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Profit Optimization for Solar Plant Electricity Production

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***Abstract*—**In this paper a stochastic scenario-based model predictive control applied to molten salt storage systems in concentrated solar tower power plant is presented. The main goal of this study is to build up a tool to analyze current and expected future resources for evaluating the weekly power to be advertised on electricity secondary market. This tool will allow plant operator to maximize profits while hedging the impact on the system of stochastic variables such as resources or sunlight shortage.

Solving the problem first requires a mixed logic dynamic modeling of the plant. The two stochastic variables, respectively the sunlight incoming energy and electricity demands from secondary market, are modeled by least square regression. Robustness is achieved by drawing a certain number of random variables realizations and applying the most restrictive one to the system. This scenario approach control technique provides the plant operator a confidence interval containing a given percentage of possible stochastic variable realizations in such a way that robust control is always achieved within its bounds. The results obtained from many trajectory simulations show the existence of a ''reliable'' interval, which experimentally confirms the algorithm robustness.

***Keywords*—**Molten Salt Storage System, Concentrated Solar Tower Power Plant, Robust Stochastic Model Predictive Control.

I. Introduction

THE intention in present study is to design a tool giving molten salt solar plant operator and to evaluate his opportunities on secondary energy market M_s, regulated so that nominal power production is fixed on a week period. Offer and demand being unequal, only a fraction of projected nominal output will be used. Operator receives a volatile signal re-actualized every 10sec. and oscillating between –100% and +100%. So aside power input from sun radiation which may unpredictably vary with climatic conditions the electricity network demand is also another stochastic variable. So, common framework for an optimal control of such a system is the Stochastic Model Predictive Control (SMPC) [1] which has been showing useful robustness in various industrial applications [2].

For thermal concentration solar plants, various techniques are used to concentrate solar rays toward thermal receiver [3]. This part will not be considered and the ''system'' discussed here will be concerning the following part up to electricity production, see Fig. 1.

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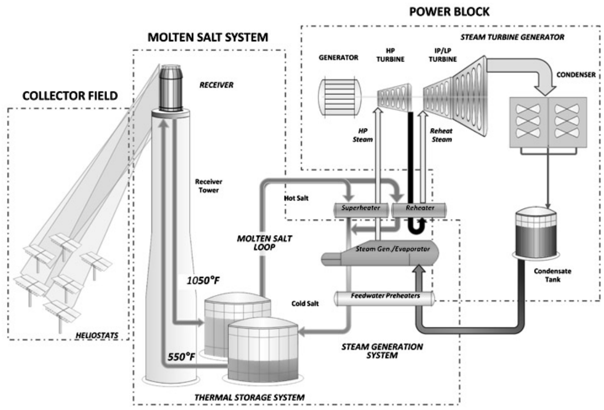

Fig. 1 Molten Salt Solar Plant

Amongst the various performance control algorithms, the minimal variance performance model based control will be used here because it is a non intrusive one [4]. So in a first part system model will be described and mathematically formulated, in the second one optimal control algorithms are presented with the use of deterministic and stochastic MPC. The third part is concerning exploration of different strategies with associated robustness, and conclusion will open on further possible researches.

II. Solar Plant Model

1. From Fig. 1, solar rays are directed through heliostats to the receiver on top of the tower in order to heat up molten salt liquid which is stored in a thermally isolated tank. The hot molten salt liquid is released to a steam turbine generating the electricity on demand, and cold exiting molten salt liquid is recycled into another storage tank before being heated again in the receiver. Main problem is thus for solar plant operator to optimally release hot molten salt to fit at best (fluctuating) expected electricity demand. The following hypotheses will be set to keep model coherence and to provide the frame within which system equations can be formulated:

1) Stochastic solar energy input $w_1(t)$ and electricity kWh price P(t) are de-correlated.
2) Minimal plant reaction time being typically 1min, working time step when discretizing the equations is taken as Δ t = 15 min
3) Measurements of molten salt level x in the tank and of (stochastic) solar power are perfect and do not require further processing such as filtering

