

# PSO Based Weight Selection and Fixed Structure Robust Loop Shaping Control for Pneumatic Servo System with 2DOF Controller

Randeep Kaur, Jyoti Ohri

**Abstract**—This paper proposes a new technique to design a fixed-structure robust loop shaping controller for the pneumatic servosystem. In this paper, a new method based on a particle swarm optimization (PSO) algorithm for tuning the weighting function parameters to design an  $H_\infty$  controller is presented. The PSO algorithm is used to minimize the infinity norm of the transfer function of the nominal closed loop system to obtain the optimal parameters of the weighting functions. The optimal stability margin is used as an objective in PSO for selecting the optimal weighting parameters; it is shown that the proposed method can simplify the design procedure of  $H_\infty$  control to obtain optimal robust controller for pneumatic servosystem. In addition, the order of the proposed controller is much lower than that of the conventional robust loop shaping controller, making it easy to implement in practical works. Also two-degree-of-freedom (2DOF) control design procedure is proposed to improve tracking performance in the face of noise and disturbance. Result of simulations demonstrates the advantages of the proposed controller in terms of simple structure and robustness against plant perturbations and disturbances.

**Keywords**—Robust control, Pneumatic Servosystem, PSO,  $H_\infty$  control, 2DOF.

## I. INTRODUCTION

IN recent times pneumatic servosystems have got wide acceptance in industries and other special systems. Current researches focus on improving the disturbance rejection properties of pneumatic servosystems to work well in industrial environment. The pneumatic actuator is an attractive choice in industrial and non-industrial applications over conventional electrical and hydraulic actuators due to its reliability, low cost, light weight, self-cooling, high power-to-weight ratio, etc [1]. So far many controllers have been developed to control the system, of which the  $H_\infty$  controller is found to guarantee robustness and performance. But its inherent highly nonlinear dynamic system and the effects of time delay are significant, so the development of a good performance control technique for this system is difficult. Robust  $H_\infty$  control can provide a perfect control to linear systems and high robustness to stabilize in adverse operating conditions like parameter change, high disturbance environment, actuator saturation and model uncertainty [2], [3].

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But in implementation point of view, H Infinity controllers are of very high order and approaches have been proposed to control this system to achieve good performance and robustness. One among them is robust control through which the controlled system can perform well even under the conditions of disturbance and uncertainties. In the control design problem, several linear mathematical equations need to be solved to find the optimal robust controller. Unfortunately, the resulting controller from the conventional techniques is usually complicated with high order. In practical work, the model reduction methods such as Hankel Norm model reduction technique, Balance Trunc Realization, etc. have been used for reducing the controller order. However, in many cases, the stability margin obtained from the reduced order controller is not satisfied. Moreover, the structure of controller is not selectable; in practical control. To overcome this problem, this paper applies the technique called structure specified robust controller to design a robust PID controller which gains both high stability margin and performance. The simple structure controllers PI or PID are today's most commonly used controllers in servosystems. To reduce the gap between the theoretical and practical approaches mentioned above, the proposed technique adopts the particle swarm optimization technique for solving the robust stabilization control problem with specified controller structure [4]-[6].

Now a day's, H infinity loop shaping is gaining very high acceptance since the performance requirements can be incorporated in the design stage as performance weight. In the H infinity loop shaping technique, a linear plant model is augmented with certain weight functions so that the closed loop transfer function of the plant will have the desired performances. From the literature [4]-[9] it has been reviewed that there exists no specific criterions for the selection of these weights and most of the time they are system specific. It requires high analytical skills for a control engineer to design these weights which makes H infinity control to be inferior to other control strategies. In this paper an automatic weight selection algorithm using PSO is proposed to design robust  $H_\infty$  controller automatically for pneumatic servosystem.

A two-degree-of-freedom (2DOF) control configuration may be used in order to achieve a control system with both a performance specification, e.g. through a reference model and some guaranteed stability margins. The approaches found in the literature are mainly based on optimization problems. The approach presented in [10] expands the role of  $H_\infty$  optimization tools in 2DOF system design. The one-degree-of-

freedom (1DOF) loop shaping design procedure [11] is extended to a 2DOF control configuration by means of a parameterization. A feedback controller is designed to meet robust performance requirements in a manner similar as in 1DOF loop shaping design procedure and a pre-filter controller is added to the overall compensated system to force the response of the closed loop to follow that of a specified reference model. The approach is carried out by assuming uncertainty in the normalized coprime factors of the plant [12]. Such uncertainty description allows a formulation of the  $H_\infty$  robust stabilization problem providing explicit formulae. The use of a 2DOF controller must allow the control-loop designer to take into consideration the regulatory control performance and control effort requirements in conjunction with the control system robustness and then improve the servo-control performance. In this paper, a new tuning procedure for structure specified PID Controller in 1DOF structure tuning the parameters with PSO is proposed [13], [14]. In addition, a new tuning procedure for Polynomial Controller in 2DOF structure is also proposed. The aim is to have good set point tracking and disturbance rejection and also maximum robustness to model uncertainties.

