Revolving Ferrofluid Flow in Porous Medium with Rotating Disk

Paras Ram, Vikas Kumar

***Abstract*—**An attempt has been made to study the effect of rotation on incompressible, electrically non-conducting ferrofluid in porous medium on Axi-symmetric steady flow over a rotating disk excluding thermal effects. Here, we solved the boundary layer equations with boundary conditions using Neuringer-Rosensweig model considering the z-axis as the axis of rotation. The non linear boundary layer equations involved in the problem are transformed to the non linear coupled ordinary differential equations by Karman's transformation and solved by power series approximations. Besides numerically calculating the velocity components and pressure for different values of porosity parameter β with the variation of Karman's parameter α, we have also calculated the displacement thickness of boundary layer, the total volume flowing outward the z-axis and angle between wall and ferrofluid. The results for all above variables are obtained numerically and discussed graphically.

***Keywords*—**Ferrofluid, magnetic field porous medium, rotating disk.

I. Introduction

FERROFLUIDS are stable suspensions of colloidal ferromagnetic particles of the order of 10nm in suitable non-magnetic carrier liquids. These colloidal particles are coated with surfactants to avoid their agglomeration. Because of the industrial applications of ferrofluids, the investigation on them fascinated the researchers and engineers vigorously since last five decades. One of the many fascinating features of the ferrofluids is the prospect of influencing flow by a magnetic field and vice-versa [1], [2]. Ferrofluids are widely used in sealing of the hard disc drives, rotating x-ray tubes under engineering applications. Sealing of the rotating shafts is the most known application of the magnetic fluid. The major applications of ferrofluid in electric field is that controlling of heat in loudspeakers which makes its life longer and increases the acoustical power without any change in the geometrical shape of the speaker system. Magnetic fluids are used in the contrast medium in X-ray examinations and for positioning tamponade for retinal detachment repair in eye surgery. Therefore, ferrofluids play an important role in bio-medical applications also.

There are rotationally symmetric flows of incompressible ferrofluids in the field of fluid mechanics, having all three velocity components radial, tangential and vertical in space, different from zero. In such types of flow, the variables are independent of the angular coordinates. We consider this type of flow for an incompressible ferrofluid when the plate is subjected to the magnetic field $[H_r, 0, H_z]$, using Neuringer-Rosensweig model [3]. This model has been used by Verma et al. [4]–[6] for solving paramagnetic couette flow, helical flow with heat conduction by taking into account the interactions of external magnetic field with ferrofluid. A detail account of magneto-viscous effects in ferrofluids has been given in a monograph by Odenbach [7]. Rosensweig [8] has given an authoritative introduction to the research on magnetic liquids in his monograph and study of the effect of magnetization yields the interesting information.

The pioneering study of ordinary viscous fluid flow due to the infinite rotating disk was first carried out by Karman [9]. He introduced the famous similarity transformation which reduces the governing partial differential equations into the ordinary differential equations. Cochran [10] obtained asymptotic solutions for the steady hydrodynamic problem formulated by Von Karman. Benton [11] improved Cochran’s solutions and solved the unsteady case. Attia [12] studied the unsteady state in the presence of an applied uniform magnetic field. Ram et al. [13] investigated the ferrofluid flow in a porous medium due to an infinite rotating disk.

Attia [14] investigated steady laminar flow of an incompressible viscous non-Newtonian fluid due to the uniform rotation of porous disk of infinite extent in porous medium with heat transfer. Sunil and Mahajan [15] studied the nonlinear stability analysis of magnetized ferrofluid heated and soluted from below with MFD viscosity via generalized energy method. Ram et al. [16] solved the non-linear differential equations under Neuringer-Rosensweig model for ferrofluid flow by using power series approximations and discussed the effect of magnetic field-dependent viscosity on velocity components and pressure profile. Further, the effect of porosity on velocity components and pressure profile has been studied by Ram et al. [17]. The magnetic effects on heat fluid and entropy generation interactions in a porous medium for a laminar incompressible flow in an inclined channel have been studied by Komurgoz et al. [18]. Ram and Kumar [19] investigated the effects of field dependent viscosity on ferrofluid flow saturating the porous medium over a rotating disk. And, ferrofluid flow with heat transfer over a stretchable rotating disk with magnetic field dependent viscosity has been investigated by Ram and Kumar [20].

