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Pseudo-polynomial motion commands for vibration suppression of belt-driven rotary platforms

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Abstract—The motion planning technique described in this paper has been developed to eliminate or reduce the residual vibrations of belt-driven rotary platforms, while maintaining unchanged the motion time and the total angular displacement of the platform. The proposed approach is based on a suitable choice of the motion command given to the servomotor that drives the mechanical device; this command is defined by some numerical coefficients which determine the shape of the displacement, velocity and acceleration profiles. Using a numerical optimization technique, these coefficients can be changed without altering the continuity conditions imposed on the displacement and its time derivatives at the initial and final time instants. The proposed technique can be easily and quickly implemented on an actual device, since it requires only a simple modification of the motion command profile mapped in the memory of the electronic motion controller.

Keywords—Command shaping, Residual vibrations, Belt transmission, Servomechanism.

I. Introduction

The belt drives are widely used in industrial machines, because they allow to achieve optimal design solutions, in particular as regards low noise emissions during operation, high mechanical efficiency, low maintenance costs and high reliability. In cases where it is essential to eliminate slippage, the use of timing belts is recommended, in order to obtain a perfect synchronization between the shafts, even in presence of unexpected load changes.

Nevertheless, as a result of rapid accelerations and/or high loads applications, a belt drive may show serious dynamic problems, such as vibration of some elements of the mechanical transmission; this is essentially due to the elasticity of the material used for the construction of the transmission belt. This problem can be particularly evident on a multi-stage belt drive, where the elasticity of the individual branches can degrade the global dynamic performance of the system.

In cases where a vibration-free movement and high positioning accuracy are required, the transmission must be carefully designed; for this purpose, it is very useful to carry out computer simulations, using mathematical models able to reproduce, with good reliability, the dynamics of an actual belt transmission. Through the numerical simulation it is also possible to verify the effects of a given motion command on a transmission having a some degree of elasticity.

The purpose of this paper is to provide a contribution to the design of positioning systems driven by low stiffness belt transmissions, in particular as regards the reduction of mechanical vibrations; the approach here presented is based on the selection of an appropriate motion strategy, which is defined through an optimization procedure, in accordance with the design constraints (motion time, total displacement, speed and torque limits of the motor, etc.).

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The proposed method seems to be particularly advantageous, since it is easily implementable on an actual machine: in fact it does not require mechanical changes aimed to improve the belt transmission stiffness, but it acts on the motion command of the servomotor that drives the system. So a simple modification of the profile stored into the motion controller is sufficient to improve the dynamic performances of the machine. To implement this technique the following steps are necessary:

1) definition of a mathematical model that allows to simulate the actual behaviour of the servomechanism with good accuracy;
2) definition of a parametric motion command, whose shape (i.e. the displacement, velocity and acceleration profiles) can be modified by changing a set of numerical parameters (called *shape parameters*);
3) definition of a performance index, that allows a simple and practical evaluation of the vibratory effects;
4) use of a numerical algorithm which is able to determine the "optimal" motion profile, that is the profile which generates the best performance index.

II. Mathematical modeling

To perform the dynamic analysis of an elastic belt transmission, lumped parameter models with one or more degrees of freedom are frequently adopted.

In general, the deduction of the equations of motion of a belt transmission does not present particular difficulties and it can be carried out through dynamical equilibrium considerations or through the Lagrangian approach. The following hypotheses are usually assumed:

- each branch of the belt is modeled by a linear spring in parallel with a viscous damper; the stiffness k of each branch can be determined by the well-known relationship $k = EA/L$, where E, is the Young's modulus of the belt material, A is the cross sectional area of the branch and L its length; the damping coefficient c can be experimentally determined through free vibration tests;
- the mass of the belt is negligible, if compared to the pulleys mass;
- no slippage of the belt on pulleys is considered in the model;
- the belt has no flexural stiffness.