The paper is organized as mentioned below. The pneumatic servosystem is described in Section II. Section III describes conventional  $H_\infty$  loop-shaping control. Section IV describes the proposed automatic weight selection by PSO and particle swarm optimization based fixed-structure  $H_\infty$  loop shaping control followed by its design. In Section V the concept of 2DOF approach is specified. Design examples are shown in Section VI followed by its analysis in Section VII. Finally, the conclusion is in Section VIII.

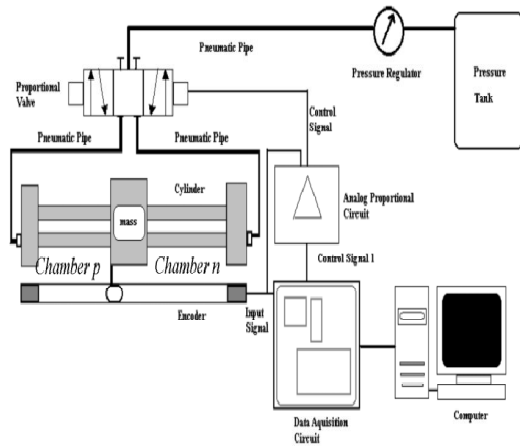


Fig. 1 Pneumatic Servosystem [1]

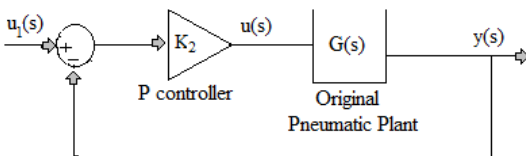


Fig. 2 (a) Modified Plant Model

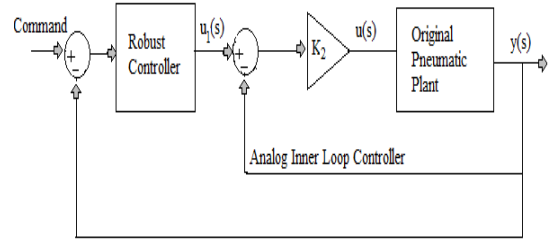


Fig. 2 (b) Controllers

II. DYNAMIC MODEL

Pneumatic Dynamic Model and Modified Plant Model:

The dynamic model of a pneumatic system is difficult to determine due to its nonlinearity and large variation in plant parameters such as actuator saturation, non linearity in sensors etc. More over the nominal model of the plant will be highly perturbed with disturbances and model uncertainties. General linearization is applied at an operating point to obtain the linear dynamic model. The following mathematical model represents a pneumatic plant [1]:

$$\frac{y(s)}{u(s)} = \frac{k_1}{s(s^2 + \frac{C}{M}s + k_2)} \tag{1}$$

where

$$k_1 = \frac{\gamma RT_s G_i}{M} \left( \frac{S_p}{V_{po}} + \frac{S_n}{V_{no}} \right) \tag{2}$$

$$k_2 = \frac{\gamma}{M} \left( \frac{S_p^2 P_{po}}{V_{po}} + \frac{S_n^2 P_{no}}{V_{no}} \right) \tag{3}$$

where  $y(s)$  is position output,  $u(s)$  is the input valve voltage,  $\gamma$  is the ratio of specific heat,  $S_p$  and  $S_n$  are areas of piston of chamber  $p$  and  $n$ ,  $C$  is the viscous friction coefficient,  $M$  is piston mass,  $T_s$  is temperature,  $R$  is ideal gas constant, and  $V_p$  and  $V_n$  are the volume of chambers  $p$  and  $n$ .  $P_p$  and  $P_n$  are pressure in chambers 1 and 2.  $o$  is a subscript denoting the operation point and  $G_i$  is the coefficient of the linearized air mass flow rate. Fig. 1 shows the setup of the pneumatic system. Equation (1) shows the model. For simplicity, a modified plant model introducing an analog proportional controller is approximated as a stable plant with time delay (Fig. 2 (a)). Fig. 2 (b) shows the inner loop, outer loop, and controller.  $G(s)$  is the pneumatic plant. The following equations are derived to obtain the model of the modified plant, whose dynamic model is, from Fig. 2 (a):

$$y(s) = \frac{\frac{k_1 K_2}{s(s^2 + (\frac{C}{M})s + k_2)}}{1 + \frac{k_1 K_2}{s(s^2 + (\frac{C}{M})s + k_2)}} u_1(s) \tag{4}$$

$$\frac{y(s)}{u_1(s)} = \frac{k_1 K_2}{(s^2 + (\frac{C}{M})s^2 + k_2 s + k_1 K_2)} \tag{5}$$

$$= \frac{k_1 K_2}{(s+T_1)(s+T_2)(s+T_3)} \tag{6}$$

where  $u_1(s)$  is a new defined input, the dynamic model in (4) is approximated as a lower order model with time delay [1].