In the present paper, we take cylindrical coordinates r, θ, z where the z-axis is normal to the plane and this axis is considered as the axis of rotation. We have presented the

Paras Ram is with the National Institute of Technology, Kurukshetra, Haryana 136119 India (phone: 91-94165-39758; fax: 91-1744-238050; e-mail: parasram_nit@yahoo.co.in).

Vikas Kumar is with National Institute of Technology, Kurukshetra, Haryana 136119 India (phone: 91-94660-19417; fax: 91-1744-238050; e-mail: vksingla.nitkkr@yahoo.com).

boundary layer equations along with boundary conditions. These equations along with Maxwell's relations are solved theoretically as well as numerically. Also, it is found that there is a large variation in the boundary layer thickness as compared to the ordinary viscous fluid flow case. We have also given the expression for the total volume flowing outwards the axis taken over a cylinder of radius R around the z-axis by using the description given on page 229 in Schlichting [21]. The effect of vertically applied magnetic field in a circular layer of ferrofluid within the rotating disk is studied within the framework of the Neuringer-Rosensweig approach. This problem, to the best of our knowledge, has not been investigated yet.

II. Formulation and Solution of the Problem

The ferrofluid flow under consideration is represented by the following basic governing equations.

Equation of continuity

$$\nabla \cdot \vec{V} = 0 \tag{1}$$

Equation of motion

$$\rho\left(\frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}\right) = -\nabla p' + \mu_0 (\vec{M}\cdot\nabla\vec{H}) + \mu\nabla^2\vec{V}$$

$$+2\rho(\varpi\times\vec{V}) + \frac{\rho}{2}\nabla|\varpi\times r|^2 - \frac{\mu}{K}\vec{V} \tag{2}$$

The effect of rotation includes two terms: (a) Centrifugal force $-\frac{1}{2}grad|\varpi\times r|^2$ and (b) Coriolis acceleration $2(\varpi\times\vec{V})$. In (2), $p' - \frac{\rho}{2}|\varpi\times r|^2 = p$ is the reduced pressure, where p' stands for fluid pressure.

Maxwell's relations

$$\nabla\times\vec{H} = \vec{0};\quad \nabla\cdot(\vec{H}+\vec{M}) = 0 \tag{3}$$

Assumptions

$$\vec{M} = \chi\vec{H},\ \vec{M}\times\vec{H} = \vec{0} \tag{4}$$

The revolving ferrofluid flow is subjected to following boundary conditions

$$\left.\begin{array}{l} V_r = 0,\ V_\theta = 1,\ V_z = 0 \quad at \quad z = 0 \\ V_r, V_\theta \to 0 \ and \ V_z \to -C \ \ as \ z\to\infty \end{array}\right\} \tag{5}$$

On considering the assumptions that the flow is steady $\left(i.e.\ \frac{\partial}{\partial t}(\,) = 0\right)$ and axisymmetric $\left(i.e.\ \frac{\partial}{\partial\theta}(\,) = 0\right)$, negligible variation in magnetic field in axial direction, the boundary layer approximation $-\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu_0}{\rho}|\vec{M}|\frac{\partial}{\partial r}|\vec{H}| = -r\omega^2$ in radial direction and using the Karman's similarity transformations

$$\begin{array}{l} V_r = r\omega E(\alpha),\ V_\theta = r\omega F(\alpha), V_z = \sqrt{\upsilon\omega}\, G(\alpha), \\ p = \rho\omega\upsilon P(\alpha) \ \ where \ \ \alpha = \sqrt{\frac{\omega}{\upsilon}}\, z. \end{array} \tag{6}$$

