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Proportionally Damped Finite Element State-Space Model of Composite Laminated Plate with Localized Interface Degeneration

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Abstract**—**In the present work, the finite element formulation for the investigation of the effects of a localized interfacial degeneration on the dynamic behavior of the [90˚/0˚] laminated composite plate employing the state-space technique is performed. The stiffness of the laminate is determined by assembling the stiffnesses of sub-elements. This includes an introduction of an interface layer adopting the virtually zero-thickness formulation to model the interfacial degeneration. Also, the kinematically consistent mass matrix and proportional damping have been formulated to complete the free vibration governing expression. To simulate the interfacial degeneration of the laminate, the degenerated areas are defined from the center propagating outwards in a localized manner. It is found that the natural frequency, damped frequency and damping ratio of the plate decreases as the degenerated area of the interface increases. On the contrary, the loss factor increases correspondingly.

Keywords**—**Dynamic finite element, localized interface degeneration, proportional damping, state-space modeling.

I. INTRODUCTION

IN recent years, composite materials has become one of the mainly employed materials in advanced engineering, primarily as components in civil engineering, aerospace, automotive and other structural applications. Usually, they are fabricated as laminated structures where two or more laminae are bonded by a layer of adhesive material.

Although laminated composite has almost unlimited potential in satisfying the strength requirement, they may exhibit several peculiar modes of failure such as matrix crazing, delamination, fiber failure and interfacial bond failure due to debonding. In reality, it is impossible to have a perfect interfacial bond especially during manufacturing process or the actual service life of composite laminates. One of the most common failures, the delamination, is an interlayer separation damage mode, which possibly occurs in the interface of a laminated composite. It may result in a reduction of the stiffness of material. Therefore, a model of laminated composite with capability of describing the interfacial imperfection should be adopted to study such condition.

The dynamic response such as natural frequency, modal damping and loss factor depend in general on the material density, elastic constants, damping properties, geometry and layers orientations. Therefore, damping has become one of the important parameters related to the study of dynamic behavior of laminated composite structures. In general, damping can be divided into two main types: viscous damping and structural damping. Damping usually occurs as a mixture of two mechanisms in a composite laminate. One of the mechanisms is damping between the fiber and adhesive layer within the laminated plies and the other mechanism is damping between the plies or between the laminae. The damping properties of laminated plate can be affected during the manufacturing process of gluing all layers. In relation, the manufacturing faults may cause the presence of debonding areas in the glued region. The presence of bonding-free areas could reduce the natural frequency and hence disturb the damping properties of material. Due to high labor and cost demands of experimental studies, the predictions of changes in structural dynamic properties can be investigated by using the finite element method. With the modeling of degeneration of localized interfacial in composite laminated plate with an inclusion of damping, the accuracy to predict the failure will be improved and more realistic. An accurate modeling expression for damping is essential in describing better the dynamic response of laminated composite structures.

Thus far, several interface elements for modeling the behavior of the interfacial layers of laminated plates have been proposed. Sun and Pan [1] proposed a method to characterize the mechanical behavior of composite laminate interfaces, which is based on the generalized composite laminate theory. Bui, Marechal and Nguyen-Dang [2] presented a numerical analysis for laminated composite plate with imperfect interlaminar interfaces. In relation to damping modeling, Rikards et al. [3] applied two methods of damping analysis, complex eigenvalues and energy methods, for evaluating the dynamic performance of a sandwich structure. Hu and Dokainish [4] presented two damping models, the viscoelastic damping (VED) and the specific damping capacity (SDC) models, to assess the damping behavior of composites. By considering the damping and delamination, Oh et al. [5] performed a dynamic analysis of laminated composites with multiple delaminations according to higher-order cubic zigzag theory.

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Note that aforementioned modeling efforts have been centralized on single site interfacial imperfection rather than various locations. A discreet imperfection area has been possible since the introduction of numerical method such as the well-established finite element approach. In utilizing this technique, Abo Sabah and Kueh [6] studied the effects of localized interface delamination on the behavior of laminated composite plates subjected to low velocity impact loading for different fiber orientations by means of virtually zero-thickness interface definition. They found that when the local delamination area increases, displacement increases. They also found that as top and bottom fiber orientations deviation increases, both central deflection and energy absorption increase.

II. Numerical Procedure

A. Model Description

The main structure studied here is a rectangular cross-ply laminate plate with two composite laminae. The top lamina is of 90 degrees fiber direction and the bottom lamina is aligned at 0 degree. They are of constant thickness and an interfacial layer is considered in between. Each lamina is formed by unidirectional E-glass fibers and Epoxy (3501-6) matrix material with a volume fraction of 0.4. The laminated composite plate is considered to be thin and flat such that the shear deformation is neglected.