Here, we approximate the modified plant model as a second order with time delay, which is more correct than the first-order model. The following equation shows approximation of the modified plant model:

$$\frac{y(s)}{u_1(s)} \approx \frac{Ae^{-\theta_2 s}}{(s^2 + b_1 s + c_1)} \quad (7)$$

where  $\theta_2$  is delay time.  $A$ ,  $b_1$  and  $c_1$  are unknown parameters that must be identified. The identified plant model [1] is as

$$G_p(s) \approx \frac{551.3e^{-0.12s}}{(s^2 + 43.26s + 536.9)} \quad (8)$$

### III. CONVENTIONAL $H_\infty$ LOOP-SHAPING CONTROL

$H_\infty$  loop-shaping control, proposed by McFarlane and Glover [11], is an efficient way to design a robust controller and has been applied to a variety of control problems. Uncertainties in this approach are modeled as coprime factor uncertainty. This uncertainty model does not represent actual physical uncertainty, which, in fact, is unknown. This approach requires only a desired open loop shape in the frequency domain. Two weighting functions,  $W_1$  (pre-compensator) and  $W_2$  (post-compensator), are specified to shape original plant  $G$  so that the desired open loop shape is achieved. In this approach, the shaped plant is formulated as a normalized coprime factor that separates plant  $G_s$  into normalized nominator  $N_s$  and denominator  $M_s$  factors. In any plant model  $G$ , the shaped plant  $G_s$  is formulated as

$$G_s = W_2 G W_1 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (9)$$

$$G_s = (N_s + \Delta_{N_s})(M_s + \Delta_{M_s})^{-1} \quad (10)$$

where  $A, B, C, D$  represent plant  $G_s$  in the state-space form,  $\|\Delta_{N_s}, \Delta_{M_s}\|_\infty \leq \varepsilon$ ,  $N_s$  and  $M_s$  are nominator and denominator normalized coprime factors.  $\Delta_{N_s}$  and  $\Delta_{M_s}$  are uncertainty transfer functions in nominator and denominator factors.  $\varepsilon$  is an uncertainty boundary, called a stability margin. To obtain these normalized coprime factors, the following equation is applied:

$$[N_s \ M_s] = \begin{bmatrix} A + HC & B + HD & H \\ R^{-1/2}C & R^{-1/2}D & R^{-1/2} \end{bmatrix} \quad (11)$$

where  $H = -(BD^T + ZC^T)R^{-1}$ ,  $R = I + DD^T$  and matrix  $Z \geq 0$  is the unique positive definite solution to the algebraic Riccati equation:

$$(A - BS^{-1}D^T C)Z + Z(A - BS^{-1}D^T C)^T - ZC^T R^{-1} CZ + BS^{-1} B^T = 0 \quad (12)$$

where  $S = I + D^T D$ .

Once the desired loop shape is achieved, the  $\infty$ -norm of the transfer function from disturbances  $w$  to states  $z$  is subjected to be minimized over the stabilizing controllers  $K$ . Fig. 3 shows the block diagram of  $H_\infty$  loop shaping control.

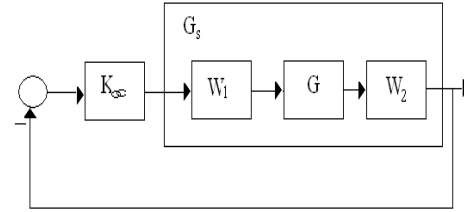


Fig. 3 Block diagram of  $H_\infty$  loop shaping control.

Based on standard  $H_\infty$  loop-shaping, the following steps are proposed for an SISO plant [2]:

1. Shape singular values of nominal plant  $G$  by using pre-compensator  $W_1$  and/or post-compensator  $W_2$  to get the desired loop shape.  $W_1$  and  $W_2$  should be chosen so that  $G_s$  contain no hidden modes.  $W_1$  is used to meet tracking performance and disturbance attenuation and  $W_2$  to attenuate sensor noise. Practically, we select  $W_1$  as an integral action weighting function, which make a zero steady state error.  $W_2$  can be chosen as an identity matrix because we can neglect sensor noise effect when a good sensor is used.

$$W_1 = K_w \frac{s+a}{s+b}, \quad W_2 = 1 \quad (13)$$

where  $K_w$ ,  $a$  and  $b$  are positive numbers.  $b$  is selected as a small number ( $\ll 1$ ) for integral action.

2. Minimize the  $\infty$ -norm of transfer matrix  $T_{zw}$  over all stabilizing controllers  $K$  to obtain optimal cost  $\gamma_{opt}$ , as

$$\gamma_{opt} = \varepsilon_{opt}^{-1} = \inf \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + G_s K)^{-1} M_s^{-1} \right\|_\infty \quad (14)$$

The resulting  $\varepsilon_{opt}$  is a measure of robustness of the desired loop shape. It also indicates compatibility of weighting functions with robust control of the plant.  $\varepsilon_{opt} < 0.25$  (or  $\gamma_{opt} > 4$ ) indicate that  $W_1$  or  $W_2$  designed in step 1 is incompatible with robust stability. We must return to step (1) and readjust  $W_1$  or  $W_2$ .  $\varepsilon_{opt}$  is determined using the unique method

$$\gamma_{opt} = \varepsilon_{opt}^{-1} = (1 + \lambda_{\max}(XZ))^{1/2} \quad (15)$$

where  $X$  and  $Z$  are the solutions of Riccati equations (12) and (16), and  $\lambda_{\max}$  is the maximum eigenvalue.

