Reduced Order Modeling of Natural Gas Transient Flow in Pipelines

M. Behbahani-Nejad, and Y. Shekari

Abstract—A reduced order modeling approach for natural gas transient flow in pipelines is presented. The Euler equations are considered as the governing equations and solved numerically using the implicit Steger-Warming flux vector splitting method. Next, the linearized form of the equations is derived and the corresponding eigensystem is obtained. Then, a few dominant flow eigenmodes are used to construct an efficient reduced-order model. A well-known test case is presented to demonstrate the accuracy and the computational efficiency of the proposed method. The results obtained are in good agreement with those of the direct numerical method and field data. Moreover, it is shown that the present reduced-order model is more efficient than the conventional numerical techniques for transient flow analysis of natural gas in pipelines.

Keywords—Eigenmode, Natural Gas, Reduced Order Modeling, Transient Flow.

I. INTRODUCTION

THE dynamic behavior of long pipelines is characterized by large time constants, sometimes of as much as several hours, due to the resistance to flow in pipes and the large storage capacity of the pipelines. Transients in such complex and large scale systems can be satisfactorily described by the nonhomogeneous, nonlinear hyperbolic, inviscid Euler system of conservation laws in one dimensional form [1]. Under isothermal conditions the continuity and momentum equations together with an equation of state constitute the governing equations describing transient flow in natural gas pipelines.

Traditional methods for the numerical analysis of system of governing equations are the Method of Characteristics (MOC) [2] and several finite difference schemes such as explicit finite differences [3] and implicit schemes [4]. Recent relevant studies used higher resolution explicit TVD Methods for the solution of sharp discontinuities fronts [5]. More recently the Method of Lines has been used with an adaptive mesh for solution system of governing equations of transient natural gas

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flow [1]. However, one prefers a numerical method which is not only accurate but also with low computational cost.

Reduced-order modeling (ROM) is recently known as a computational efficient technique for analysis of unsteady flows. Eigenmodes of the flow are used to construct reduced-order models similar to the normal mode analysis commonly used in structural dynamics. The advantage of a modal approach is that one may construct a reduced-order model by retaining only a few of the original modes. This method has been used for unsteady aerodynamics and aeroelastic problems by several researchers [6] – [14].

Although ROM based on the flow eigenmodes is a well-known numerical technique, it is not yet applied for transient compressible flow analysis in the pipelines. In the present work this approach is chosen to achieve an efficient computational scheme for natural gas transient pipeflows. The nonhomogeneous Euler equations under isothermal condition are numerically solved using the implicit Steger-Warming flux vector splitting method (FSM) and their results are compared with the available experimental results. Next, they are linearized about the steady state condition and the linearized flow results are compared with the corresponding nonlinear ones. Then, the eigensystem of the linearized transient flow is derived and the eigenvalues and eigenvectors are calculated. Based on the above eigenanalysis, a few dominant eigenmodes are used to construct a reduced order model. Next, the results of the present ROM are compared with those of the direct numerical schemes and its accuracy and efficiency is discussed. Finally, the paper is concluded with some comments about the present eigenanalysis and ROM for natural gas transient flow in pipelines.

II. GOVERNING EQUATIONS

Under isothermal conditions the Euler equations along with a source term due to the pipe friction effect are governed the dynamics of the natural gas in a long pipeline [5]. In conservative form they are

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{E}(\mathbf{Q})}{\partial \mathbf{x}} - \mathbf{H}(\mathbf{Q}) = 0 \qquad (1)$$

where

$$\mathbf{Q} = \begin{pmatrix} \rho \\ \rho u \end{pmatrix}$$

$$\mathbf{E}(\mathbf{Q}) = \begin{pmatrix} \rho u \\ \rho u^2 + c^2 \rho \end{pmatrix} \tag{2}$$

$$\mathbf{H}(\mathbf{Q}) = \begin{pmatrix} 0 \\ -\dfrac{\rho f_g u |u|}{2D} \end{pmatrix}$$

In the above equations c is isothermal sound speed, ρ is the gas density, u is the axial gas velocity, D is diameter of the pipe and f_g is the pipe friction factor.