The lamina is modeled and discretized by using a 4-node rectangular plate finite element. Also, the laminated composite plate is considered as a transversely isotropic solid material. There are five degrees of freedom at each node, which are displacement in *x*-direction (u), displacement in *y*-direction (v), displacement in *z*-displacement (w), rotation about *y*-direction ($\o_x$) and rotation about *x*-direction ($\o_y$).

Besides, the interfacial layer is considered as an orthotropic material with null normal stress in *x*- and *y*-directions ($\sigma_x = 0$ and $\sigma_y = 0$) and in-plane shear stress in *x-y* plane ($\tau_{xy} = 0$). It is modeled using a quadrilateral zero-thickness solid element with 8 nodes. However, there are only three degrees of freedom for each node, which are the displacement in *x*-direction (u), displacement in *y*-direction (v), and displacement in *z*-displacement (w). The stiffness matrix of the lamina and interfacial element is computed using a 2 × 2 Gauss quadrature rule.

Proportional damping model is applied to describe the damping in this study. Eigenvalue analysis is conducted by means of state-space approach since the model is under free vibration environment. It is worth noting that a full degeneration is considered in a localized manner. From the eigenvalue analysis, natural frequency, damped natural frequency, loss factor and damping ratio of laminated composite plate, with the application of degeneration of localized degeneration, are to be determined.

B. Numerical Model Construction

The configuration of the [90/0] cross-ply laminated composite rectangular plate is shown in Fig. 1. The governing equation for the dynamic finite element in terms of the eigenvalue problem is

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = 0 \tag{1}$$

where $\{M\}$ is the global mass matrix, $\{\ddot{q}\}$ is the nodal acceleration, $[C]$ is the global damping matrix, $\{\dot{q}\}$ is the nodal velocity, $\{K\}$ is the global stiffness matrix, and $\{q\}$ is the nodal displacement.

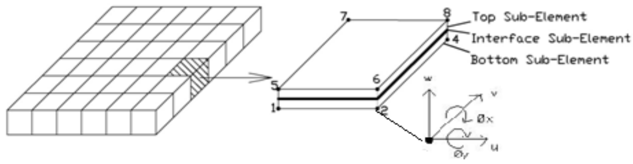


Fig. 1 The two lamina sub-elements, interface element and the arrangement of nodes and the degrees of freedom (DOF) of each node in the lamina sub-element

C. Stiffness Matrix of Elemental Lamina

The local stiffness matrix of the lamina is developed by combining the element strain-displacement matrix with the ABD matrix of the lamina shown below.

$$K = \iint [B_i^T (A)_{ABD} B_i + B_i^T (B)_{ABD} B_o + B_o^T (B)_{ABD} B_i + B_o^T (D)_{ABD} B_o] |J| d\zeta d\eta \tag{2}$$

where Bi is the in-plane element strain-displacement matrix, Bo is the out-of-plane element strain-displacement matrix, (A)ABD is the extensional stiffness, (B)ABD is the coupling stiffness, and (D)ABD is the bending stiffness of the lamina.

D. Stiffness Matrix of Elemental Interface

The stiffness matrix of the zero-thickness interface element is computed by

$$K = \frac{1}{h} \int \int B^T D B \, |J| d\zeta d\eta \tag{3}$$

where h is the thickness of the interface, B is the combined element strain-displacement matrix, D is the elasticity matrix, and $|J|$ is the determinant of Jacobian matrix.

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \tag{4}$$

$$D = \begin{bmatrix} G_{xz}(1-R) & 0 & 0 \\ 0 & G_{yz}(1-R) & 0 \\ 0 & 0 & E_z(1-R) \end{bmatrix} \tag{5}$$

where G_{xz} and G_{yz} are the out-of-plane shear modulus, E_z is the Young's Modulus in the *z*-direction and R is the imperfection factor.

E. Mass Matrix

The kinematically consistent mass matrix $[M]$ is defined to describe the mass contribution by summing all element mass matrices $[m]$.

$$[M] = \sum_{e=1}^{n}[m] \tag{6}$$

where

$$[m] = \iint_A \rho h N^T N \, dA \tag{7}$$

n is the total number of elements. ρ and N are the density and the combined shape functions, respectively.