$$(A - BS^{-1}D^T C)^T X + X(A - BS^{-1}D^T C) - XBS^{-1}B^T X + C^T R^{-1} C = 0 \quad (16)$$

3. Select  $\varepsilon < \varepsilon_{opt}$ , then synthesize controller  $K_\infty$  that satisfies

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + G_s K_\infty)^{-1} M_s^{-1} \right\|_\infty = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + G_s K_\infty)^{-1} \begin{bmatrix} I & G_s \end{bmatrix} \right\|_\infty \leq \varepsilon^{-1} \quad (17)$$

Controller  $K_\infty$  is obtained by solving the optimal control problem in (17).

4. Final controller ( $K$ ) follows

$$K = W_1 K_\infty W_2 \quad (18)$$

#### IV. PARTICLE SWARM OPTIMIZATION BASED FIXED-STRUCTURE $H_\infty$ LOOP SHAPING

PSO is a well known algorithm that can be applied to any optimization problem [15]. This algorithm applies the concept of particles fly around the problem space until the stopping criteria are met. In the proposed technique, PSO is adopted in both weight selection and control synthesis. The PSO technique can generate a high quality solution within shorter calculation time and stable convergence characteristic than other stochastic methods. PSO is a population based search process where individuals, referred to as particles, are grouped into a swarm. Each particle in swarm represents a candidate solution to the optimization problem.

##### A. Weight Selection:

Weight selection is an important procedure for  $H_\infty$  loop shaping. Some researchers incorporated the performance specifications for selecting the appropriated weight. The selection purely depends on the plant model. There are no hard and fast rules for selecting the performance and the robustness weighting functions. It is very difficult to simultaneously achieve all the requirements for the synthesis of robust controller. Even though there are no methods available for selecting the transfer function for weight functions, certain generalization can be done by understanding the loop shaping procedure [2]. The main draw back in this is that there are so many parameters to be fixed for determining the weight functions. Based on (14),  $\varepsilon_{opt}$  can be used for indicating the compatibility of the selected weight with the robust stability requirement. However, in some cases, time domain response of closed loop system at nominal plant is not satisfied although  $\varepsilon_{opt}$  is satisfied. In this paper, we specify the performance specifications and then evaluate the optimal weight  $W_1$  by using PSO. The fitness function for the weight selection is given as

$$\text{Fitness} = \varepsilon_{opt} \text{ if the performance specifications are satisfied} \\ = 0.01 \text{ (or a small value) otherwise.}$$

##### B. Controller Synthesis

The proposed technique fixes the structure of the controller ( $K(p)$ ) and then the PSO is adopted to find the optimal parameter,  $p$ , to achieve the maximum stability margin. In the proposed technique, the stability margin ( $\varepsilon$ ) is a single index to indicate the robust performance of the designed controller.  $K_\infty$  can be found by  $K_\infty = W_1^{-1} K(p) W_2^{-1}$ . Suppose that  $W_1$  and  $W_2$  are invertible. Generally  $W_2$  is chosen as identity matrix I. Therefore, the objective can be written in the form:

$$\text{Objective function} = \varepsilon = \|T_{zw}\|_\infty^{-1} \quad (19)$$

$$= \left\| \left[ \begin{array}{c} I \\ W_1^{-1} K(p) \end{array} \right] (I - G_s W_1^{-1} K(p))^{-1} \left[ \begin{array}{c} I \\ G_s \end{array} \right] \right\|_\infty^{-1} \quad (20)$$

For the design, [6] the controller  $K(p)$  will be designed to minimize the infinity norm from disturbance to state or maximize the stability margin by the PSO method. The PSO is based on the concept of swarm's movement. A bird represents a particle and the position of each particle represents the candidate solution. When applying the PSO; PSO parameters, i.e. the population of swam ( $n$ ), lower and upper boundary ( $p_{min}$  ,  $p_{max}$ ) of the problem, minimum and maximum velocity of particles ( $v_{min}$  ,  $v_{max}$ ), minimum and maximum iteration ( $i_{max}$ ), minimum and maximum inertia weight, need to be specified. In an iteration of the PSO, the value of fitness or objective function of each particle is evaluated. The particle which gives the highest fitness value is kept as the answer of current iteration. The inertia weight ( $Q$ ), value of velocity ( $v$ ) and position ( $p$ ) of each particle in the current iteration ( $i$ ) are updated by using (21), (22) and (23), respectively.

$$Q = Q_{max} - \left( \frac{Q_{max} - Q_{min}}{i_{max}} \right) i \quad (21)$$

$$v_{i+1} = Qv_i + \alpha_1 [\gamma_{1i} (P_b - p_i)] + \alpha_2 [\gamma_{2i} (U_b - p_i)] \quad (22)$$

$$p_{i+1} = p_i + v_{i+1} \quad (23)$$

where  $\alpha_1, \alpha_2$  are acceleration coefficients,

$\gamma_{1i}, \gamma_{2i}$  are any random numbers in (0-1) range.

