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Radiation Effect on MHD Casson Fluid Flow over a Power-Law Stretching Sheet with Chemical Reaction

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***Abstract*—**This article addresses the boundary layer flow and heat transfer of Casson fluid over a nonlinearly permeable stretching surface with chemical reaction in the presence of variable magnetic field. The effect of thermal radiation is considered to control the rate of heat transfer at the surface. Using similarity transformations, the governing partial differential equations of this problem are reduced into a set of non-linear ordinary differential equations which are solved by finite difference method. It is observed that the velocity at fixed point decreases with increasing the nonlinear stretching parameter but the temperature increases with nonlinear stretching parameter.

***Keywords*—**Boundary layer flow, nonlinear stretching, Casson fluid, heat transfer, radiation.

I. Introduction

OVER last few decades, a large number of researchers have shown their interest to investigate the boundary layer flow behaviours of non-Newtonian fluids over a stretching sheet for its great importance in industrial applications. This type of the flow occurs in industrial process like manufacturing coated sheets, drilling muds, polymer sheet extruded unendingly from a die, spinning of fibres, etc. The rate of cooling or heating at the nonlinear stretching sheet has an expectant significant to produce the quality of final product. Therefore, studies on convective heat transfer for non-Newtonian fluids presents a great compact of challenges to the engineers and researchers as the constitutive equations of non-Newtonian fluid cannot be expressed uniquely. It also shows more complexity to construct the governing equation to describe the viscoelastic feathers in non-Newtonian fluid compared with Navier-Stokes for viscous fluid. The Cason fluid is a class of non-Newtonian fluid which has definite advantages across the other non-Newtonian fluid model. Although this fluid can be assumed as shear thinning fluid. No flow occurs due to having an infinite viscosity at zero shear rate but it exhibits the zero viscosity at an infinite shear rate. Such type of fluid model can be conceived for the flow of tomato sauce, honey, different kind concentrated soups, jelly etc. For certain cases to analysis the human blood flow, it is addressed with Casson fluid model.

The velocity distribution of an incompressible viscous fluid over stretching surface was firstly investigated by [1]. Here the flow was made exclusively due to linear stretching of the sheet. Carragher and Crane [2] and Gupta and Gupta [3] investigated the momentum and heat transfer of viscous fluid over stretching surface with constant and variable temperature at the surface, respectively. Later, a large number of researchers, even now examined the flow analysis over a stretching with Casson fluid model based on various geometrical models [4]-[8]. Another side, the problem during the production of synthetic sheet requires to control the rate of heat transfer of the sheet by casting them through electrically conducting fluid in the presence of magnetic field. As a results, the final product will become a good quality with having desire characteristics. Yang and Chen [9] investigated the flow of electrically conducting fluid over continuously moving flat surface in the presence of magnetic field. Engineers apply the suction and blowing in many cases like, design of thrust bearing and thermal oil recovery. Suction is also used in chemical processes to take out the reactants [10], [11]. Other side, blowing is applied to bring the reactants. It is also used to control the cooling the surface and prevent corrosion or drag reduction on the surface [12].

The effects of heat and mass transfer on non-Newtonian fluid with chemical reactions are of significant in many chemical processes. Whereas, thermal radiation effect on fluid flow and heat transfer process has great meaning for project many energy conservation system at a high temperature. It is also observed that the effect of radiation due to heat transfer in the boundary layer are examined for process in space technology at high temperature. Thus thermal radiation is one of the factor to control the heat transfer in the process. Thermal radiation effect with viscous dissipation in the fluid flow on nonlinear stretching sheer was investigated by Chambre and Young [13]. The boundary layer flow over a flat plate was studied to removing the chemically reactive species in the boundary layer by destroying or generating the reactant [14].

The purpose of this work is to investigate the momentum and heat transfer characteristic in boundary layer flow of Casson fluid over a nonlinearly stretching sheet in the presence of a transverse magnetic field. Combined effects of suction/blowing and thermal radiation are considered to control the rate of heat transfer at the surface. The governing partial differential equations are reduced into set of nonlinear ordinary differential equations by suitable similarity transformation. It is interesting to note that the velocity distribution and rate of heat transfer are regulated appreciably by other parameters (nonlinear stretching sheet and applied magnetic field) in presence of suction and injection at the

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surface. The solutions for velocity, temperature, concentration distribution are plotted in graphs for various values of physical parameters and have been analysed its physical significant.