The system model will be taken as a linear discrete time one, in the form

$$x(\mathrm{k}+1) = A\mathrm{x}(\mathrm{k}) + B\mathrm{u}(\mathrm{k}) + \mathrm{w}(\mathrm{k}) \tag{1}$$

with tank volume state vector $x(t) = [x_1(t), x_2(t)]^T$, extracted volume control vector $u(t) = [u_1(t), u_2(t)]^T$, and energy random input $w(t) = [w_1(t), 0]$, see Fig. 2, and where index *1* refers to hot fluid circuit and *2* to cold one.

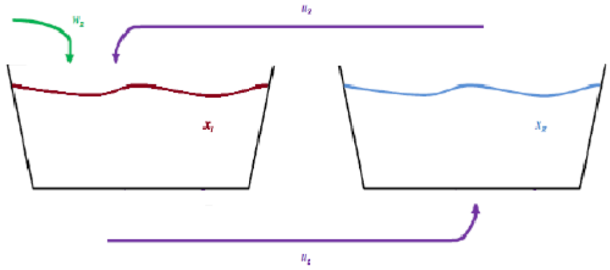

Fig. 2 Source-Sink Model of Solar Power Plant

Other system parameters are M the heliostat surface (m²), γ_n the steam turbine power (MWh) limiting electricity production at time k, and x_{jmax} the upper bound of level x_j. Matrices A and B are:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad B = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

Finally, control vector components are supposed to be bounded above inside the interval $[u_{jmin}, u_{jmax}]$ so that control space $\mathcal{U}$ is defined by $\{u \in \mathcal{U} \mid u_j \in (0, [u_{jmin}, u_{jmax}])\}$.

2. Input vector $w_1(t)$ has been evaluated from sunshine data recorded every hour between year 2000 and year 2005 representing over 52400 values. From a representative week selected for each trimester of the years, a theoretical "perfect" sunshine has been reconstructed by collecting its maximal value for each of the 24 hourly intervals in the 2184 collected days, see Fig. 3. Comparison of a typical day with previous theoretical one gives by mean square method a reducing factor for each trimester due to cloud random influence [5]. On the other side, demand w_d from the network is defined from two signals $<w>_{\mathrm{av}}$ and $<w>_{\mathrm{fluc}}$, respectively representing the week projected production and the real demand expressed as a percentage of week projected production. Volatile real demand given by the sector to operator has been collected over a year period with 15min frequency representing an array of 34365 values, out of which difference with predicted demand can be plotted as a percentage (note that this difference can have either sign), see Fig. 4. To simplify the numerical burden, the relation between energy output from the tank u and delivered electricity Γ_{out} from steam turbine is taken as a linear one $\Gamma_{out} = \Gamma_n u$ where Γ_n elements are > 0.

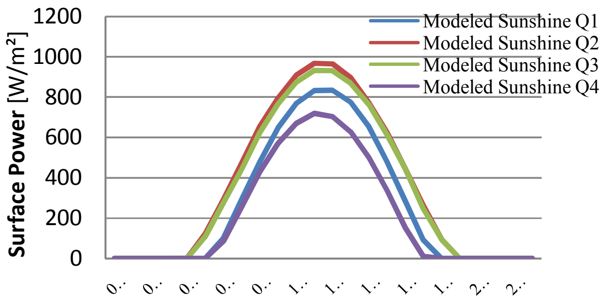

Fig. 3 Averaged Theoretical Sunshine per Trimester

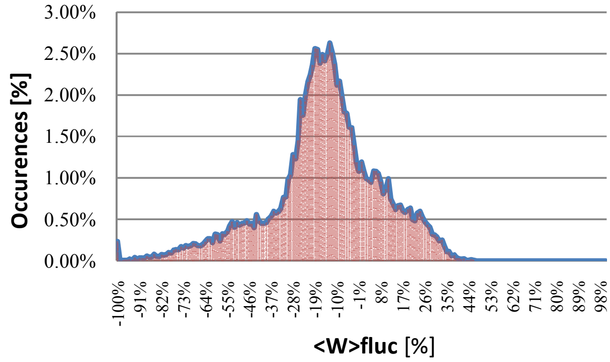

Fig. 4 Distribution Function of Demand Difference in % vs. Occurrence in %

From these elements the problem of optimized production can be formulated.