Based on the PSO technique, a set of controller parameters  $p$  is formulated as a particle and the fitness can be written as:

$$\text{Fitness function} = \left\| \left[ \begin{array}{c} I \\ W_1^{-1} K(p) \end{array} \right] (I - G_s W_1^{-1} K(p))^{-1} \left[ \begin{array}{c} I \\ G_s \end{array} \right] \right\|_\infty^{-1} \quad (24)$$

Fitness value is specified as a very small value if the controlled system is unstable.

##### C. Steps of Weight Selection

- 1) Select a weight structure  $W_1$ , normally done by using (14). Specify the parameters of PSO such as population size of swarm, lower and upper bound values of problem space, minimum and maximum velocity of particles, minimum and maximum inertia weights, maximum iteration and acceleration coefficients.
- 2) Initialize several sets of weight parameters as particles in the 1st generation. By using (14) as the weight structure, the weight parameters are  $x_1, x_2, x_3$  and  $x_4$ . The weight parameters are particles in this problem.
- 3) Evaluate the fitness function ( $f_s$ ) of each particle; find the best position found by particle  $i$ , call it as  $P_b$ , and find the best position found by swarm, call it as  $U_b$ .
- 4) Update the inertia weight ( $Q$ ).
- 5) Increment the iteration for a step ( $i = i + 1$ ). If the current iteration is the maximum iteration  $i = i_{max}$ , stop. If not, go to Step 3.
- 6) Check the optimal fitness value ( $\varepsilon_{opt}$ ). If  $\varepsilon_{opt} < 0.25$ , then back to step 1 to change the weight structure and/or adjust the performance specifications if possible.

#### D. Steps of Controller Synthesis:

- 1) Select controller structure  $K(p)$  and initialize several sets of parameters  $p$  as population in the 1<sup>st</sup> generation. Specify the parameters of PSO such as population size of swarm, lower and upper bound values of problem space, minimum and maximum velocity of particles, minimum and maximum inertia weights, maximum iteration and acceleration coefficients.
- 2) Evaluate the fitness value of each particle by using (24). Find the best position found by particle  $i$ , call it as  $P_b$ , and find the best position found by swarm, call it as  $U_b$ .
- 3) Update the inertia weight ( $Q$ ).
- 4) Increment the iteration for a step ( $i = i+1$ ). If the current iteration is the maximum iteration  $i = i_{max}$ , stop. If not, go to Step 2.
- 5) Check performances in both frequency and time domains. If the performance is not satisfied such as too low  $\epsilon$  (too low fitness function), then go to step 1 to change the structure of controller. Low  $\epsilon$  indicates that the selected control structure is not suitable for the problem.

#### V. 2DOF CONTROLLER

Structure of controller is a challenging problem in control theory. In a control system, the degree of freedom is defined as the number of closed-loop transfer functions that can be adjusted independently. In 1DOF structure, if the disturbance rejection is desired, the set-point response is often found to be poor, and vice versa. So in some researches on the optimal tuning of PID controllers, two tables to tune controller is given, one for the 'optimal disturbance rejection', and the other one for the 'optimal set point response'. The 2DOF controller handles such a problem, that is, in this structure both set point tracking and disturbance rejection optimization is possible. A great number of tuning methods are presented in new researches in the structure of 2DOF.

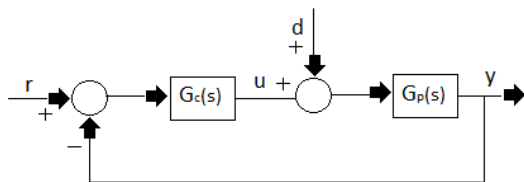


Fig. 4 Block Diagram of 1DOF controller

It is well known that 2DOF control design which combines the feedforward control and feedback control to achieve the desired tracking performance has been widely applied in trajectory tracking control system. Several robust 2DOF control design approaches take account into  $H_\infty$  performance specifications in worst system uncertainties and solve the  $H_\infty$  optimization problem to improve the tracking performance in the face of noise, disturbances, modelling uncertainty and robustness. These can be due to nonlinearities, unmodeled dynamics, and changes in plant parameters. This is accomplished by reducing the loop bandwidth for better noise attenuation and disturbance rejection and increasing the

transmission bandwidth, using simultaneous synthesis of the pre-filter, for improved tracking performance. The 2DOF controllers present the advantage of a complete separation between feedback and reference tracking properties: the feedback properties of the controlled system are assured by a feedback controller, i.e. the first degree of freedom; the reference tracking specifications are addressed by a prefilter controller, i.e. the second degree of freedom, which determines the open-loop processing of the reference commands. Generally, good set point response and disturbance rejection is the primary objective [16].