F. Damping Matrix

The proportional damping, $[C]$, also known as Rayleigh damping, is adopted and defined as a linear combination of global mass matrix and global stiffness matrix.

$$[C] = \alpha[M] + \beta[K] \tag{8}$$

The coefficients, α and β, are computed by considering the required levels of proportional damping at two different frequencies, which are the first and second modes of free vibration

G. State-Space Form

The linear equation of motion (1) is solved employing the state-space method. For free vibration cases, the governing equation becomes an eigenvalue problem shown below.

$$([A] - \bar{\omega}[B])\hat{q} = 0 \tag{9}$$

where $\bar{\omega}$ is the frequency of the natural vibration in complex solution ($\bar{\omega} = \bar{\omega}_r + i\bar{\omega}_i$), $\hat{q}$ is the mode shape vector and

$$[A] = \begin{bmatrix} [0] & -[K] \\ -[K] & -[C] \end{bmatrix}$$
$$[B] = \begin{bmatrix} -[K] & [0] \\ [0] & [M] \end{bmatrix} \tag{10}$$

H. Damping Properties

In terms of the structural response, the dynamic behavior of laminated composite plate is considered. Therefore, two important parameters, which are damping ratio, ζ, and loss factor, η, are formulated in the followings.

$$\zeta = \frac{\alpha + \beta\omega_n^2}{2\omega_n} \tag{11}$$

$$\eta = \frac{\omega_n}{\omega_n \zeta} \tag{12}$$

where ω_n = natural frequency and ω_d = damped frequency $= \omega_n \times \sqrt{\omega_n \times \zeta}$

I. Degeneration Pattern

To model the degeneration of interface, an area of interface degeneration is implemented at the center of the laminate. This degeneration region is extended radially throughout the interface of the laminated composite plate as shown in Fig. 2.

J. Validation

The present model is made similar with the work done by Hu and Dokainish [4] for validation purpose. We compare their computed natural frequencies and loss factor with those from the present study. Material properties used in the validation are: E_1= 42.62 GPa, E_2= 12.50 GPa, G_{12}= 5.71 GPa, G_{21}= 2.855 GPa, v_{12}=0.30, and ρ = 1971.0 kg/m^3. It is obvious that a good agreement is achieved in the comparison.

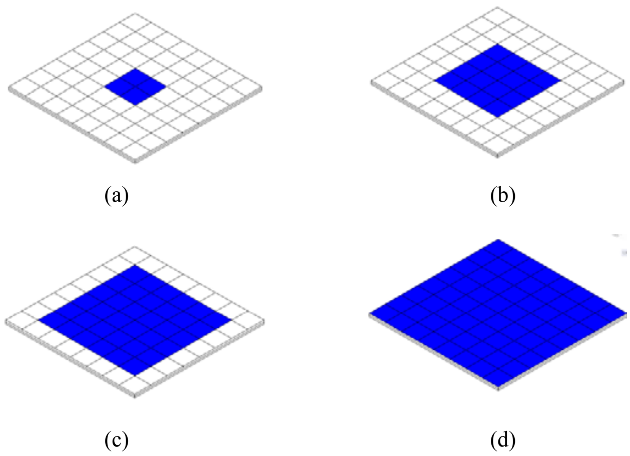


Fig. 2 The degeneration is initially implemented at the center and extended radially throughout the interface of the plate (shaded area indicate those degenerated)

TABLE I
COMPARISON OF NATURAL FREQUENCY AND LOSS FACTOR

	Natural Frequency	Loss Factor
Hu and Dokainish [4]	9.3744	6.75
Present	10.1535	6.07

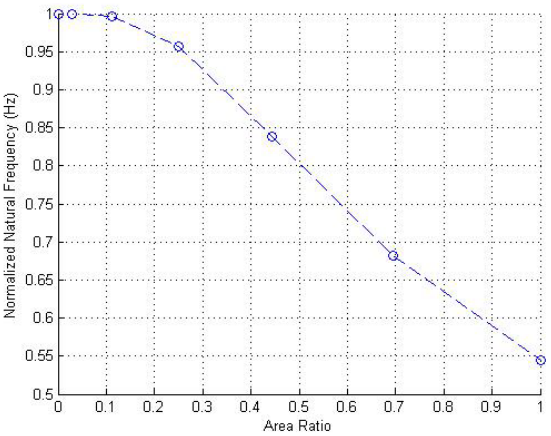
III. RESULTS AND DISCUSSION

The dynamic performances of the composite plate with degeneration from the center that is extending radially throughout the interface of the laminated composite plate in terms of natural frequency, damped frequency, loss factor and damping ratio are shown in Figs. 3 (a)-(d).