III. Model Predictive Control Application

Briefly stated, deterministic predictive command consists in reassessing discrete command problem as a minimization problem which is here solved by linear programming (LP) method. System cost function depending on state and input variables, on system constraints, is researched by solving LP problem at each time step for a certain number n of time steps in advance representing the receding horizon $H(n)$ [6]. Opposite to usual PID type control, next command value is determined from actual and future system values requiring the knowledge of future system evolution and of inputs behavior. The problem then formulates in the general form

$$\textbf{(P)} \quad Min_u c^T x \geq 0 \tag{2}$$

$$\begin{aligned} s.t. \quad & x = x_- + \lambda_- w_- - u_-, \\ & \Gamma_n u \geq w_d - \varepsilon_d, \\ & x_{min} \leq x \leq x_{\max} - \varepsilon_{\max}, \\ & u \in \mathcal{U} \end{aligned}$$

where $x_- = x(j-1)$ and command input space $\mathcal{U}$ is constrained by $\{u \in \mathcal{U} \mid u_j \in (0, [u_{jmin}, u_{jmax}])\}$. Because space $\mathcal{U}$ is non convex, usual methods [7] do not apply directly and stability issue may not be fully satisfied [8]. In the following a more direct destocking strategy minimizing a cost function by only considering $x(n), x_{min}, x_{max}$ will be developed. Let the cost function

$$C_j = \lambda_j \omega_j + \mu_j \eta_j \quad (3)$$

where $\omega_j = \frac{1}{x_{max}-x_{min}} \sum_{i=1}^{T} w_i$ and $\eta_i = \frac{1}{x_{max}-x_{min}} \sum_{i=1}^{T} u_i$ represents respectively normalized cumulative sun incoming energy and off-loaded cumulative energy during T periods. $\lambda_j(x)$ and $\mu_j(x)$ are antagonistic state dependent weighting factors on ω_j and η_j such that when x gets close to x_{min} then λ_j is significantly higher than μ_j resulting in restoring x up to its optimal level. Conversely, when x gets close to x_{max} then μ_j is higher than λ_j. They can for instance in simplest linear case be described as follows:

$$\begin{cases} \lambda_j = \xi_0 + \omega_j - \eta_j \\ \mu_j = 1 - \lambda_j = 1 - \xi_0 - \omega_j + \eta_j \end{cases} \quad (4)$$

where

$$\xi_0 = \frac{x_j - x_{max}}{x_{max} - x_{min}} \quad (5)$$

By introducing $X_j = \eta_j$ and $Y_j = \omega_j - \eta_j$, it is clear that (**P**) can be reformulated as a trivial system

$$\begin{aligned} &(\mathbf{P'}) \quad Min_{j>0} \{C_j = X_j + \xi_0 Y_j + Y_j^2\} \geq 0 \quad (6) \\ &s.t. \quad u_i \leq P_t/(4 \times P_v^2), \\ &\qquad \xi_0 - 1 \leq Y_j \leq \xi_0 \end{aligned}$$

However such system would not guarantee robust control in the long run since it does not take into account future states of ω_j and η_j to establish optimal weighting factors λ_j^* and μ_j^*. Predictive control is obtained by considering three time steps and pondering the impact of future states through a relevant sequence. The improved cost function becomes as follows

$$C'_j = \lambda_T^T \omega_T + \mu_T^T \eta_T + I^T \varepsilon_T \quad (7)$$

where I = [0, 1, …, 1] is a vector counting T+1 parameters, and $\varepsilon_T = [\varepsilon_{j+1}, \ldots, \varepsilon_{j+T}]$ is a relaxation vector representing the gap between expected demanded power and actual generated power given the resources at hand. Improvement of model robustness requires smoothing antagonist linear factors λ_j and μ_j. As research for optimum smoothing factors is not in the scope of this paper, the following trigonometric looped system is empirically chosen:

$$\begin{cases} \lambda_j = \frac{1}{2} + \frac{1}{2}\cos(\pi[1 - \xi_j]) \\ \mu_j = 1 - \lambda_j \end{cases} \quad (8)$$