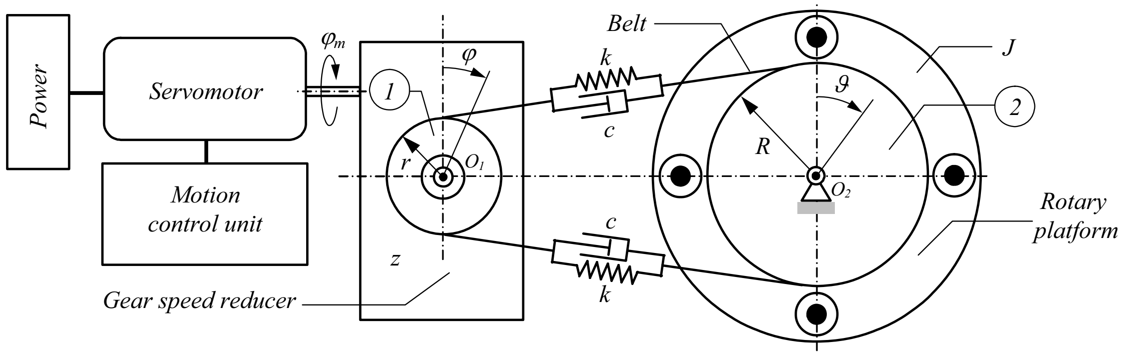

Fig. 1. Rotary platform driven by a servomotor through a gear speed reducer and a belt transmission.

As in all cases where a mathematical modeling of an actual device or machine is formulated, the values of physical parameters of the system should be determined with good accuracy in order to obtain a correct simulation of the experimentally observable phenomena.

To demonstrate the effectiveness of this approach, this paper presents a practical example, where the previously described technique is employed to reduce the residual vibrations of the system represented in Fig. 1.

The device consists of a rotary platform driven by a servomotor through a gear speed reducer and a belt transmission. The system parameters and their corresponding symbols are listed in Table I.

If we suppose that the servomotor is able to correctly execute the motion command assigned by the electronic control unit (this hypothesis is usually satisfied, when a position and/or a velocity feed-back loop is implemented inside the motion controller), the rotation of the motor shaft $\varphi_m(t)$ and its time derivatives $\dot{\varphi}_m(t)$ and $\ddot{\varphi}_m(t)$ are known; through the gear ratio z it is immediate to calculate the angular displacement of the pulley (1). The rotation $\vartheta(t)$ of the pulley (2), which drives the rotary platform, can be calculated by solving the following motion equation:

$$J\ddot{\vartheta} + 2cR(R\dot{\vartheta} - r\dot{\varphi}) + 2kR(R\vartheta - r\varphi) = 0 \qquad (1)$$

which expresses the dynamic equilibrium of the rotating masses about the pivot O_2. Since $\varphi = z\varphi_m$, Eq. (1) can be rewritten as:

$$J\ddot{\vartheta} + 2cR^2(\dot{\vartheta} - \lambda\dot{\varphi}_m) + 2kR^2(\vartheta - \lambda\varphi_m) = 0 \qquad (2)$$

where $\lambda = zr/R$.

TABLE I
PARAMETERS OF THE SYSTEM IN FIG. 1.

SYMB.	DESCRIPTION
J	Moment of inertia of the platform
k	Stiffness of each belt branch
c	Damping const. of each belt branch
r	Radius of pulley (1)
R	Radius of pulley (2)
z	Gear ratio of the speed reducer

At this point it is convenient to introduce the variable $\alpha = \vartheta - \lambda\varphi_m$, which represents the difference between the actual position ϑ of the platform and its theoretical position $\vartheta^* = \lambda\varphi_m$, corresponding to a perfectly rigid behaviour of the belt transmission.

Using this new variable we obtain from Eq. (2):

$$J(\ddot{\alpha} + \lambda\ddot{\varphi}_m) + c_t\dot{\alpha} + k_t\alpha = 0 \qquad (3)$$

where $k_t = 2kR^2$ and $c_t = 2cR^2$ are respectively the equivalent stiffness and the equivalent damping constant of the transmission.

Introducing the natural angular frequency of the mechanical system $\omega_n = \sqrt{k_t/J}$ and the non-dimensional damping ratio $\xi = c_t/2J\omega_n$, Eq. (3) can be rearranged as follows:

$$\ddot{\alpha} + 2\xi\omega_n\dot{\alpha} + \omega_n^2\alpha = -\lambda\ddot{\varphi}_m \qquad (4)$$

Knowing the analytical expression of the motor angular acceleration $\ddot{\varphi}_m$ and starting from null initial conditions (that is $\alpha(0) = 0$, $\dot{\alpha}(0) = 0$), the solution of the differential equation (4) can be calculated through the convolution integral [1]. If the system is underdamped ($\xi < 1$) we have:

$$\alpha(t) = -\frac{\lambda}{\omega_d}\int_0^t f(t,\tau)\,d\tau \qquad (5)$$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$ is the damped natural frequency of the system and $f(t,\tau)$ is defined as:

$$f(t,\tau) = \ddot{\varphi}_m(\tau)e^{-\xi\omega_n(t-\tau)}\sin[\omega_d(t-\tau)] \qquad (6)$$

The angular position of the rotary platform can be now easily calculated through the following relationship:

$$\vartheta(t) = \alpha(t) + \lambda\varphi_m(t) \qquad (7)$$

Through a proper selection of the acceleration command of the motor $\ddot{\varphi}_m(t)$ it is possible to eliminate or strongly attenuate the residual vibrations of the rotary platform. The next section gives some mathematical details about a class of functions which seem to be particularly suitable to this aim.