III. FINITE DIFFERENCE SCHEME

The implicit Steger-Warming flux vector splitting method (FSM) has been used as the numerical scheme. This method is chosen, because it doesn't have the problem of numerical instability. In delta formulation, the finite difference form of the method is [15]

$$-\left(\frac{\Delta t}{\Delta x}\mathbf{A}^+_{j-1}\right)\Delta\mathbf{Q}_{j-1}$$
$$+\left(\mathbf{I} + \frac{\Delta t}{\Delta x}\left(\mathbf{A}^+_j - \mathbf{A}^-_j\right) - \Delta t\mathbf{B}_j\right)\Delta\mathbf{Q}_j$$
$$+\left(\frac{\Delta t}{\Delta x}\mathbf{A}^-_{j+1}\right)\Delta\mathbf{Q}_{j+1} = \tag{3}$$
$$-\frac{\Delta t}{\Delta x}\left(\mathbf{E}^+_j - \mathbf{E}^+_{j-1} + \mathbf{E}^-_{j+1} - \mathbf{E}^-_j\right) + \Delta t\,\mathbf{H}_j$$

where subscript j indicates the spatial grid point, superscript n indicates the time level and

$$\Delta\mathbf{Q} = \mathbf{Q}^{n+1} - \mathbf{Q}^n \tag{4}$$

In relation (3), $\mathbf{I}$ is the identity matrix, $\mathbf{A}$ and $\mathbf{B}$ are Jacobean matrices which are defined as

$$\mathbf{A} = \frac{\partial\mathbf{E}}{\partial\mathbf{Q}} \qquad , \qquad \mathbf{B} = \frac{\partial\mathbf{H}}{\partial\mathbf{Q}} \tag{5}$$

Moreover, $\mathbf{A}^+$ and $\mathbf{A}^-$ are respectively the positive and negative parts of the Jacobean matrix $\mathbf{A}$, namely

$$\mathbf{A}^+ = \begin{pmatrix} \dfrac{c^2 - u^2}{2c} & \dfrac{u+c}{2c} \\ \dfrac{(u+c)^2(c-u)}{2c} & \dfrac{(u+c)^2}{2c} \end{pmatrix}$$
$$\mathbf{A}^- = \begin{pmatrix} \dfrac{u^2 - c^2}{2c} & \dfrac{c-u}{2c} \\ \dfrac{(u+c)(c-u)^2}{2c} & -\dfrac{(c-u)^2}{2c} \end{pmatrix} \tag{6}$$

In addition, $\mathbf{E}^+$ and $\mathbf{E}^-$ are the positive and negative parts of the flux vector $\mathbf{E}$ which are represented as:

$$\mathbf{E}^+ = \begin{pmatrix} \dfrac{\rho(u+c)}{2} \\ \dfrac{\rho(u+c)^2}{2} \end{pmatrix}$$
$$\mathbf{E}^- = \begin{pmatrix} \dfrac{\rho(u-c)}{2} \\ \dfrac{\rho(u-c)^2}{2} \end{pmatrix} \tag{7}$$

When equation (3) is applied to each grid point, a block tridiagonal system of algebraic equations is obtained. This equations system is solved at each time step, which results in $\Delta\mathbf{Q}$. Next, $\mathbf{Q}$ at the advanced time step can be calculated using Eq. (4).

IV. LINEARIZED FINITE DIFFERENCE EQUATIONS

To do the eigenanalysis and construct an eigenmode based reduced order model, it is necessary to linearize the equations. For this purpose, the flow field variables at each time step are considered as

$$Q^{n+1} = Q^o + \hat{Q}^{n+1} \tag{8}$$

where Q^o is the corresponding steady state value and $\hat{Q}$ is a small perturbation about it. Substituting equation (8) into (3) and doing some manipulations yields

$$-\left(\frac{\Delta t}{\Delta x}\mathbf{A}^{o+}_{j-1}\right)\hat{\mathbf{Q}}^{n+1}_{j-1} +$$
$$\left(\mathbf{I} + \frac{\Delta t}{\Delta x}\left(\mathbf{A}^{o+}_j - \mathbf{A}^{o-}_j\right) - \Delta t\mathbf{B}^o_j\right)\hat{\mathbf{Q}}^{n+1}_j \tag{9}$$
$$+\left(\frac{\Delta t}{\Delta x}\mathbf{A}^{o-}_{j+1}\right)\hat{\mathbf{Q}}^{n+1}_{j+1} = \hat{\mathbf{Q}}^n_j$$

The above linearized equation can be represented as

$$\mathbf{W}^o\hat{\mathbf{Q}}^{n+1} = \mathbf{I}\hat{\mathbf{Q}}^n + \mathbf{V}^{n+1} \tag{10}$$

where $\mathbf{V}$ is a vector consisting the imposed values by the boundary conditions and $\mathbf{W}^o$ is made by the left hand side factors of Eq. (9).

