

# Probability and Instruction Effects in Syllogistic Conditional Reasoning

Olimpia Matarazzo, Ivana Baldassarre

**Abstract**—The main aim of this study was to examine whether people understand indicative conditionals on the basis of syntactic factors or on the basis of subjective conditional probability. The second aim was to investigate whether the conditional probability of  $q$  given  $p$  depends on the antecedent and consequent sizes or derives from inductive processes leading to establish a link of plausible co-occurrence between events semantically or experientially associated. These competing hypotheses have been tested through a  $3 \times 2 \times 2 \times 2$  mixed design involving the manipulation of four variables: type of instructions (“Consider the following statement to be true”, “Read the following statement” and condition with no conditional statement); antecedent size (high/low); consequent size (high/low); statement probability (high/low). The first variable was between-subjects, the others were within-subjects. The inferences investigated were Modus Ponens and Modus Tollens. Ninety undergraduates of the Second University of Naples, without any prior knowledge of logic or conditional reasoning, participated in this study.

Results suggest that people understand conditionals in a syntactic way rather than in a probabilistic way, even though the perception of the conditional probability of  $q$  given  $p$  is at least partially involved in the conditionals’ comprehension. They also showed that, in presence of a conditional syllogism, inferences are not affected by the antecedent or consequent sizes. From a theoretical point of view these findings suggest that it would be inappropriate to abandon the idea that conditionals are naturally understood in a syntactic way for the idea that they are understood in a probabilistic way.

**Keywords**—Conditionals, conditional probability, conditional syllogism, inferential task.

## I. INTRODUCTION

CONDITIONAL reasoning – based on the “if  $p$  then  $q$ ” statements - has been investigated in several research fields, e.g. logic, philosophy, psychology, and linguistic (for review, see [1]- [3]). Conditional statements occur frequently in daily discourses to express opinions and predictions, and to make inferences. Nevertheless, the way with which conditional clauses are understood in natural language is still unclear, while their logical and philosophical interpretation is controversial. According to propositional logic, conditionals are conceived as material implications ( $p \rightarrow q$ ), that is as statements formed by two clauses connected by an asymmetric relationship.

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The antecedent ( $p$ ) is a sufficient condition for the consequent ( $q$ ), which, in turn, is a necessary condition for the antecedent. Material implication has an *extensional* nature: its truth-value is determined by the truth values of its component propositions,  $p$  and  $q$ . Thus, conditionals are false only when  $p$  is true and  $q$  is false, and true otherwise, as the truth table of material implication clearly shows (see Table 1). Some logicians and philosophers [1], [4]-[6] have considered material implication as an unsatisfactory interpretation for ordinary conditionals used in natural discourse. This interpretation, in fact, leads to certain counterintuitive conclusions, the so-called *material implication paradoxes*. They can result from the acceptance of the truth of the conditional when the antecedent is false or by virtue of the truth of the consequent. In more formal terms, given *not- $p$* , it follows that “if  $p$ , then  $q$ ” (P1); given  $q$ , it follows that “if  $p$ , then  $q$ ” (P2). For example the following statement “If the moon is a star, the earth is a planet” is true by virtue of P1 (the moon is not a star) and by virtue of P2 (the earth is a planet). To handle the difficulties of the *material implication paradoxes*, Quine [4] has proposed a defective truth table, including a third truth value, indeterminate or irrelevant (I), together with the two values (true and false) of the propositional logic. According to this table, a conditional is irrelevant or indeterminate if the antecedent is false whereas it is true when antecedent and consequent are both true, and it is false when the antecedent is true and the consequent is false (see Table I).

Nevertheless, the defective implication table raises other types of problems: e. g., it is unable to explain why people tend to consider a conditional to be true when both antecedent and consequent are false. A third approach to conditionals is the suppositional point of the view, proposed by Ramsey [5], according to which a conditional statement of the form “if  $p$  then  $q$ ” can be interpreted as expressing the conditional probability that  $q$  occurs given  $p$ , i.e. as  $P(q|p)$ . In this way, conditional reasoning is no longer considered as a deductive reasoning form but, rather, as a probabilistic reasoning form.

In past decades, the two main theoretical approaches in the psychology of reasoning, mental logic (e.g. [7], [8]) and mental model (e.g.[9], [10]) theories, adopted an extensional conception of conditionals, although they differed about the syntactic vs. semantic features attributed to inferential processes.