III. PARAMETRIC MOTION COMMANDS

As it is well known, the preshaping technique employs motion commands, whose analytical expressions are calculated in order to satisfy some design requirements. Therefore it is important to use parametric functions, i.e. particular mathematical expressions that depend on a set of numerical coefficients, which play the role of *control variables*. Furthermore, it must be possible to change the shape of the function without affecting the boundary conditions (that is the conditions at the initial and final time instants).

A particularly simple analytical expression used to define these motion commands is the following:

$$\varphi_m(t) = \Phi\left[1 - \sum_{i=0}^{2} C_i\left(1 - \frac{t}{T}\right)^{\gamma_i}\right] \tag{8}$$

where φ_m is the displacement and t the time. The symbols Φ and T indicate the total displacement and the motion time respectively, whereas the exponents γ_i are the shape parameters.

Differentiation of Eq. (8) yields:

$$\dot{\varphi}_m(t) = \frac{\Phi}{T}\left[\sum_{i=0}^{2} \gamma_i C_i\left(1 - \frac{t}{T}\right)^{\gamma_i - 1}\right] \tag{9}$$

$$\ddot{\varphi}_m(t) = -\frac{\Phi}{T^2}\left[\sum_{i=0}^{2} \gamma_i(\gamma_i - 1) C_i\left(1 - \frac{t}{T}\right)^{\gamma_i - 2}\right] \tag{10}$$

We observe that the expressions (8), (9) and (10) satisfy the following conditions:

$$\varphi_m(T) = \Phi \quad \dot{\varphi}_m(T) = 0 \quad \ddot{\varphi}_m(T) = 0 \tag{11}$$

In order to calculate the coefficients $C_i\,(i = 0, 1, 2)$, we impose that the displacement, the velocity and the acceleration are null for $t = 0$; therefore we can write:

$$\varphi_m(0) = 0 \quad \dot{\varphi}_m(0) = 0 \quad \ddot{\varphi}_m(0) = 0 \tag{12}$$

In this way we obtain the following linear system of equations:

$$\begin{cases} \sum_{i=0}^{2} C_i = 1 \\ \sum_{i=0}^{2} \gamma_i C_i = 0 \\ \sum_{i=0}^{2} \gamma_i(\gamma_i - 1) C_i = 0 \end{cases} \tag{13}$$

which can be rewritten in matrix form as:

$$\mathbf{Z}\mathbf{c} = \mathbf{u} \tag{14}$$

where $\mathbf{c} = (C_0\ C_1\ C_2)^T$, $\mathbf{u} = (1\ 0\ 0)^T$ and $\mathbf{Z}$ is a 3×3 matrix given by the following expression:

$$\mathbf{Z} = \begin{pmatrix} 1 & 1 & 1 \\ \gamma_0 & \gamma_1 & \gamma_2 \\ \gamma_0(\gamma_0 - 1) & \gamma_1(\gamma_1 - 1) & \gamma_2(\gamma_2 - 1) \end{pmatrix} \tag{15}$$

The symbolic solution of the system (13) gives the values of the constants C_0, C_1 and C_2 as function of the exponents γ_0, γ_1 and γ_2:

$$\begin{aligned} C_0 &= \frac{\gamma_1\gamma_2}{(\gamma_2 - \gamma_0)(\gamma_1 - \gamma_0)} \\ C_1 &= \frac{\gamma_0\gamma_2}{(\gamma_1 - \gamma_0)(\gamma_1 - \gamma_2)} \\ C_2 &= \frac{\gamma_0\gamma_1}{(\gamma_0 - \gamma_2)(\gamma_1 - \gamma_2)} \end{aligned} \tag{16}$$