II. Problem Statement and Governing Equations

Let us consider the steady flow of an incompressible viscous Casson fluid over a non-linearly stretching sheet which coincides with horizontal plane y=0. The fluid is electrically conducting under the influence of applied variable magnetic field B(x) which is normal to the stretching sheet. Two equal and opposite forces are applied along the x axis so that the sheet is stretched with the origin fixed (see Fig. 1).

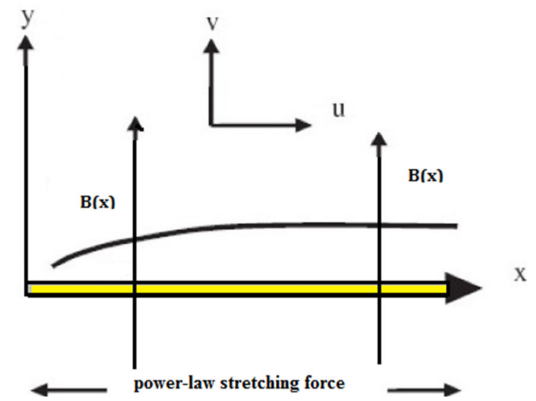


Fig. 1 Physical sketch of the problem

The governing boundary layer equations are written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial x}=\kappa\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k(C-C_\infty) \tag{4}$$

where u and u are the velocity components in the x and y directions respectively, ν is the kinematic viscosity, ρ is the fluid density, σ is the electrical conductivity of the fluid, $\beta=\sqrt{2\pi_c}\mu_B/p_y$ is the parameter of the Casson fluid, and κ is the thermal diffusivity of the fluid, q_r is the radiative heat flux, and c_p is the specific heat. C is the concentration, D is the diffusion coefficient, C_∞ is the value of the concentration in the free stream, k(x) is the variable reaction rate given by k(x) $=\frac{cx^{n-1}L}{V_0}$, where L is the reference length and V_0 is a constant. In (2), the external electric field and the polarization effects are negligible but variable magnetic field, $B(x)=B_0x^{(n-1)/2}$ is considered.

The boundary conditions corresponding to non-linear stretching of sheet are given by:

$$u=U=cx^n, v=V(x),\ T=T_w, C=C_w\ at\ y=0 \tag{5}$$

$$u\to 0,\quad T\to T_\infty,\ C\to C_\infty\quad as\ \ y\to y_\infty \tag{6}$$

Here, c (c > 0) is a parameter related to the surface stretching speed, $T_w\ (=T_o+T_o x^m)$, is the temperature at the sheet and $C_w\ (=C_0+C_o x^t)$, is the concentration at the sheet, T_∞ is the free stream temperature, and n is the power index related to the surface stretching speed and the suction velocity is $V(x)=V_0x^{\frac{n-1}{2}}$. Here C_o and T_o are constants.

Introducing the following transformations:

$$\eta=y\sqrt{\frac{c(n+1)}{2\nu}}x^{\frac{n-1}{2}},\qquad u=cx^nf'(\eta),$$
$$v=-\sqrt{c\nu\left(\frac{n+1}{2}\right)}x^{\frac{n-1}{2}}\left[f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right],$$
$$\frac{T-T_w}{T_w-T_\infty}=\theta(\eta),\ \frac{C-C_w}{C_w-C_\infty}=\varphi(\eta) \tag{7}$$

Substituting (7) in (2) and (3), we find the reduced governing equations as:

$$\left(1+\frac{1}{\beta}\right)f'''+ff''-\frac{2n}{n+1}f'^2-\left(\frac{2}{n+1}\right)Mf'=0, \tag{8}$$

$$R^*\theta''-\frac{2m}{(n+1)}Pr\theta f'+Prf\theta'=0 \tag{9}$$

$$\varphi''-\frac{2t}{n+1}Sc\varphi f'+Scf\varphi'-\frac{k_1}{n+1}Sc\varphi=0 \tag{10}$$

where the prime denotes the differentiation with respect to η and

$$M=\frac{\sigma B_0{}^2}{c\rho};\ R^*=\left(1+\frac{1}{R}\right);\ R=\frac{3\kappa k^*}{16\sigma T_\infty{}^3},\ Pr=\frac{\mu C_p}{k};\ Sc=\frac{\nu}{D};\ k_1=\frac{L}{V_0}.$$

Here k^* is the mean absorption co-efficient.