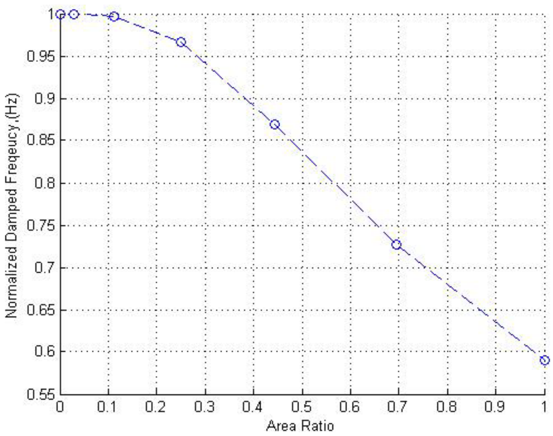
As the degenerated area ratio increases, the natural frequency decreases. Therefore, the whole laminate accordingly becomes weaker. Similarly, the damped frequency follows the same dropping trend and has a large reduction when the degenerated area ratio is more than 0.1. The loss factor is a good way to express the damping characteristics of a material. The higher the loss factor, the more damping a material has. From the results, the loss factor increases as the degenerated area increases. This implies that the occurrence of the degenerated interface in the laminated composite plate promotes higher damping. This indicates also that a greater imperfection improves the damping behavior of a laminated material, owing to higher energy absorption by means of friction. Since the damping ratio and natural frequencies are dependent to each other, the decrease of damping ratio can be seen in Fig. 3 (d).

IV. Conclusion

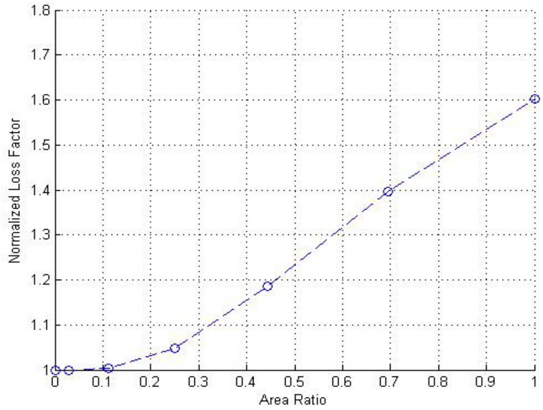
Finite element formulation for a two-layer cross-ply laminated composite plate prescribed with an interfacial element and a proportional damping was developed. A state-space approach was employed to investigate the dynamic behaviors of the plate. From the study, it is found that the natural frequency, damped frequency and damping ratio of the plate decrease as the degenerated area of the interface increases. However, the loss factor increases when the degenerated area of the interface increases, due to greater damping provided by higher friction.



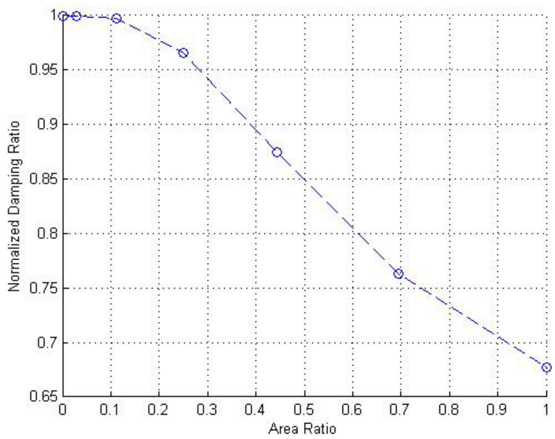
(a)



(b)



(c)



(d)

Fig. 3 The change in (a) normalized natural frequency, (b) normalized damped frequency, (c) normalized loss factor, and (d) normalized damping ratio of cross-ply composite laminated plate due to various interfacial degenerated areas

References

[1] H. Sun and N. Pan. Mechanical Characterization of the Interfaces in Laminated Composites. *Journal of composite structures*, 74, 25–29. Elsevier, 2005.

[2] V. Q. Bui, E. Marechal, H. Nguyen-Dang. Imperfect interlaminar interfaces in laminated composites: delamination with the R-curve effect. *Composites Science and Technology, 60*, 2619-2630, 2000.

[3] R. Rikards, A. Chate and E. Barknov. Finite Element Analysis of Damping, The vibrations of Laminated Composites. *Computers & Structures, 47*, 1005 -1015, 1993.

[4] B. G. Hu and M. A. Dokainish. Damped Vibrations of Laminated Composite Plate- Modelling and Finite Element Analysis. *Finite Element in Analysis and Design, 25,* 103-124, 1993.

[5] J. Oh, M. Cho, J.-S. Kim. Dynamic analysis of composite plate with multiple delaminations based on higher-order zigzag theory. *International Journal of Solids and Structures, 42*, 6122–6140, 2005.

[6] S. H. Abo Sabah and A. B. H. Kueh. Finite Element Modeling of Laminated Composite Plates with Locally Delaminated Interface Subjected to Impact Loading. *The Scientific World Journal*, 2014.