Clearly the procedure can be generalized, by extending the boundary conditions up to the n^{th} derivative of the displacement; in this case the unknowns to be calculated are the $n+1$ coefficients $C_0, C_1, \ldots, C_n$ and the expression of $\varphi_m(t)$ must be modified as follows:

$$\varphi_m(t) = \Phi\left[1 - \sum_{i=0}^{n} C_i\left(1 - \frac{t}{T}\right)^{\gamma_i}\right] \tag{17}$$

The generalized formula for the k^{th} time derivative $(1 \le k \le n)$ is:

$$\varphi_m^{(k)}(t) = (-1)^{k+1}\frac{\Phi}{T^k}\left\{\sum_{i=0}^{n} P(\gamma_i, k) C_i\left(1 - \frac{t}{T}\right)^{\gamma_i - k}\right\} \tag{18}$$

where

$$P(\gamma_i, k) = \prod_{r=0}^{k-1}(\gamma_i - r) \tag{19}$$

Even in this case, for $t = T$, the function φ_m is equal to the total displacement Φ and its time derivatives up to order n are null; the coefficients C_i can be also calculated by imposing the conditions:

$$\varphi_m(0) = \dot{\varphi}_m(0) = \ddot{\varphi}_m(0) = \cdots = \varphi_m^{(n)}(0) = 0 \tag{20}$$

In this way we obtain the following linear system of equations:

$$\begin{cases} \sum_{i=0}^{n} C_i = 1 \\ \sum_{i=0}^{n} P(\gamma_i, k) C_i = 0 \quad k = 1, 2, \ldots n \end{cases} \tag{21}$$

which can be rewritten in matrix form as:

$$\bar{\mathbf{Z}}\bar{\mathbf{c}} = \bar{\mathbf{u}} \tag{22}$$

where $\bar{\mathbf{c}} = (C_0\ C_1\ C_2 \ldots C_n)^T$, $\bar{\mathbf{u}} = (1\ 0\ 0 \ldots 0)^T$ and $\bar{\mathbf{Z}}$ is a square matrix of order $n+1$ having the following structure:

$$\bar{\mathbf{Z}} = \begin{pmatrix} 1 & 1 & \ldots & 1 \\ \gamma_0 & \gamma_1 & \ldots & \gamma_n \\ \gamma_0(\gamma_0 - 1) & \gamma_1(\gamma_1 - 1) & \ldots & \gamma_n(\gamma_n - 1) \\ \vdots & \vdots & \ldots & \vdots \\ P(\gamma_0, n) & P(\gamma_1, n) & \ldots & P(\gamma_n, n) \end{pmatrix} \tag{23}$$

We note that the matrix $\bar{\mathbf{Z}}$ can be automatically compiled by a computer program, without particular difficulties.

It is important to observe that the function $\varphi_m(t)$ and its time derivatives depend on the values assigned to the coefficients γ_i.

If these exponents assume integer positive values, we obtain the classical polynomial functions, widely cited in the technical literature and frequently employed to design cam mechanisms [2] or to define the reference motion commands for electro-mechanical actuators employed in robotic manipulators or in other automatic machines.

On the contrary, if these parameters assume non integer (but always positive) values, the corresponding functions are no more polynomial, even if, from the analytical point of view, they can be still calculated with the previous described procedure.

In this case we can introduce the concept of *pseudo-polynomial* function, to put in evidence, at the same time, the analogies and the differences with a standard polynomial function.

New motion commands can be generated, if the shape parameters are suitably changed.

As an example, Fig. 2 show a comparison between a standard polynomial function (with integer exponents) and a pseudo-polynomial function (with non integer positive exponents).

The diagrams have been calculated by means of Eqs. (8), (9) and (10), using the control parameters listed in Table II; the same table shows the values of the coefficients C_i ($i = 0, 1, 2$), given by Eqs. (16). It is evident the deformation of the diagrams, when non-integer values of the parameters are used.

These parameters can be automatically selected by a numerical procedure, in order to optimize a suitable target function, as it will be explained in the next sections.

IV. Suppression of the residual vibration

As mentioned above, the residual vibration is a free vibration that appears at the end of the motion interval, that is for $t > T$; in this situation the shaft of the servomotor is kept locked on the final position, whereas the rotary platform oscillates due to the elasticity of the belt transmission.

The vibration is possible because some energy is still present in the mechanical device at the time instant $t = T$.

Through a proper selection of the reference motion command imposed to the servomotor it is possible to set at zero or to minimize the value of this energy; consequently the residual oscillations will be eliminated or strongly reduced in amplitude.

Starting from these considerations, it is clear that the final mechanical energy of the system plays the role of a target function, whose value can be set to zero (or minimized), by means of suitable optimization techniques, available in literature [3].