The boundary conditions take the following form:

$$f'=1,\ f=S,\quad \theta=1,\quad \varphi=1\ \text{at}\quad \eta=0 \tag{11}$$

$$f'\to 0,\quad \theta\to 0,\ ,\quad \varphi\to 0\quad \text{as}\quad \eta\to\infty, \tag{12}$$

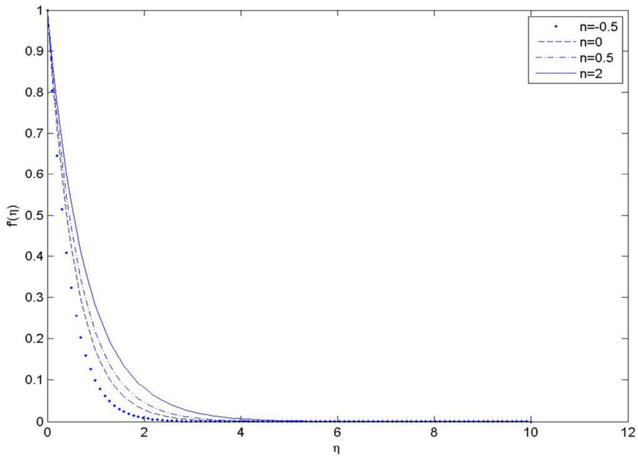
where the prime denotes the differentiation with respect to η, and $Pr=\frac{\nu}{\kappa}$ is the Prandtl number. $S=\frac{V_0}{\sqrt{\frac{c\nu(n+1)}{2}}}$ is suction parameter.

III. Results and Discussions

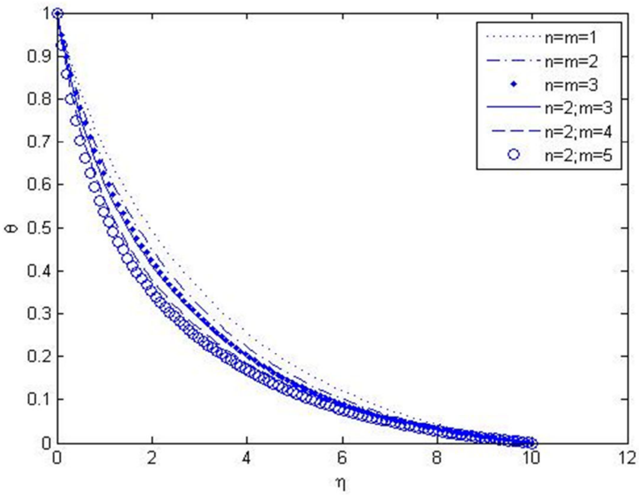
Equations (8)-(10) along with the boundary conditions (11) and (12) are solved numerically by finite-difference method using Thomas algorithm [15]. In order to analyse the results, numerical computations have been carried out for various values of power law factor n and m, Magnetic parameter M, radiation parameter R, suction parameter S, Schmidt number Sc and Prandtl number Pr.

Figs. 2 (a), (b), (c) display the velocity, temperature and concentration distribution respectively inside the boundary layer for different power law factor n and m. Other parameters are considered as Pr=0.7, Sc=0.3, β=2, M=2, R=2. Suction parameter varies with the exponent n as described above. In Fig. 2 (a), it is noted that the velocity increases as increasing n for fixed value of η. Similarly, Fig. 2 (b) shows that the temperature decreases with increasing the value of n and m. On the other hand, Fig. 2 (c) shows that when n=t and both power-law index n and t increase, the concentration increases but when n≠t, concentration decreases for increasing the value of t with constant n.

Figs. 3 (a), (b), and (c) show the variation of velocity, temperature and concentration distributions, respectively, for various value of magnetic parameter. It is observed that as magnetic parameter increases, the velocity decreases. On the other hand, with increase in magnetic parameter, temperature and concentration increases when other all parameters are considered as constant.