To introduce stochasticity in predictive command, trajectory theory will be used to generate scenarios while implementing non necessary normal probabilistic constraints [9]. It can be shown that N_{traj} random variable realizations represent β_{traj} % of possible realization values in the system [10], and $\beta_{traj} \in$ [0,1] can be parametrized according to desired robustness degree. This means in turn that if operator can control N_{traj} then he will control β_{traj} % of anticipated random variable realizations. Expression of a (sufficient) lower bound for N_{traj} [11] in terms of various system parameters and constraints will be used here as a robust lower limit.

$$N_{traj} \geq \frac{2}{\varepsilon_{traj}} \ln \frac{1}{\beta_{traj}} + N_d \quad (9)$$

However application of scenario method generates very large dimension stochastic variables, as $w_d, w_I \in \mathbf{R}^{n \times N_{traj}}$ and linear programming problem has to be solved N_{traj} times.

IV. Optimal Control Objectives and Results

Two main objectives can be assigned for optimal control. Primary one is dealing with weekly command, determined from energetic potential $x(k)$ in tanks, energetic contribution of previous time period $w_I(k-1)$, the running week $t(k)$ and the demand of previous week $w_d(k-1)$. The second one, closely linked to the first, consists in determination of $\langle w \rangle_{av}$ to be weekly published by operator, in fact an upper bound in terms of x_0, w_0 and week t_0. From a data base refreshed every 15min on electricity network demand and on sunlight, primary objective is to generate the production level w_d with fixed robustness while respecting constraints on state and command variables. Robustness ε_{traj} (β_{traj}) is first determined over a one-week step. w_I and w_d are simulated and N_{traj} most restrictive trajectories are selected corresponding to largest week demand and most unfavorable sunlight.

To achieve robustness through scenario approach, the values ε_{traj} = 10% (β_{traj} = 10%) is considered. Chosen horizon $H(n)$ is here fixed to $H(n)$ = 3 for computational simplicity. Calculations must be performed every 15 minutes, so there are n = 672 iterations. Finally, one gets $N_{traj} \geq N_{traj}$ (ε_{traj},β_{traj}) = 286 from (8), which corresponds to a balanced compromise between computational burden and reliability of predictions. All the results are worked out and displayed with *SolarPlantTool.xlsm* software which generates all optimization loop and collects minimized costs obtained from linear programming (LP) tool. 90% robust control is achieved with the following determined samples: molten salt capacity over 1400000kWh is considered, expected demanded power is set between 15000 and 35000kW, and U_{max} = 55556kW.

The following histogram shows end values of x, at midnight. For 89% of the time, molten salt tank is more than half full, providing enough stock to satisfy expected demanded power until sunrise

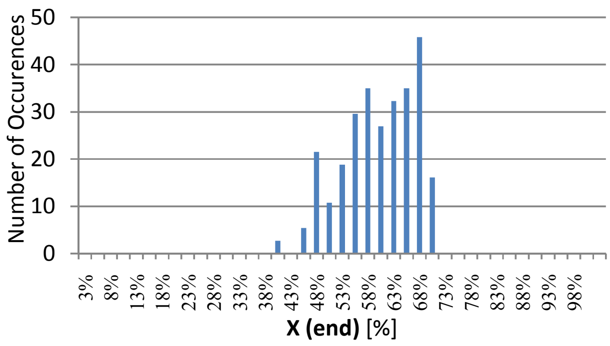

Fig. 5 Histogram of Simulated End Values of x vs. Occurence

For clarity purposes, one may now focus on {2 days – 2 nights} period. Fig. 6 below shows four significant events. First the red curve, representing generated power, matches $<w>_d$, represented by the blue surface, as there is enough power in the tank. Around time step #21, *ie* at 5:00am, the LP knows the tank is running low on power, u is therefore adapted to minimize c. The resulted penalty which only amounts to 3563kW is then mitigated over the following time step #22. At 11:45am, the tank is full, even if P_v matched with $<w>_d$ for the whole morning. P_v is therefore far greater than $<w>_d$ since LP must satisfy the stock constraint x_{max}.