We get a system of non linear coupled ordinary differential equations in the dimensionless variables *E, F, G* and *P* as:

$$E'' - GE' - E^2 + F^2 + 2F - \beta E - 1 = 0 \tag{7}$$

$$F'' - GF' - 2EF - 2E - \beta F = 0 \tag{8}$$

$$P' - G'' + GG' + \beta G = 0 \tag{9}$$

$$G' + 2E = 0 \tag{10}$$

The boundary conditions for the flow are

$$\left.\begin{array}{l} E(0) = 0,\ F(0) = 1,\ \ G(0) = 0,\ P(0) = P_0 \\ E, F\to 0 \ and \ G\to -c \ \ as \ \alpha\to\infty \end{array}\right\} \tag{11}$$

Cochran indicated that formal asymptotic expansions (for large α) of the system of (7)-(10) are the power series in $exp\,(-c\alpha)$, i.e.

$$E(\alpha) \approx \sum_{i=1}^{\infty} A_i e^{-ic\alpha} \tag{12}$$

$$F(\alpha) \approx \sum_{i=1}^{\infty} B_i e^{-ic\alpha} \tag{13}$$

$$G(\alpha) \approx G(\infty) + \sum_{i=1}^{\infty} C_i e^{-ic\alpha} \tag{14}$$

$$(P - P_0)(\alpha) \approx \sum_{i=1}^{\infty} D_i e^{-ic\alpha} \tag{15}$$

Let $E'(0) = a$ and $F'(0) = b$. Using this supposition and (12)-(15), we get the following boundary conditions for the approximate solution:

$$E''(0) = -2;\ E'''(0) = a\beta - 4b;\ E^{iv}(0) = -2(b^2 + 3\beta) \tag{16}$$

$$F''(0) = \beta;\ F'''(0) = 4a + b;\ F^{iv}(0) = 2ab + \beta^2 - 8 \tag{17}$$

$$G'(0) = 0;\ G''(0) = -2a;\ G'''(0) = 4;$$
$$G^{iv}(0) = 2(4b - a\beta) \tag{18}$$

$$P'(0) = -2a;\ P''(0) = 4;\ P'''(0) = 8b;$$
$$P^{iv}(0) = 4(b^2 + 2\beta - 3a^2) \tag{19}$$

First five coefficients in (11) are calculated with the help of (10) and the boundary conditions for $E(\alpha)$ in (15), which are as follows:

$$A_1 = \left(-\frac{b^2+3\beta}{12c^4} - \frac{7(4b-a\beta}{12c^3} - \frac{71}{12c^2} + \frac{77}{12}\frac{a}{c}\right)$$

$$A_2 = \left(-\frac{b^2+3\beta}{3c^4} + \frac{13(4b-a\beta)}{6c^3} + \frac{59}{3c^2} - \frac{107}{6}\frac{a}{c}\right)$$

$$A_3 = \left(-\frac{b^2+3\beta}{2c^4} - \frac{3(4b-a\beta)}{c^3} - \frac{49}{2c^2} + \frac{39}{2}\frac{a}{c}\right)$$

$$A_4 = \left(\frac{b^2+3\beta}{3\,c^4} + \frac{11(4b-a\beta)}{6c^3} + \frac{41}{3c^2} - \frac{61}{6}\frac{a}{c}\right)$$

$$A_5 = \left(-\frac{b^2+3\beta}{12c^4} - \frac{5(4b-a\beta)}{12c^3} - \frac{35}{12c^2} + \frac{25}{12}\frac{a}{c}\right)$$

Similarly we can find other coefficients involving in (17) - (19). Using the values $a = 0.54$, $b = -0.62$ and $c = 0.886$ from Cochran [10], we calculate the values of the coefficients $A_1, A_2, A_3, A_4, A_5;\ B_1, B_2, B_3, B_4, B_5;\ C_1, C_2, C_3, C_4, C_5;$ D_1, D_2, D_3, D_4 and D_5, numerically. We draw the graphs of velocity components and asymptotic pressure with the dimensionless parameter α.