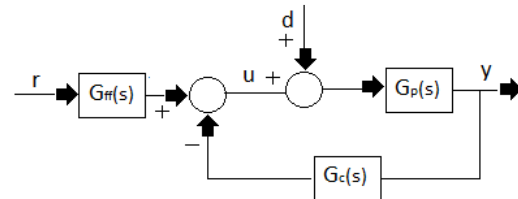


Fig. 5 Block Diagram of 2DOF controller

#### VI. SIMULATION RESULTS

In order to validate the selection procedure of weighting functions, pneumatic servosystem is taken and the structure of robust controllers was designed. To evaluate the performance and robustness of the proposed system, responses of the system from conventional  $H_\infty$  loop shaping; proposed robust 1DOF-Fixed Structure PID Controller and 2DOF-Fixed Structure Polynomial Controller are investigated. The selection procedure and the algorithm are coded in MATLAB and executed. The program written in MATLAB [17] returned the following controllers meeting the criteria. The transfer function of the plant and controllers obtained are given as following:

$$\text{Plant Transfer function (G): } \frac{5.684e-014s^2-551.3s+9188}{s^3+59.93s^2+1258s+8948} \quad (25)$$

Initialization parameters used for PSO are: population size =25, maximum no. of iterations = 2500, cognitive acceleration = 2.05, social acceleration = 2.05, minimum and maximum inertia weights are 0.6 and 0.9. The PSO algorithm aims to find the optimal value of  $[K_w, a]$  based on (13), the weight parameters range is selected as  $K_w$  as  $[0.8, 2]$ ,  $a$  as  $[1, 10]$ ,  $b = 0.001$ . The PSO in 17<sup>th</sup> iteration converges with the optimal solution  $X = [0.8000 \ 1.0000]$  which on substitution gives weighting function as

$$W_1 = \frac{0.8s+0.8}{s+0.001}, \quad W_2 = 1 \quad (26)$$

In the proposed technique, PSO is used to evaluate the weight  $W_1$ . By using PSO, the optimal stability ( $\epsilon_{opt}$ ) is founded to be 0.7395. This value indicates that the selected weights are compatible with robust stability requirement. With these weighting functions, the shaped plant is then determined as

$$G_s = W_1 G W_2 \tag{27}$$

We first design a controller by the conventional  $H_\infty$  loop shaping procedure. Then, the  $H_\infty$  loop shaping controller can be evaluated as following:

$$\text{Controller Transfer function (K): } \frac{0.7281s^4 + 40.34s^3 + 705.8s^2 + 4137s + 3471}{s^4 + 73.72s^3 + 3038s^2 + 3961s + 3.958} \tag{28}$$

The controller designed by  $H_\infty$  loop shaping controller is fourth order controller and complicated. It is not easy to implement practically.

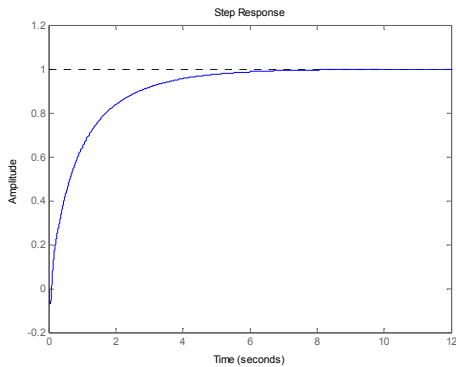


Fig. 6 Step response of  $H_\infty$  loop shaping controller

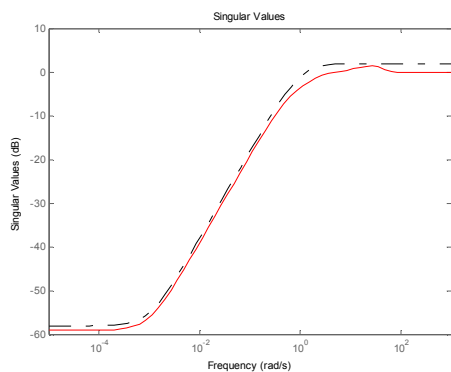


Fig. 7 Singular value plot of  $H_\infty$  loop shaping controller

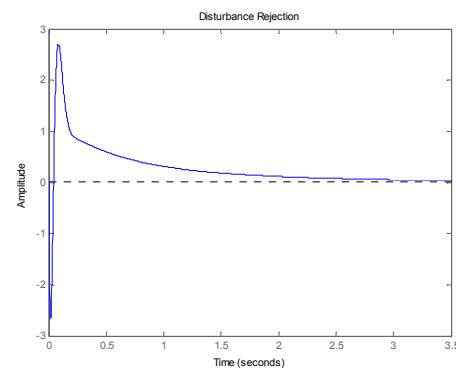


Fig. 8 Disturbance Rejection by  $H_\infty$  loop shaping controller

Next, a fixed-structure robust controller using the proposed algorithms is designed. The structure of controller is selected as PID with first-order derivative filter. The controller structure is expressed as:

$$K(p) = \left( K_p + \frac{K_i}{s} + \frac{K_d s}{\tau_d s + 1} \right) \tag{29}$$

