{b}s \\ \bar{k} \end{bmatrix},$$

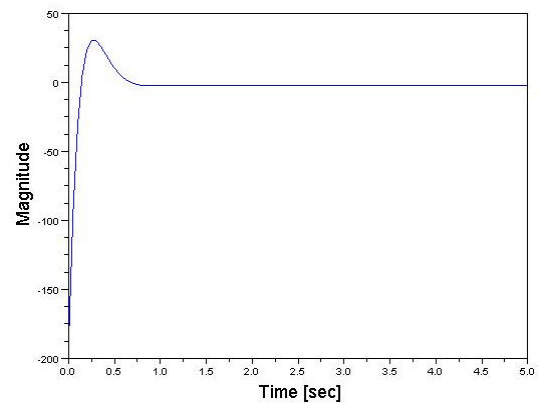
$$P_{21} = -\begin{bmatrix} w_m w_p & w_b w_p & w_k w_p \\ 0 & 0 & 0 \end{bmatrix}, \quad P_{22} = \begin{bmatrix} w_p(\bar{m}s^2 + \bar{b}s + \bar{k}) \\ 0 \end{bmatrix},$$

$$P_{23} = \begin{bmatrix} w_p \\ w_u(\bar{m}s^2 + \bar{b}s + \bar{k}) \end{bmatrix}, \quad P_{31} = [w_m \quad w_b \quad w_k],$$

$$P_{32} = \bar{m}s^2 + \bar{b}s + \bar{k} \quad \text{and} \quad P_{33} = 1.$$



(a) Unit step disturbance response



(b) Control input

Fig. 2 Closed-loop responses of the MSD system

Using the *lft* command, the  $N$  matrix in the  $N-\Delta$  structure in [7] can be obtained and subsequently its (2,2) block may be used to check the nominal performance:

```
-> N = lft(P, -K); N22 = syslin('c', N(4 : 5, 4));
-> frq = logspace(-3, 3, 100);
-> [frq np1] = repfreq(N22(1,1), frq);
-> [frq np2] = repfreq(N22(2,1), frq);
-> for i = 1 : 100,
-> NPmatrix = [np1(i) np2(i)]';
-> np(i) = norm(NPmatrix, 2);
-> end ; -> plot2d1("oln", frq, np);
```

The above code yields Fig. 3 (a) in which one can verify that the maximum value of the graph is close to  $\gamma = 1.9883$ . The (1,1) block of the  $N$  matrix may be used to check the robust stability:

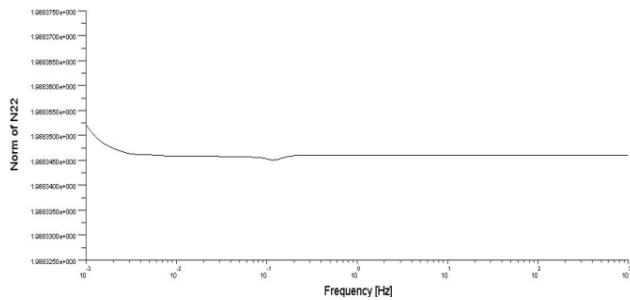
```
-> N11 = syslin('c', N(1 : 3, 1 : 3));
-> //need the next line for each i, j = 1, 2, 3
-> [frq rs_ij] = repfreq(N11(i, j), frq);
```

```

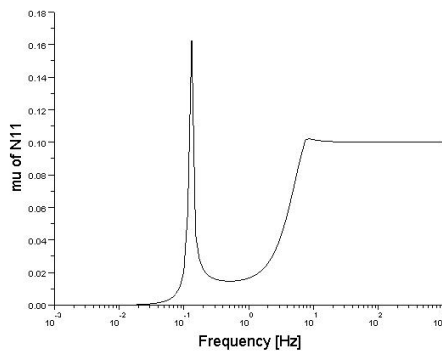
-> for k = 1 : 100,
-> RSmatrix = [rs_11(k) rs_12(k) rs_13(k);
-> rs_21(k) rs_22(k) rs_23(k); rs_31(k) rs_32(k) rs_33(k)];
-> rs(k) = mucomp(RSmatrix, [1 1 1], [1 1 1]);
-> end ;
-> plot2d1(" oln", frq, rs);

```

Note that the second and third parameters of *mucomp* indicate that the uncertainty block  $\Delta$  of present interest is real and diagonal. Fig. 3 (b) shows the  $\mu$ -plot for the  $H_\infty$  controller and suggests that the closed-loop system is robust stable with respect to real and diagonal perturbations as the maximum value of the graph is less than 1.



(a) Nominal performance (NP) check



(b) Robust stability (RS) check

Fig. 3 NP and RS for the MSD system

### III. CONCLUSION

This paper introduced an open-source software package SCILAB, an alternative of MATLAB, which can be used for robust control design and analysis of a typical mass-spring-damper system. Using the previously published ideas in [1], [2], this popular mechanical system was considered to provide another example of usefulness of SCILAB for advanced control design.

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