(a)



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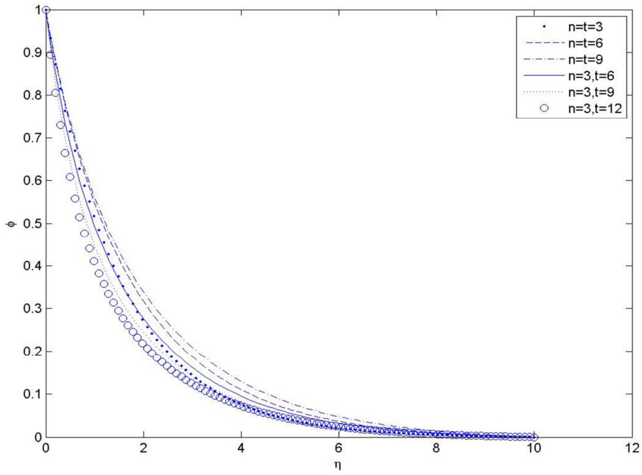
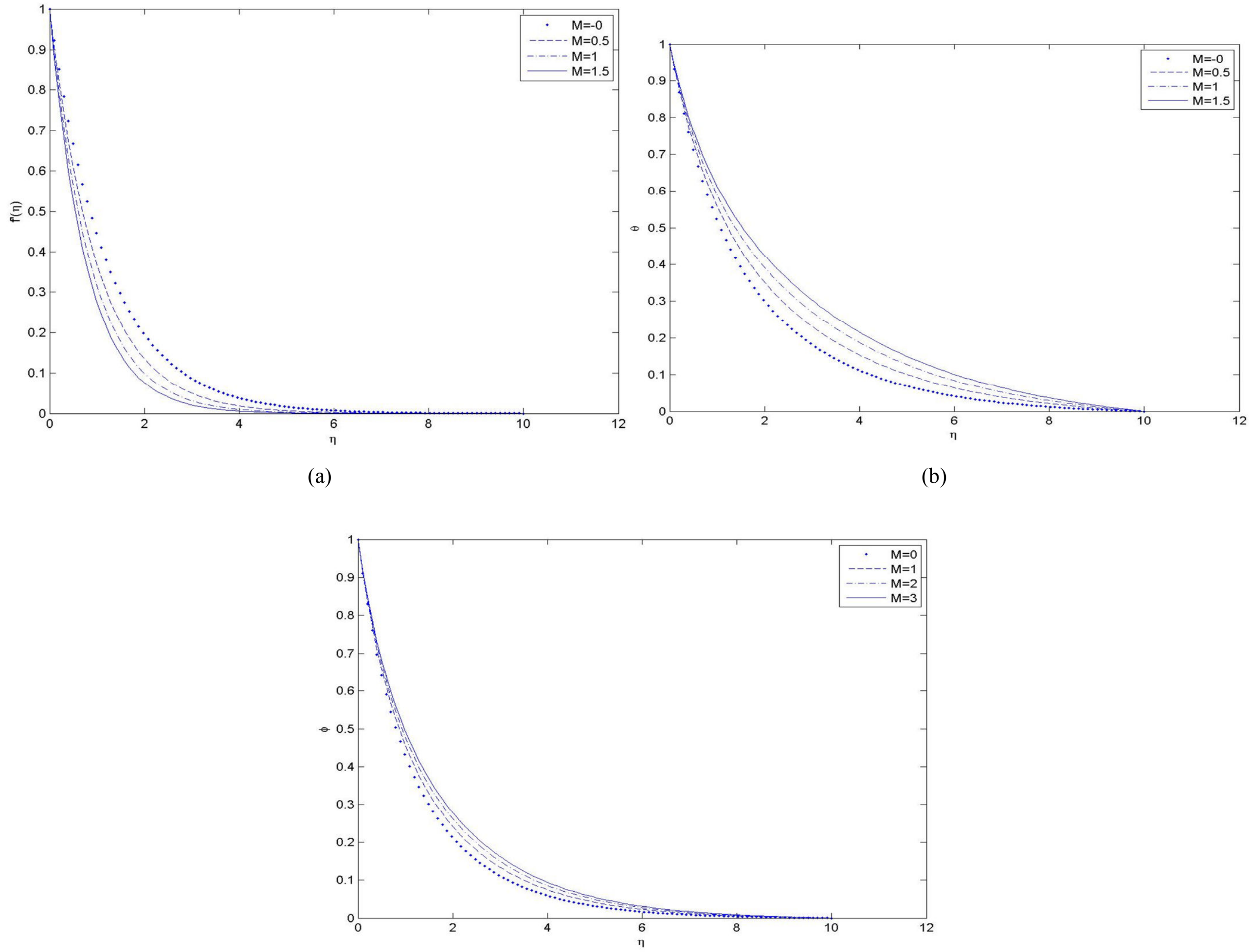