The boundary layer displacement thickness is calculated as:

$$d = \frac{1}{r\varpi}\int_{z=0}^{\infty} v_\theta\, dz = \int_{\alpha=0}^{\infty} F(\alpha)\, d\alpha \tag{20}$$

Total volume flowing outward the z-axis,

$$Q = 2\pi R\int_{z=0}^{\infty} v_r\, dz = 2\pi R^2 \int_{\alpha=0}^{\infty} \varpi E(\alpha)\sqrt{\nu/\varpi}\, d\alpha$$
$$= -\pi R^2 \sqrt{\varpi\nu}\, G(\infty) = 2.786094 R^2 \sqrt{\varpi\nu}$$
$$= 2.786094 R^2 \nu \frac{\alpha}{z}$$

The fluid is taken to rotate at a large distance from the wall, the angle becomes

$$\tan\varphi_0 = -\left(\frac{\partial v_r}{\partial z}\Big/\frac{\partial v_\theta}{\partial \theta}\right) = -\frac{E'(0)}{F'(0)} = \frac{0.54}{0.62} = 0.870967$$

$$\Rightarrow \varphi_0 = 41^0$$

III. Discussion of Results

The problem considered here involves a number of parameters, on the basis of which, a wide range of numerical results have been derived. A brief summary of these results is presented here. If we remove the terms $2\rho(\varpi \times \vec{V}) + \frac{\rho}{2}\nabla|\varpi \times r|^2$ from the momentum (2), the problem reduces to the case of disk driven ferrofluid flow in porous medium already discussed by Ram et al. [13], and the result thus obtained are found to be in proper accordance with the results presented in [13]. The numerical results for the velocity profile, for (r, θ, z) components of velocity, commonly known as radial, tangential, axial velocities, are shown graphically in Figs. 1-3, respectively.

In Fig. 1, E_1, E_2, E_3 and E_4 show the radial velocity profiles with the variation of Karman's parameter α for porosity parameter $\beta = 0, 1, 2$ and 3, respectively. Clearly $\beta = 0$ indicates the absence of porous medium. Here, the radial velocity E_1, E_2, E_3 and E_4 have the maximum values 0.085052, 0.0866, 0.088238 and 0.089961 at α = 0.35, 0.36, 0.37 and 0.38, respectively. From the graph, we conclude that the peak values for radial component of velocity are increasing in accordance with porosity parameter β. Also, in the present case, the radial velocity profile shows an oscillation in positive and negative region due to revolution of ferrofluid. In the nut shell, we can say that the oscillating trend of values of radial velocity due to revolution of ferrofluid in a porous medium leads to its slow convergence.

Fig. 2 shows the tangential velocity profiles for different values of porosity parameter. Here, for α = 2, the tangential velocity components are 0.249515, 0.558861, 0.879119 and 1.210286 for different values of porosity parameter β = 0, 1, 2 and 3, respectively. Hence, the tangential velocity component is increasing with an increase in Karman's parameter α due to increase in porosity parameter. Also, there is an appearance of instant jump in tangential velocity as we increase the porosity parameter β from 1 to 3. This instant jump is due to the fact that increasing porosity parameter favors the revolution of ferrofluid and thus increases the resultant of revolution of the disk and the fluid.

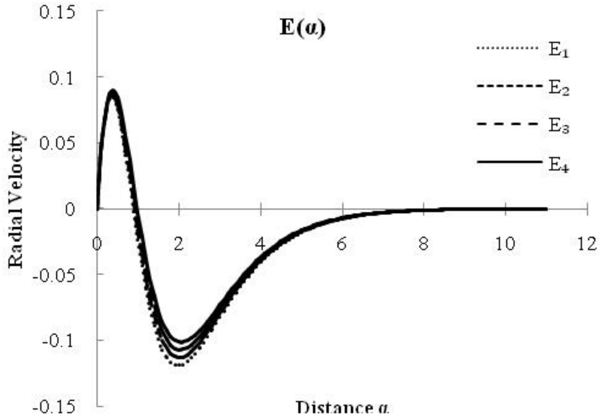

Fig. 1 Effect of rotation on radial velocity for variation in porosity parameter β