V. EIGENANALYSIS AND ROM

For zero forcing function, $\mathbf{V}$, one can set $\hat{\mathbf{Q}}_i = \mathbf{x}_i \exp(\lambda_i t)$ and $z_i = \exp(\lambda_i \Delta t)$ to obtain the following generalized eigenvalue problem

$$z_i\mathbf{W}^o\mathbf{x}_i = \mathbf{I}\mathbf{x}_i \tag{11}$$

where λ_i and z_i are i^{th} eigenvalues in λ-plane and z-plane, respectively, and $\mathbf{x}_i$ is the corresponding eigenvector. More generally Eq. (11) can be written as

$$\mathbf{ZW}^o\mathbf{X} = \mathbf{IX} \tag{12}$$

where $\mathbf{Z}$ is a diagonal matrix containing the eigenvalues and $\mathbf{X}$ is a matrix with columns that are the right eigenvectors. On the other hand, the left eigenvectors satisfy the following relation

$$(\mathbf{W}^\circ)^{\mathbf{T}}\mathbf{YZ} = \mathbf{IY} \qquad (13)$$

where $\mathbf{Y}$ is a matrix with rows that are the left eigenvectors. If the eigenvectors are normalized suitable, they satisfy the following orthogonality conditions

$$\mathbf{Y}^{\mathbf{T}}\mathbf{W}^\circ\mathbf{X} = \mathbf{I}$$
$$\mathbf{Y}^{\mathbf{T}}\mathbf{IX} = \mathbf{Z} \qquad (14)$$

The dynamic behavior of the fluid can be represented as the sum of the individual eigenmodes, that is,

$$\hat{\mathbf{Q}} = \mathbf{Xc} \qquad (15)$$

where $\mathbf{c}$ is the vector of normal mode coordinates. Substitution of (15) into (10), premultiplying by $\mathbf{Y}^T$, and making use of the orthogonality condition gives a set of N uncoupled equations for the modal coordinates $\mathbf{c}$,

$$\mathbf{c}^{n+1} = \mathbf{Zc}^n + \mathbf{Y}^T\mathbf{V}^{n+1} \qquad (16)$$

Now, one may construct a reduced-order model by retaining only a few of the original modes.

VI. RESULTS AND DISCUSSIONS

A 72259.5 m long pipeline of 0.2 m diameter is considered as a test case to verify the results of the present method. Figure 1 shows the test case schematically. The above test case which its experimental results are available, has been studied by Taylor et al. [16], Zhou and Adewumi [5], and also by Tentis et al. [1]. The pipeline transports natural gas of 0.675 specific gravity at $10^\circ\mathrm{C}$. The gas viscosity is 11.84×10^{-6} kg/ms while the pipeline wall roughness is 0.617mm and isothermal sound speed equals 367.9 m/s. At the pipeline's inlet, the gas pressure is kept constant at 4.21MPa, whereas the pipe's mass flow rate at the outlet varies with a 24-hour cycle, corresponding to changes in consumer demand within a day as is shown in Fig. 2.

$$p_i = 4.21\,\mathrm{MPa} \qquad\qquad p_O(t) = ?$$
$$m_i(t) = ? \qquad\qquad m_O = m_O(t)$$
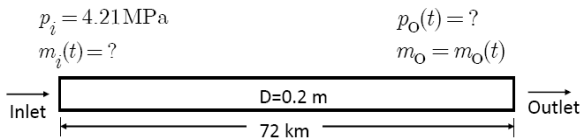
Inlet — D=0.2 m — Outlet

72 km

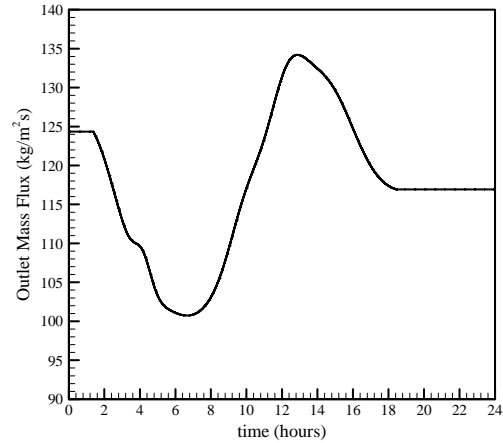Fig. 1 Schematic of the pipeline and its B.Cs.