For the 1-DOF system described by Eq. (1) the total mechanical energy E_{tot}, at the generic time instant t, can be easily calculated by adding the kinetic energy of the rotating masses $E_{kin} = \frac{1}{2}J\dot{\vartheta}^2$ to the potential energy $E_{pot} = k(R\vartheta - r\varphi)^2$ due to the elastic deformation of the belt branches; so we have:

$$E_{tot} = \frac{1}{2}[J\dot{\vartheta}^2 + 2k(R\vartheta - r\varphi)^2] = \frac{1}{2}[J\dot{\vartheta}^2 + k_t(\vartheta - \lambda\varphi_m)^2] \qquad (24)$$

If we introduce the variable α, Eq. (24) can be rewritten as:

$$E_{tot} = \frac{1}{2}J[(\dot{\alpha} + \lambda\dot{\varphi}_m)^2 + \omega_n^2\alpha^2] \qquad (25)$$

where $\omega_n^2 = k_t/J$.

TABLE II
CONTROL PARAMETERS OF THE FUNCTIONS REPRESENTED IN FIG. 2.

	γ_0	γ_1	γ_2	C_0	C_1	C_2
POLYNOMIAL	3	4	5	10	-15	6
PSEUDO-POLYN.	4.5	6.2	8.4	7.855	-10.107	3.252

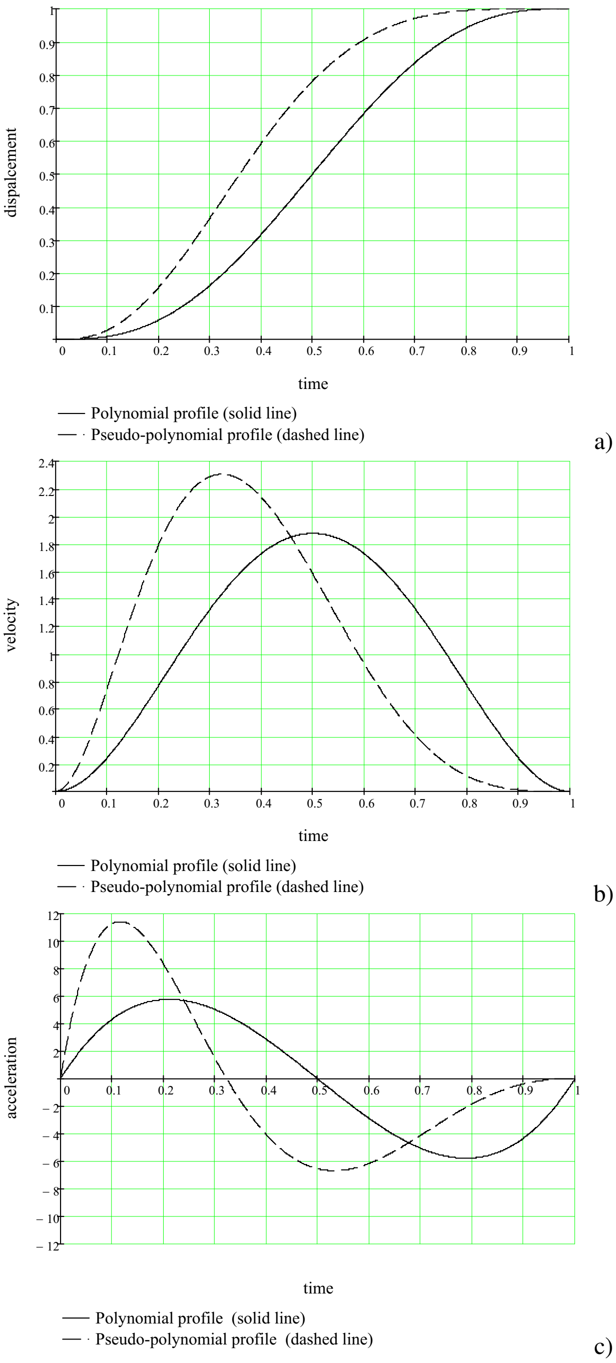

Fig. 2. Comparison between a polynomial motion command (solid line) and a pseudo-polynomial motion command (dashed line). a) displacement; b) velocity; c) acceleration. The functions have been calculated for $T = 1\,\mathrm{s}$ and $\Phi = 1\,\mathrm{rad}$.