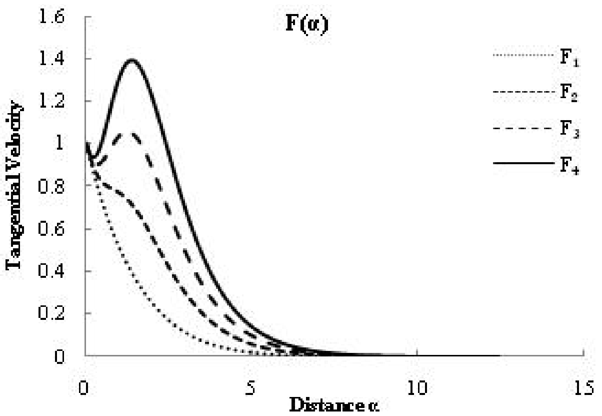

Fig. 2 Effect of rotation on tangential velocity for variation in porosity parameter β

Fig. 3 shows the axial velocity profiles, which are zero in the beginning and tends to a finite negative value -0.886 in the last as shown by G_1, G_2, G_3 and G_4 for different values of porosity parameter. It is clear that when we increase the magnetic field, the axial velocity goes to more negative region. Here, G_1 shows the axial velocity for the flow without porous medium. When we increase the value of α, it decreases continuously in the negative region. There is negligible variation in the axial velocity for revolution of ferrofluid for different values of porosity parameter i.e. the curves for the axial velocities G_1, G_2, G_3 and G_4 are almost same for porosity parameter $\beta = 0, 1, 2$ and 3, respectively. When we increase the value of α, these decrease continuously in the negative region and converges to -0.886 onwards α = 10.

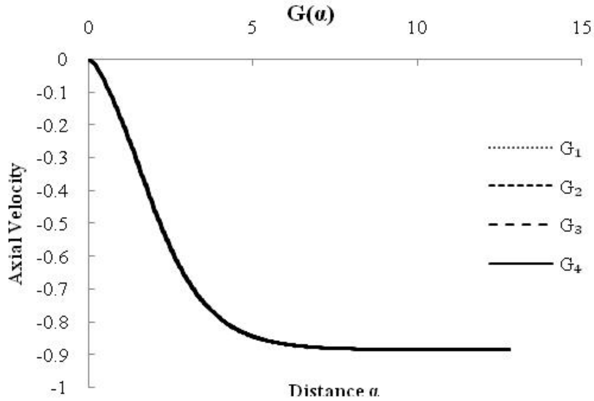

Fig. 3 Effect of rotation on axial velocity for variation in porosity parameter β

Fig. 4 presents the pressure profiles with initial pressure P_0 at α = 0 for different values of porosity parameter. The pressure goes to negative region for first few values of α. At α = 0.4, $(P_1 - P_0)$ goes to maximum negative value -0.16912 for β = 0, whereas $(P_2 - P_0), (P_3 - P_0)$ and $(P_4 - P_0)$ reaches to maximum negative values -0.16647, -0.16521 and -0.16398 at α = 0.3 for porosity parameter β = 1, 2 and 3, respectively. After continuously increasing the values of α, the pressure also increases and at around α = 1, it enters in positive region. When we increase the values of α continuously α = 1 onwards, the pressure $(P_1 - P_0)$, $(P_2 - P_0)$, $(P_3 - P_0)$ and $(P_4 - P_0)$ attain their peak values 0.219005, 0.262645, 0.306288 and 0.349931 at α = 2 for β = 0, 1, 2 and 3, respectively.