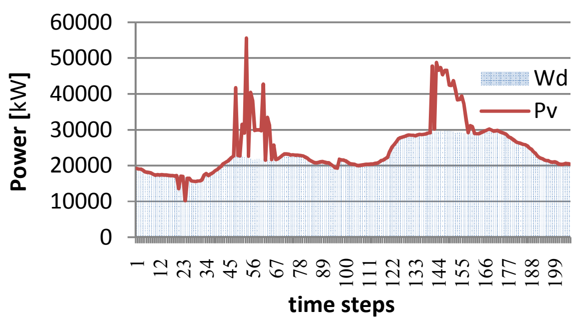

Fig. 6 Expected Demanded Electricity Production vs. Generated Electricity

Fig. 7 shows simulated levels of x corresponding to 286 trajectories of $<w>_d$. When x gets close to x_{max} the LP adapts u such that storage curve evolves in a saw tooth manner, allowing P_v to be as close to $<w>_d$ as possible in order to lower losses impacting potential future profits.

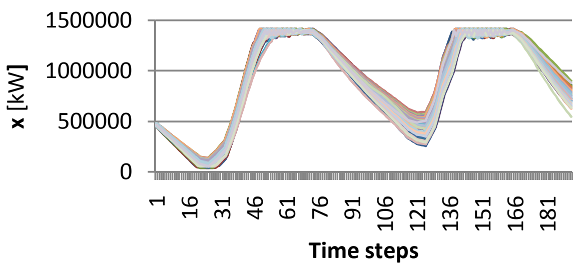

Fig. 7 Stored Resources

ε is plotted in Fig. 8 below, through [286x96]=26880 time steps. The following chart only takes into account non-zero values of ε, *i.d.* 2122 values which represents 7% of simulated ε values. The histogram shows that heavy penalties due to breach of demanded power constraint only appears .4% of the time which again proves actual control robustness.

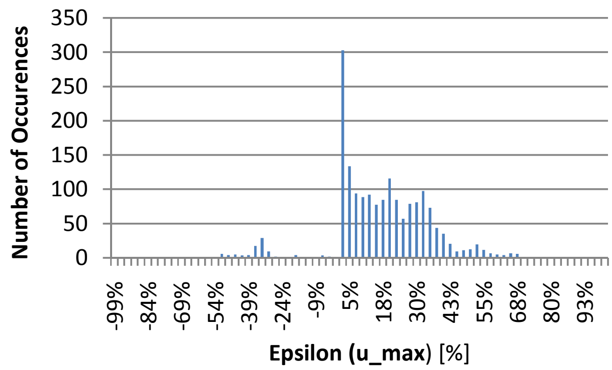

Fig. 8 Histogram of Simulated ε (< 0and > 0) over a Whole Week

V. Conclusion

The problem of determination of optimal electricity production from a solar molten plant system with storage has been analyzed. The main difficulty of best matching production with demand both of random nature has been framed in an optimal predictive control model which allows plant operator to satisfy all constraints while optimizing financial return. From this result, conditions for decision to undertake construction of such plant are easily set up according to technical operating costs and money market conditions. It is also shown that the problem is numerically tractable with modest open and free software available in a educational environment in which present study has been developed, with simulations implemented in VBA Excel. Linear Programming parts have been solved with Excel Solver. Precision of present model can be easily improved by using quantile regression to evaluate more accurate demand prediction, and by extension of observation horizon, both being paid by a more sophisticated computing hardware.

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