Fig. 2 Imposed mass flux at the outlet

Fig. 3 illustrates the present results of FSM for pressure time changes at the pipe outlet, along with those of the others [1, 5, 16] and the experiments. There are some differences between the present nonlinear FSM results with those obtained by the others. However, when they are compared with the experiments, it seems that all of the numerical methods have the nearly similar errors. The interesting point is the accuracy of the results of the present linearized FSM. As is shown in Fig. 3, the linearized FSM can predict the transient behavior of the outlet pressure as nearly accurate as the nonlinear models. Thus, one can construct a reduced order model based on the linearized equations to estimate the transient gas pipeflows more efficiently. In figure 4 the gas pressure at some different points are presented and compared with those by Zhou and Adewumi [5]. It is observed that the present results are in relatively good agreement with those obtained by the above authors. Moreover, Fig. 4 shows that the present linearized FSM results in the gas pressure as accurate as the nonlinear model.
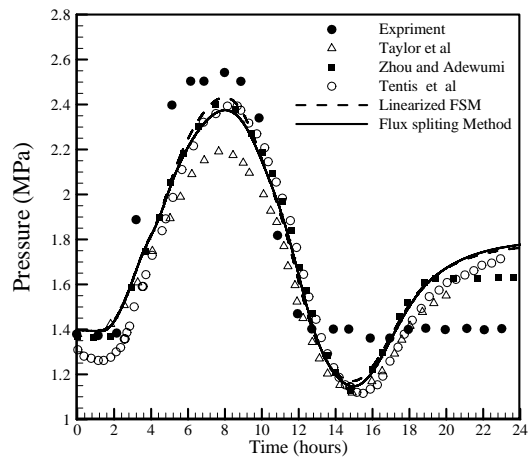

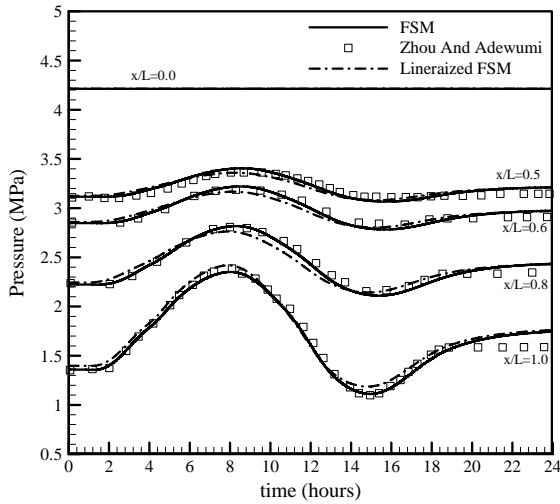Fig. 3 Comparison of pressure time history at the outlet

Fig. 4 Pressure time history at different points of the pipe

Fig. 6 The Eigenvalues in λ-plane

Next, the results of the present eigenanalysis are discussed. The eigenvalues of the present method are shown in Figs. 5 and 6 in λ-plane and z-plane, respectively. In addition, the first 60 eigenvalues in λ-plane are illustrated in Fig. 7. As is shown in Fig. 5, this eigensystem has no any zero eigenvalue in the z-plane. Behbahani-Nejad et al. [17] have been shown that when there is no zero eigenvalue, there is no any quasisteady eigenmode and thus, it is likely to construct a reduced order model without the static correction requirement. On the other hand, Fig. 6 illustrates that the real part of all eigenvalues are negative. In physical point of view, it means that the present numerical technique is stable. Moreover, as is shown in Fig. 7, a few first eigenvalues corresponds the dominant eigenmodes, because their absolute real and imaginary parts are relatively small and therefore they are activated before the other eigenmodes.
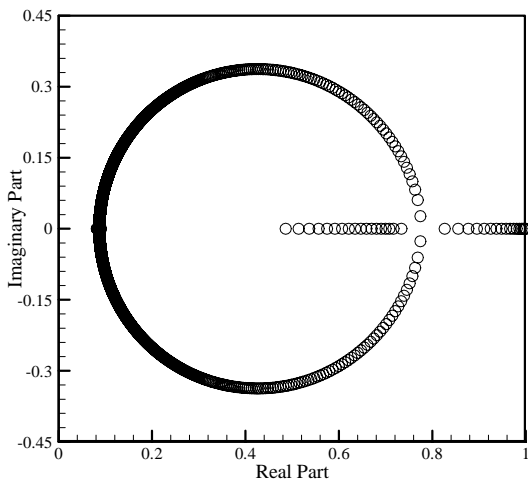
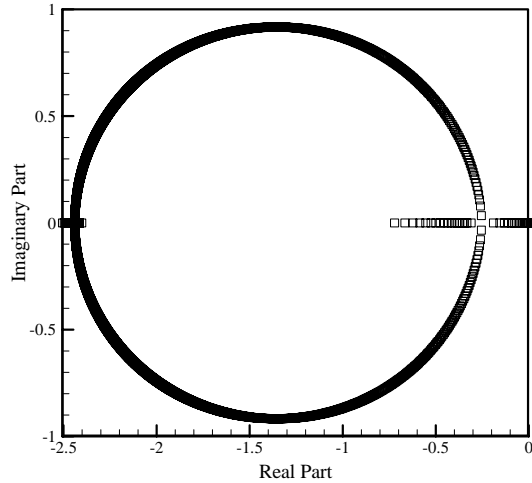The eigenvalues discussed in the preceding paragraph are used to construct a reduced order model. As there is no any zero eigenvalue, it is expected that the present reduced order model for natural gas transient flow gives satisfactory results without the static correction. Fig. 8 shows the pressure time history at the pipe outlet. It is observed that the results of the present ROM with only 4 or 5 eigenmodes are in excellent agreement with those obtained by the direct numerical method. The similar results are obtained for the pressure time changes at the other points along the pipe and are shown in Fig. 9. Finally, the efficiency of the present ROM are confirmed when its CPU times are compared with the direct numerical method. Table I indicates the computational times for the present ROM and the direct method. It is declared that there is about 70% reduction in CPU time when the present ROM is used.
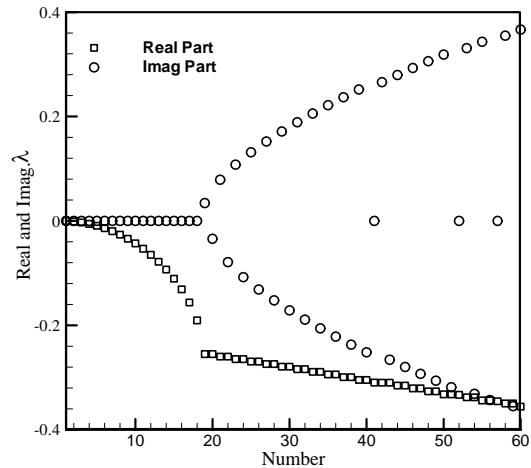