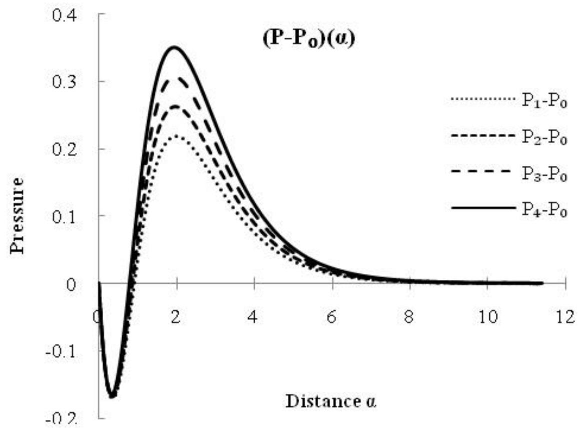

Fig. 4 Effect of rotation on pressure for variation in porosity parameter β

Comparing Figs. 1 and 4, we conclude that when the radial velocity increases, the pressure decreases and when the radial velocity decreases, the pressure increases. These figures have converse behavior to each other. The change in the curve for the radial velocity is faster due to external magnetic field, and magnetic field reduces the time required for velocity profile to reach their convergence level. In the present work, we have calculated the displacement thicknesses numerically for various values of porosity parameter. Here, the displacement thicknesses are d_1 = 1.4020582, d_2 = 2.309985, d_3 = 3.248487 and d_4 = 4.217463 for β = 0, 1, 2 and 3, respectively. Whereas in the similar study conducted by Benton [11] in absence of revolutionary effect and porous medium both, the displacement thickness was 1.27144. This much difference comes out due to the effect of porous medium. In nut shell, we can conclude that the boundary layer displacement thickness is increasing with increase in porosity parameter β.

Also, we have calculated the angle between the wall and ferrofluid, which is 41^0.

IV. Concluding Remarks

In nut shell, the porosity parameter and revolution of ferrofluid have appreciable effects on the ferrofluid flow. The values of radial, tangential velocity components and pressure get increased on increasing the porosity parameter whereas the axial velocity remains almost unaffected. Also revolution of ferrofluid in porous medium leads to a slow convergence rate of the various flow characteristics. Also, the displacement boundary layer gets thicker due to porous medium

Nomenclature

$\vec{H}$ Magnetic field intensity
$\vec{M}$ Magnetization
p' Fluid pressure
p Reduced fluid pressure
$\vec{V}$ Velocity of ferrofluid
υ Kinematic viscosity
μ_f Reference viscosity of fluid
μ_0 Magnetic permeability of free space
ρ Fluid density
χ Magnetic susceptibility
∇ Gradient operator
K Darcy Permeability
α Dimensionless parameter
β Viscosity variation parameter
r Radial direction
θ Tangential direction
z Axial direction
ω Angular velocity of disk
ϖ Velocity of fluid revolution
V_r Radial velocity
V_θ Tangential velocity
V_z Axial velocity
φ_0 Angle of fluid rotation
Q Total volume of fluid flowing outward the z-axis