(c)

Fig. 2 (a) Variations of the velocity profile $f'(\eta)$ with η for several values of power law factor n and m, (b) Variations of the temperature profile θ (η) with η for several values of power law factor n and m (c) Variations of concentration profile φ(η) with η for several values of power law factor n and m



(a) (b)

(c)

Fig 3 (a) Variations velocity profile $f'(\eta)$, with η for several values of Magnetic parameter M, (b) Variation of temperature profile θ (η) with η for several values of Magnetic parameter M, (c) Variation of concentration profile $\varphi(\eta)$, with η for several values of Magnetic parameter M

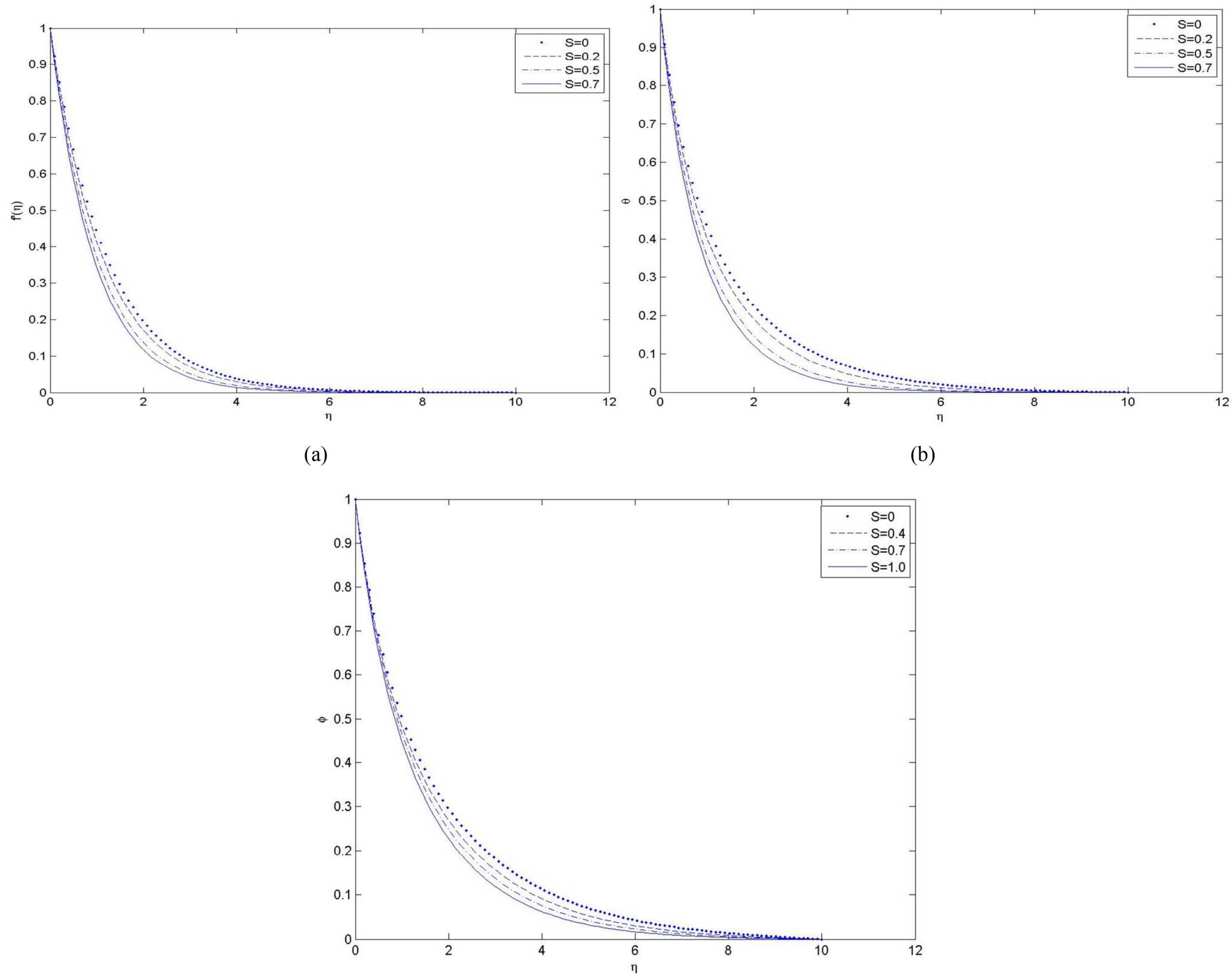
Figs. 4 (a), (b), and (c) give variation of velocity, temperature and concentration profiles inside the boundary layer with varying suction parameter. In case of velocity increases with increasing the suction parameter increases, velocity decreases. Similarly, the temperature and concentration also decrease as the suction parameter S increases.