Fig. 5 The eigenvalues in z-plane

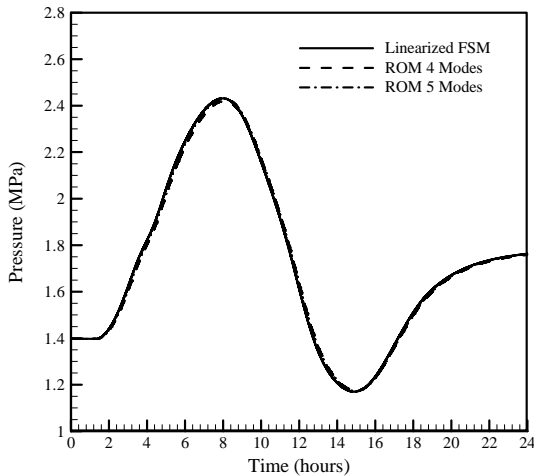Fig. 7 The first 60 eigenvalues in λ-plane

Fig. 8 Results of the present ROM and the direct method for
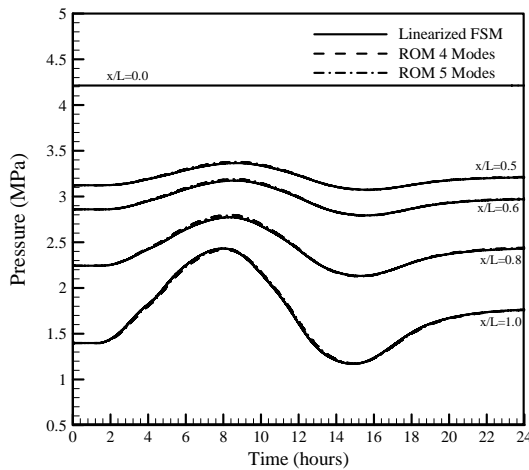pressure history at the outlet

Fig. 9 Results of the present ROM and the direct method for
pressure history at different points of the pipe

TABLE I
CPU TIMES FOR THE PRESENT ROM AND THE DIRECT METHOD

Method	Direct Simulation	ROM 4 Modes	ROM 5 Modes
CPU TIME	253 s	73 s	83 s
Time saving	–	71.14%	67.2%

VII. CONCLUSION

The present ROM can be used to analyze the transient flow of natural gas in pipelines, efficiently. Since the linearized forms of the governing equations can give satisfactory results with an enough degree of accuracy in many natural gas transient pipeflows, they can be used to construct proper reduced order models. The present eigenanalysis show that there is no any zero eigenvalue in the z-plane and therefore it is likely to construct reduced order models without the static correction requirement. It is indicated that the proposed reduced order model can efficiently gives satisfactory results for natural gas unsteady flow problems as accurate as the other direct numerical finite difference methods.

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