References

[1] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lecturers on Physics.* USA: Addison-Wesley, vol. 1, 1964.
[2] M. I. Shliomis, "Ferrofluids as thermal ratchets," *Physical Review Letters*, vol. 92, pp. 188901, 2004.
[3] J. L. Neuringer, and R. E. Rosensweig, "Magnetic fluids," *Physics of Fluids*, vol. 7, pp. 1927, 1964.
[4] P. D. S. Verma, and M. J. Vedan, "Helical flow of ferrofluid with heat conduction," *Jour. Math. Phy. Sci.*, 12, pp. 377-389, 1978.
[5] P. D. S. Verma, and M. J. Vedan, "Steady rotation of a sphere in a paramagnetic fluid," *Wear*, vol. 52, pp. 201-218, 1979.
[6] P. D. S. Verma, and M. Singh, "Magnetic fluid flow through porous annulus," *Int. J. Non-Linear Mechanics*, vol. 16, pp. 371-378, 1981.
[7] S. Odenbach, *Magnetoviscous Effects in Ferrofluids.* Berlin: Springer-Verlag, 2002.
[8] R. E. Rosensweig, *Ferrohydrodynamics*. Cambridge: Cambridge University Press, 1965.
[9] V. Karman, "Uber laminare and turbulente reibung," *Z. Angew. Math. Mech.*, vol. I, pp. 232- 252, 1921.
[10] W. G. Cochran, "The flow due to a rotating disc," *Proc. Camb. Phil. Soc.*, vol. 30, pp. 365-375, 1934.
[11] E. R. Benton, "On the flow due to a rotating disk," *Journal of Fluid Mechanics*, vol. 24, pp. 781–800, 1966.
[12] H. A. Attia, "Unsteady MHD flow near a rotating porous disk with uniform suction or injection," *Journal of Fluid Dynamics Research*, vol. 23, pp. 283-290, 1998.
[13] P. Ram, A. Bhandari, and K. Sharma, "Axi-Symmetric ferrofluid flow with rotating disk in a porous medium," *International Journal of Fluid Mechanics*, vol. 2, pp. 151-161, 2010.
[14] H. A. Attia, "Rotating disk flow and heat transfer through a porous medium of a non-Newtonian fluid with suction and injection," *Communication in Non-linear science and numerical simulation*, vol. 13, pp. 1571-1580, 2008.
[15] Sunil, and A. Mahajan, "A nonlinear stability analysis of a double-diffusive magnetized ferrofluid with magnetic field-dependent viscosity," *Journal of Magnetism and Magnetic Materials*, vol. 321, pp. 2810-2820, 2009.
[16] P. Ram, A. Bhandari, and K. Sharma, "Effect of magnetic field-dependent viscosity on revolving ferrofluid," *Journal of Magnetism and Magnetic Materials*, vol. 322, pp. 3476-3480, 2010.
[17] P. Ram, K. Sharma, and A. Bhandari, "Effect of porosity on ferrofluid flow with rotating disk," *International Journal of Applied Mathematics and Mechanics*, vol. 6, pp. 67-76, 2010.
[18] G. Komurgoz, A. Arikoglu, and I. Ozkol, "Analysis of the magnetic effect on entropy generation in an inclined channel partially filled with a porous medium," *Numerical Heat Transfer, Part A: Application: An International Journal of Computation and Methodology*, vol. 61, pp. 786-799, 2012.
[19] P. Ram, and V. Kumar, "Ferrofluid flow with magnetic field dependent viscosity due to a rotating disk in porous medium," *International Journal of Applied Mechanics*, vol. 4, pp. 1250041 (18 pages), 2012.
[20] P. Ram, and V. Kumar, "FHD flow with heat transfer over a stretchable rotating disk," *Multidiscipline Modeling in Materials and Structures,* vol. 9, pp. 524-537.
[21] H. Schlichting, *Boundary Layer Theory.* New York: McGraw-Hill Book Company, 1960.

Paras Ram was born in Haryana (India) on 20 April, 1965. The author earned Ph.D. degree in Applied Mathematics from Faculty of Engineering, R.E.C.K Kurukshetra, India in the year of 1994, and M.Sc. degree in Applied Mathematics from Kurukshetra University, Kurukshetra, India in the year of 1987. Author's major field of interests involve ferrohydrodynamics, magnetohydrodynamics and boundary layer flows.

He is presently working as Professor and Head, Department of Mathematics, National Institute of Technology Kurukshetra, India. He has 19 years of experience in teaching and research and has been associated with several reputed technical universities and institutes for curriculum design and setting of question papers in mathematics for B.Tech. programs. He has authored a book titled as 'Engineering Mathematics through Applications' (New Delhi, India: CBS Publishers and Distributors, 2011). He has participated in large number of teaching and research programs organized by various universities and has published many research papers in national and international SCI journal of considerable impact.

Dr. Ram is life member of Indian Society of Theoretical and Applied Mechanics (ISTAM) and Indian Society for Mathematical Modeling and Computer Simulation (ISMMACS).

Vikas Kumar was born in Haryana (India) on 16 December, 1984. The author has earned M.Sc. degree in Applied Mathematics from Kurukshetra University, Kurukshetra, India in the year of 2007. Author's major field of interest is boundary layer flows over infinite disk in ferrohydrodynamics.

He is presently working as Research Scholar in the Department of Mathematics, National Institute of Technolgy Kurukshetra, India. He has 6 years of experience in teaching and research. He has visited several universities and technical institutions to participate in various research activities and to present his research work. He has published 4 research papers in referred/ SCI international journal of considerable impact and 4 papers in conference proceedings.