Fig. 5 shows the change in temperature within the boundary layer for various value radiation parameter R. It is found that temperature increases with increasing the thermal radiation parameter R when other parameters are constants. Fig. 6 displays that the concentration decreases with increasing the Schmidt number at the fixed value of η when other physical parameters are taken as constants.

IV. CONCLUSIONS

The problem of MHD boundary layer flow of Casson fluid over nonlinearly stretching sheet in the presence of variable magnetic field with thermal radiation and suction effects are investigated. The numerical solutions are found with excellent agreements and the results are analysed for various physical parameters. From the present work, the following conclusions are drawn.

(i) The velocity at a fixed point increases with increase in the non-linear stretching parameter.
(ii) The temperature and concentration profile decrease in increasing the non-linear stretching parameter.
(iii) The temperature within the boundary layer increases with increasing the thermal radiation parameter R.
(iv) The concentration at point decreases with increasing the Schmidt number.



(a) (b)

(c)

Fig. 4 (a) Variations velocity profile $f'(\eta)$, with η for several values of suction parameter S, (b) Variation of temperature profile $\theta\ (\eta)$ with η for several values of suction parameter S, (c) Variation of concentration profile $\varphi(\eta\ )$, with η for several values of suction parameter S

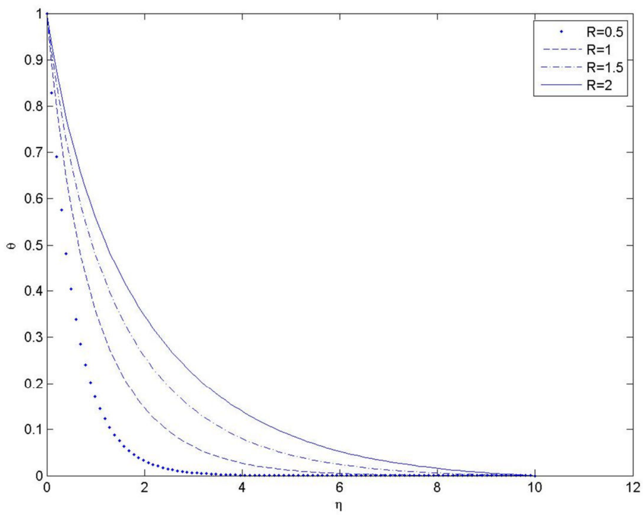


Fig. 5 Variations of temperature $\theta\ (\eta\ )\ with\ \eta$ for several values of radiation parameter R

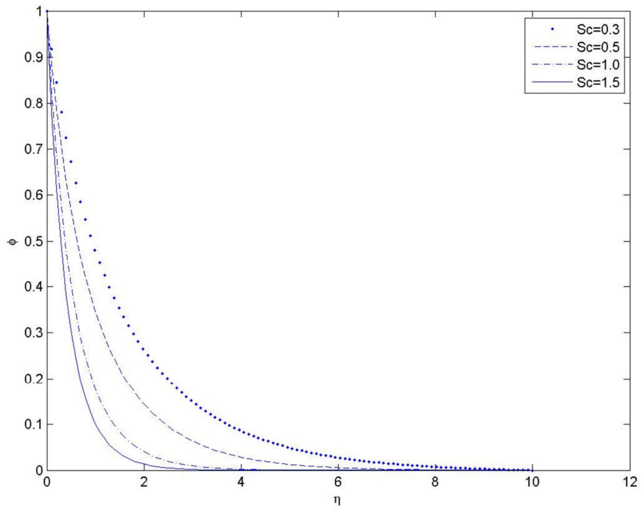


Fig. 6 Variations of $\varphi(\eta\ )\ with\ \eta$ for several values of Schmidt number

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