$K_p, K_i, K_d,$  and  $\tau_d$  are the parameters to be evaluated. In the optimization, the range of search parameters are set as follows:  $K_p$  as [5, 20],  $K_i$  as [8, 10],  $K_d$  as [0.2, 1] and  $\tau_d$  as [0.001, 0.001]. Initialization parameters used for PSO are: population size = 25, maximum no. of iterations = 2500, acceleration coefficients = 2.05, minimum and maximum inertia weights are 0.6 and 0.9. By using PSO, the optimal stability ( $\epsilon_{opt}$ ) is founded to be 0.7021. This value indicates that the selected parameters are compatible with robust stability requirement. As a result the optimal controller found to be

$$\text{Controller Transfer function K(p): } \frac{0.003135s^2 + 0.1919s + 0.9487}{0.008721s^2 + s} \tag{30}$$

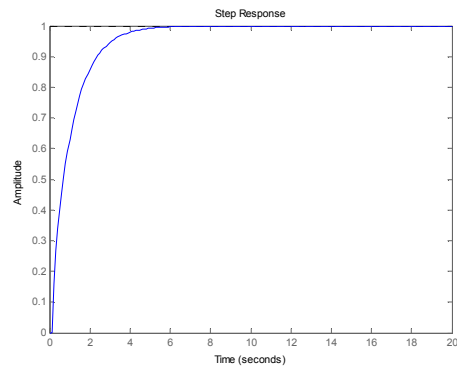


Fig. 9 Step response of 1DOF-Fixed Structure PID controller

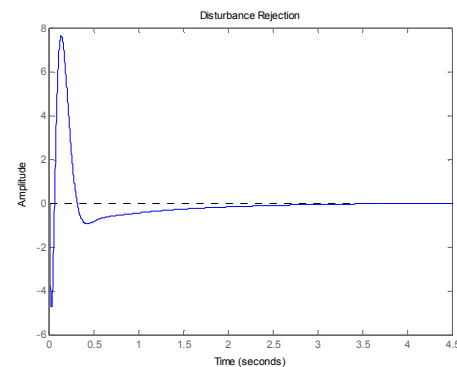


Fig. 10 Disturbance Rejection by 1DOF-Fixed Structure PID controller

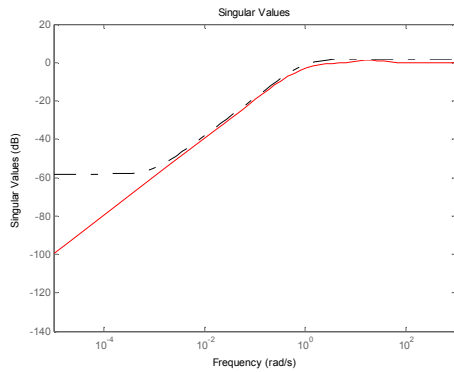


Fig. 11 Singular value plot of 1DOF-Fixed Structure PID controller

The 1DOF fixed structure PID design procedure is extended to a 2DOF control configuration by means of a parameterization. A feedback controller is designed to meet robust performance requirements in a manner similar as in 1DOF loop shaping design procedure and a pre-filter controller is then added to the overall compensated system to force the response of the closed loop to follow that of a specified reference model. In this case the specified reference model is expressed as:

$$T_{ref} = \frac{49}{s^2 + 14s + 49} \quad (31)$$

Now, in 2DOF configuration a fixed-structure robust controller using the proposed algorithms is designed. The structure of controller is selected as Polynomial Controller. The controller structure is expressed as:

$$K(s) = \left( \frac{as+b}{s^2+cs+d} \right) \quad (32)$$

$a, b, c$  and  $d$  are the parameters to be evaluated. In the optimization, the range of search parameters are set as follows:  $a$  as [1.5, 12],  $b$  as [0.01, 15],  $c$  as [1.5, 12] and  $d$  as [0.01, 15]. Initialization parameters used for PSO are: population size = 25, maximum no. of iteration = 2500, acceleration coefficients = 2.05, minimum and maximum inertia weights are 0.6 and 0.9. As a result the optimal controller found to be

$$\text{Controller Transfer function } K(s): \frac{11.84s+12.88}{s^2+3.603s+12.92} \quad (33)$$

### VII. ANALYSIS OF THE $H_\infty$ CONTROL AND THE PROPOSED CONTROL

Pneumatic servosystems are non linear systems, so the  $H_\infty$  infinity controller and the proposed controllers developed using a linear model should stabilize the actual plant under all operating conditions. The performance of pneumatic servosystems under the control analysis is split up into stability analysis, performance analysis and robustness measures.

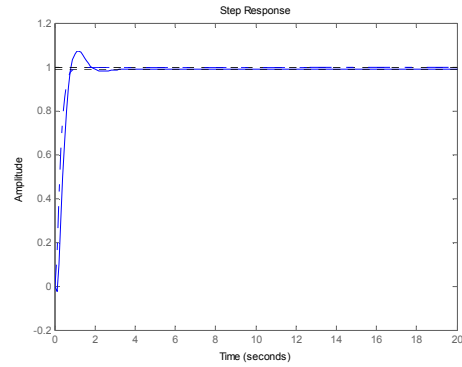


Fig. 12 Step response of 2DOF-Fixed Structure Polynomial controller tracking reference command