At the final time instant $t = T$ the angular velocity of the motor is null ($\dot{\varphi}_m(T) = 0$); therefore we obtain from Eq. (25):

$$E_{tot}(T) = \frac{1}{2}J[(\dot{\alpha}^2(T) + \omega_n^2\alpha^2(T)] \qquad (26)$$

The terms $\alpha(T)$ and $\dot{\alpha}(T)$ that appear at the right-hand side of Eq. (26) can be calculated through Eq. (5); in particular, the velocity term (for a generic time instant t) is given by the following relationship:

$$\dot{\alpha}(t) = -\frac{\lambda}{\omega_d}\frac{d}{dt}\int_0^t f(t,\tau)\,d\tau \tag{27}$$

which requires differentiation under the integral sign[1]. Since $f(t,\tau)|_{\tau=t} = 0$, the formula indicated in the footnote gives the following result:

$$\dot{\alpha}(t) = -\frac{\lambda}{\omega_d}\int_0^t g(t,\tau)\,d\tau \tag{28}$$

where $g(t,\tau) = \frac{\partial}{\partial t}f(t,\tau)$. From Eq. (6) we obtain the analytical expression of the g function:

$$g(t,\tau) = \omega_n\ddot{\varphi}_m(\tau)e^{-\xi\omega_n(t-\tau)}\cos[\omega_d(t-\tau)+\delta] \tag{29}$$

where $\tan\delta = \xi/\sqrt{1-\xi^2}$. If now we introduce the following definitions:

$$F = \int_0^T f(T,\tau)\,d\tau \qquad G = \int_0^T g(T,\tau)\,d\tau \tag{30}$$

Eq. (26) assume the form:

$$E_{tot}(T) = \frac{1}{2}J\left(\frac{\lambda}{\omega_d}\right)^2[G^2+\omega_n^2F^2] = h\mathcal{H} \tag{31}$$

where $h = \frac{1}{2}J\left(\frac{\lambda}{\omega_d}\right)^2$ and $\mathcal{H} = G^2+\omega_n^2F^2$.

The term between square brackets in Eq. (31) depends on the motor acceleration $\ddot{\varphi}_m$, through the functions $f(T,\tau)$ and $g(T,\tau)$ and therefore it depends on the values assigned to the control parameters γ_i. Hence, with mathematical formalism, we can write:

$$\mathcal{H} = \frac{E_{tot}(T)}{h} = \mathcal{H}(\gamma_0,\gamma_1,\gamma_2) \tag{32}$$

The control parameters can be automatically selected by a numerical procedure, in order to optimize the target function $\mathcal{H}$. Using this approach, it is also possible to introduce of some algebraic constraints, to reduce the computational time and to drive the optimization process towards a satisfactory solution.

V. Numerical results

This paragraph presents some numerical results, obtained by means of the motion planning technique described in the previous sections. The calculation have been performed for the system represented in Fig. 1, whose parameters are listed in Table III. The angular displacement of the motor has been set to 15 revolutions, corresponding to a rotation of 180 degrees of the platform. The total motion time is $T = 1\,\mathrm{s}$.

[1]We report here the general formula for differentiation under the integral sign:

$$\frac{d}{dt}\int_{a(t)}^{b(t)} f(t,\tau)d\tau = \\ = \int_{a(t)}^{b(t)}\frac{\partial}{\partial t}f(t,\tau)d\tau + f(t,b(t))b'(t) - f(t,a(t))a'(t)$$

If $a(t) = 0$ e $b(t) = t$ the above-mentioned formula can be simplified as follows:

$$\frac{d}{dt}\int_0^t f(t,\tau)d\tau = \int_0^t\frac{\partial}{\partial t}f(t,\tau)d\tau + f(t,\tau)|_{\tau=t}$$

The motion command for the servomotor has been generated by means of Eqs. (8), (9) and (10), using Eqs. (16) to calculate the coefficients C_0, C_1 and C_2.

In order to minimize the target function $\mathcal{H}$ an optimization procedure has been implemented using a Quasi Newton method; the initial values of the control parameters are $\gamma_0 = 3$, $\gamma_1 = 4$, $\gamma_2 = 5$ corresponding to a standard fifth degree polynomial function. At the end of the optimization process the following final values have been found: $\gamma_0 = 3.024$, $\gamma_1 = 4.565$, $\gamma_2 = 5.874$.