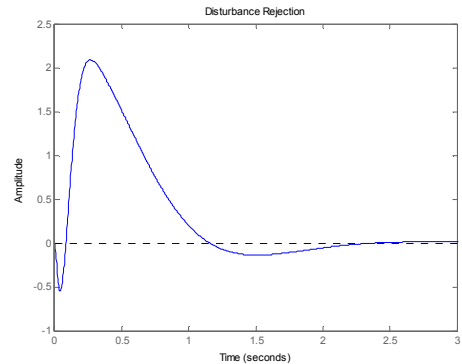


Fig. 13 Disturbance Rejection by 2DOF-Fixed Structure Polynomial controller

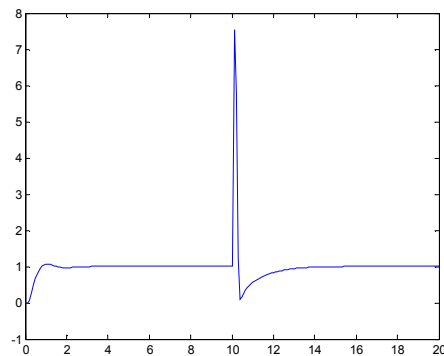


Fig. 14 Overall response of 2DOF-Fixed Structure Polynomial controller when the unit step and disturbance at 10s is entered to the plant

#### A. Stability Analysis

Stability analysis is done to ensure the stability of operation of pneumatic servosystems under various conditions. The bode plot of the sensitivity function of robust controlled pneumatic servosystem for different controllers is shown in Figs. 7 and 11. The plots show that the system is stable and Table I shows the characteristics of different controllers.

#### B. Performance Analysis

Responses from the unit step inputs by  $H_\infty$  loop shaping and the proposed robust PID and proposed robust polynomial

controller are shown in Figs. 6 and 9. Table II shows the performance obtained by the controllers. The settling time of 2DOF controller is faster than 1DOF controller. In addition, we designed a fixed-structure prefilter to achieve the tracking performance specification. Response from 2DOF controller has a maximum overshoot of 5.37% while there is no overshoot from the 1DOF-PID and  $H_\infty$  controller. Moreover, 5% settling time of the proposed controller is much faster than that of the conventional  $H_\infty$  Loop Shaping controller. The rise time of 2DOF controller is 0.48s much faster than 1DOF-PID and  $H_\infty$  controller. In case of disturbance rejection 2DOF controller is the best. Clearly, time domain specifications can be achieved by the proposed technique i.e 1DOF-Fixed structure PID controller and 2DOF-Fixed structure polynomial controller.

TABLE I  
COMPARISON OF THE CHARACTERISTICS OF CONTROLLERS

Characteristics /Controllers	$H_\infty$ -Loop shaping	1DOF-Fixed structure PID	2DOF-Fixed structure Polynomial
Phase Margin	108.0457	102.4302	102.6172
PM Frequency	1.0030	0.9352	0.0816
Delay Margin	1.8802	1.9115	0.1447
DM Frequency	1.0030	0.9352	0.0816
Stable	1	1	1

TABLE II  
COMPARISON RESULTS OF CONTROLLERS-STEP RESPONSES

Step response /Controllers	$H_\infty$ -Loop shaping	1DOF-Fixed structure PID	2DOF-Fixed structure Polynomial
Steady State	0.999	1	1
Rise Time(s)	2.59	2.2	0.487
Settling Time(s)	5.1	4.03	1.74
Peak Amplitude / At Time (s)	0.99/12	1/20	1.05/1.1

TABLE III  
COMPARISON RESULTS OF CONTROLLERS-DISTURBANCE REJECTION

Disturbance Rejection/ Controllers	$H_\infty$ -Loop shaping	1DOF-Fixed structure PID	2DOF-Fixed structure Polynomial
Settling Time(s)	3	2.1	2.07
Peak Amplitude / At Time (s)	2.7/0.08	7.65/0.131	2.09/0.268

## VIII. CONCLUSION

In this paper weight selection algorithm is proposed by using PSO. An appropriate performance weight that satisfies the time domain specifications and robustness is evaluated by PSO. The algorithm automatically synthesizes the weight functions meeting all the requirements of robust control for pneumatic servosystems. The proposed controller offers a significant improvement in control viewpoint by retaining the robust performance. Although there are many approaches for PID tuning; however, the proposed technique is an alternative method which directly considers the performance specifications and robustness in the design. In the proposed technique, the structure of controller is not restricted to PID.

The controller  $K(p)$  can be replaced by any fixed-structure controller and the proposed algorithm can still be applied functionally. A detailed analysis of the  $H_\infty$  controller and proposed controller developed for stability, performance and robustness measures has been made for pneumatic servosystem. As shown in the simulation results, the conventional  $H_\infty$  loop shaping controller performs closer to the desired loop shape as well as the proposed controller. However, because of the complicated controller in the conventional design, the proposed approach offers a significant improvement in practical control viewpoint by simplifying the controller structure, reducing the controller order and still retaining the robust performance. The results show that the proposed controllers could stabilize the pneumatic servosystem with good performance and robustness.

Other control performances will be considered in further research with multiple objective functions. The scope of design is an SISO plant, but it can be extended to an MIMO plant.

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