Fig. 3 shows the results before and after the optimization process. In the first and in the second row we have reported the angular velocity $\dot{\vartheta}(t)$ and the angular acceleration $\ddot{\vartheta}(t)$ of the rotary platform as function of time (solid line); the dashed lines indicate the corresponding motion commands. The third and the fourth row show the time histories of the kinetic and potential energy of the system.

The comparison between the velocity and the acceleration diagrams in the left and right columns clearly shows that the residual vibration disappears, when optimized motion commands are used to drive the servomotor.

VI. Conclusions

A method for reducing the residual vibration of belt-driven rotary platform has been presented in the paper. The calculation procedure employs a mathematical model of the system, a class of parametric functions and an optimization algorithm, that minimizes the total mechanical energy of the system at the final time instant. Through computer simulation, the technique has been successfully tested on a 1-DOF vibrating system, consisting of a rotary platform driven by an electric servomotor through a speed reducer and an elastic belt transmission.

The numerical results indicate that the proposed approach seems to be able to reduce (or totally eliminate) the residual vibrations, without changing the motion time, nor altering the boundary conditions of the motion command.

The proposed method does not require complex control algorithms for the servomotor or additional feedback sensors to measure the vibration amplitude and, for these reasons, it can be implemented on an actual machine with very low costs: in fact it is just sufficient a modification of the reference function memorized in the motion controller.

Since a mathematical model of the actual device is employed for motion optimization, it is necessary an accurate identification of the mechanical parameters, in particular as regards the equivalent damping coefficient. For this reason, in the future the technique will be implemented on an experimental test-bed, in order to validate the theoretical results here presented.

TABLE III
Numerical values of the system parameters.

Symb.	Val.	Unit	Symb.	Val.	Unit
J	6	$\mathrm{kg\,m^2}$	r	60	mm
k	9×10^4	N/m	R	180	mm
c	150	Ns/m	z	1:10	-

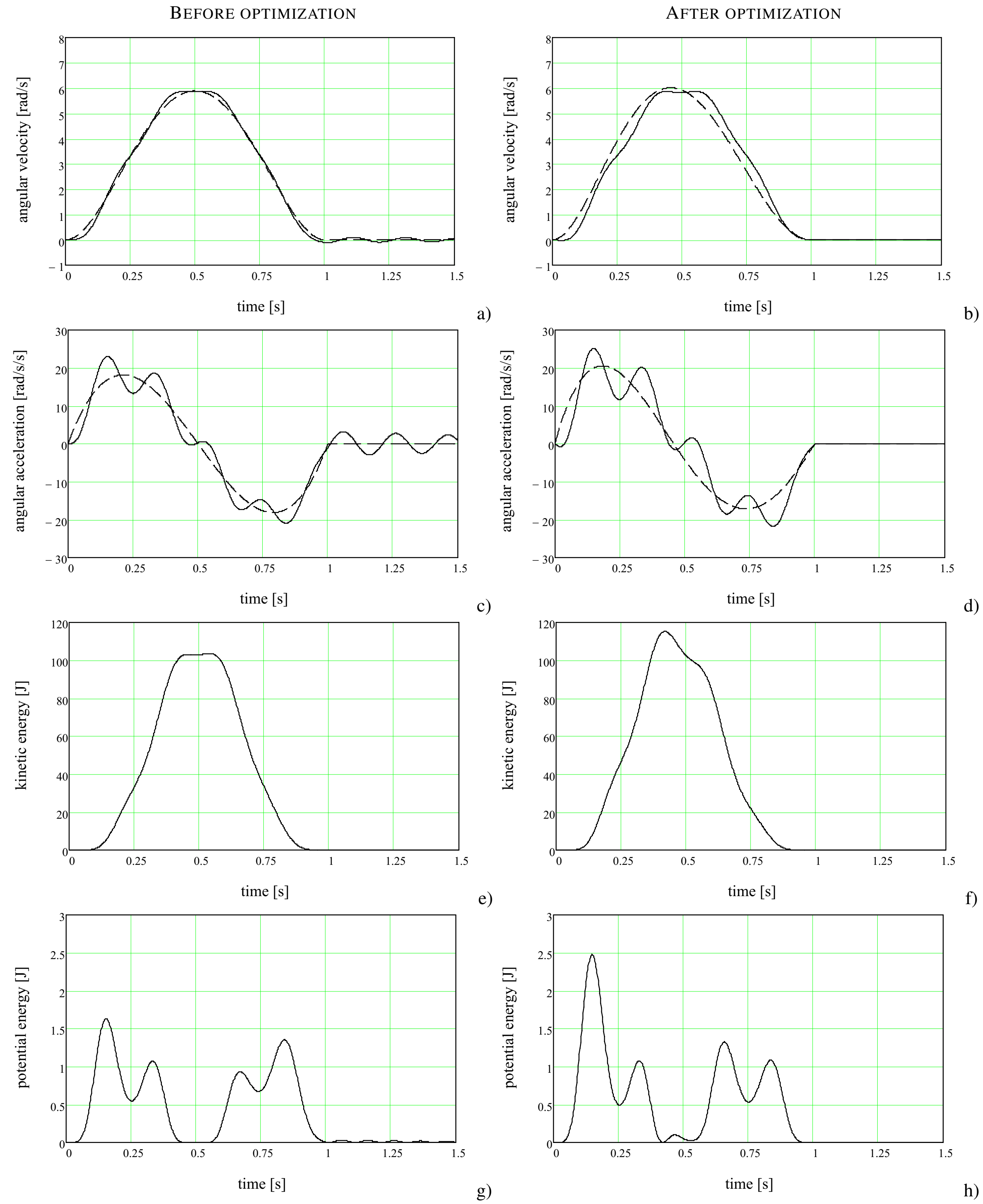

Fig. 3. Comparison between the non-optimized case (left column diagrams) and the optimized case (right column diagrams): **a, b)** angular velocity $\dot{\vartheta}(t)$ of the rotary platform; **c, d)** angular acceleration $\ddot{\vartheta}(t)$ of the rotary platform; **e, f)** kinetic energy of the masses rotating about pivot O_2. **g, h)** elastic potential energy due to belt deformation. The dashed curves in the diagrams of the 1^{st} and 2^{nd} row indicate the velocity command $\lambda\dot{\varphi}_m(t)$ and the acceleration command $\lambda\ddot{\varphi}_m(t)$.

REFERENCES

[1] Meirovich L., *Fundamentals of Vibrations*, McGraw-Hill Higher Education, 2001.
[2] Norton R.L., *Cam design and manufacturing handbook,* Industrial Press Inc., 2002.
[3] Rao S.S., *Engineering Optimization - Theory and Practice*, Wiley, New York, 1996.
[4] Singer N.C., Seering W.P., *Preshaping command inputs to reduce system vibration*, Journal of Dynamic Systems, Measurement, and Control, 112, 76-82, 1990.
[5] Singh T., Heppler G.R., *Shaped input control of a system with multiple modes*, Journal of Dynamic Systems, Measurement, and Control, 115, 341-347, 1993.
[6] Hyde J.M. and Seering W.P., *Using input command pre-shaping to suppress multiple mode vibration*, Proceedings of the IEEE Conference on Robotics and Automation, Sacramento, CA, April 1991.
[7] Singhose W.E., Crain E.A. and Seering W.P., *Convolved and simultaneous two-mode input shapers*, IEEE Proceedings Control Theory and Applications 144 (6), 515-520, 1997.
[8] Kapucu S., Gürsel A., Bayseç S., *Residual swing/vibration reduction using a hybrid input shaping method*, Mechanism and Machine Theory, 36, 311-326, 2001.
[9] Agostini M.J., Parker G.G., *Command Shaping Nonlinear Inputs Using Basis Functions*, Journal of Intelligent Material Systems and Structures, 13, 181-190, 2002.
[10] Chen K.S., Yang T.S., Ou K.S., Yin J.F., *Design of command shapers for residual vibration suppression in Duffing nonlinear systems*, Mechatronics, 19, 184198 , 2009.
[11] Park K.J., *Flexible robot manipulator path design to reduce the endpoint residual vibration under torque constraints*, Journal of Sound and Vibration, 275, 10511068, 2004.
[12] Pelaez G., Pelaez Gu., Perez J.M., Vizan A., Bautista E., *Input shaping reference commands for trajectory following Cartesian machines*, Control Engineering Practice, 13, 941958, 2005.
[13] Mimmi G., Pennacchi P., *Pre-shaping motion input for a rotating flexible link*, International Journal of Solids and Structures, 38, 2009-2023, 2001.
[14] Dharne A.G., Jayasuriya S., *Robust Adaptive Control of Residual Vibration in Point-to-Point Motion of Flexible Bodies*, Journal of Vibration and Control, 13, 951-968, 2007.
[15] Gürleyük S.S., Cinal S., *Robust Three-impulse Sequence Input Shaper Design*, Journal of Vibration and Control, 13, 1807-1818, 2007.

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