TABLE I  
MATERIAL AND DEFECTIVE IMPLICATION TRUTH TABLES

p	q	Material implication	Defective implication
T	T	T	T
T	F	F	F
F	T	T	I
F	F	T	I

T= true; F= false; I= irrelevant or indeterminate

In past decades, the two main theoretical approaches in the psychology of reasoning, mental logic (e.g.[7], [8]) and mental model [9], [10] theories, adopted an extensional conception of conditionals, although they differed about the syntactic vs. semantic features attributed to inferential processes. More specifically, mental logic theory posited the existence of a kind of mental natural deduction system that provides some basic rules to make logic inferences, such as Modus Ponens (MP) and conditional proof schema. According to the mental logic theory, the meaning of a conditional statement of the form “if p then q” is naturally understood in the following terms: if “if p then q” and “p” are given, then “q” necessarily follows. On the contrary, Modus Tollens (MT) has been ruled out from basic inference schemas, because the conclusion drawn by the negation of the consequent requires more computational steps than the one drawn by the affirmation of the antecedent. The mental model theory assumed that people create a mental model about the state of the world described by a conditional statement, on the basis of its linguistic representation. When mental models of conditional sentences are fully represented (fleshed out), reasoners interpret conditionals as material implications. Both these theories, as well as the number of other theories of human reasoning, have advocated a certain amount of pragmatic, semantic or cognitive factors to explain the discrepancy between people’s performances in reasoning tasks and the norm of propositional logic (e. g. [11]-[14]). For example, the two forms of valid inferences that one can logically endorse from conditional syllogisms - the arguments based on a conditional statement as major premise and four minor (categorical) premises: p, q, not-p and not-q – are Modus Ponens and Modus Tollens (see Table 2).

The Denial of the antecedent and the Affirmation of the consequent are invalid arguments, because they do not allow to draw any unique conclusion. Nevertheless, people often

tend to endorse the so-called fallacies of conditional reasoning, whereas the frequency of MT is far lesser than that of MP (for review see Evans, [3], [15]). The large amount of studies that have investigated the factors affecting performance in inferential task have led to divergent conclusions, even though there is an increasingly widespread tendency to abandon the idea that the human mind possesses schemes of inference and, consequently, that human deductive reasoning is based on syntactic rules (e.g. [16]-[19]). On the contrary, a growing number of authors upholds the idea that human reasoning has a fundamentally probabilistic nature and that conditionals are understood as the conditional probability of the consequent given the antecedent [2], [20]-[27]. Thus, the probability of drawing MP conclusions in a inferential task depends on the degree of belief that q occurs given p. More specifically, as Over and Evans [28] stress, in an inferential task people would apply the Ramsey test, by comparing the frequency of pq and  $p \neg q$  cases. If the frequency of pq cases is greater than that of  $p \neg q$  cases, then conditional probability is high and MP has a high probability to be drawn. Vice-versa, if the frequency of  $p \neg q$  cases is greater than that of pq cases, then conditional probability is low and MP has a low probability to be drawn. Liu [22] and Liu, Lo, & Wu, [26] have proposed a thematic approach to conditional reasoning according to which reasoners judge the probability of a conditional premise on the basis of the semantic association between antecedent and consequent. Thus, the probability of drawing an inference from a conditional statement depends on how often p and q occur simultaneously in our daily life.

The probabilistic approach to conditionals proposed by Oaksford and colleagues [24]-[27] assumes that people tend to prefer highly probable conclusions and lower probable minor premises. This means that reasoners would endorse a conclusion more easily when the inferred proposition has a large set size and the categorical premise has a low set size. Indeed, according to these authors, the probability of the events designated by the antecedent and consequent of a conditional statement corresponds to the size of the classes to which the events belong: the higher is the class size, the more probable is the event occurrence. Consequently, since a large class of events should be more probably implied by a small class of events than the opposite, the most probable conclusions of conditional syllogisms should be those concerning large sets of events.

TABLE II  
CONDITIONAL SYLLOGISMS

Inference form	Major premise	Minor premise	Conclusion
Modus Ponens (MP)	If p then q	p	q
Affirmation of the consequent (AC)	If p then q	q	no certain conclusion
Denial of the antecedent (DA)	If p then q	not-p	no certain conclusion
Modus Tollens (MT)	If p then q	not-q	not-p

## II. OVERVIEW OF THE PRESENT STUDY

The main aim of this study was to test whether people understand indicative conditionals on the basis of syntactic factors or on the basis of subjective conditional probability. If conditionals were understood in a probabilistic way, then the conclusion of a conditional syllogism with the major premise of the form “if p then p” and the minor premise of the form “p” would depend on the degree of belief that the consequent (q) is implied by the antecedent (p). If degree of belief is high, the conclusion “q” is endorsed, otherwise no certain conclusion is drawn. If conditionals were understood in a syntactic way, then the conclusion of the above-presented conditional syllogism would always be “q”, because the conditional connective “if....then” would mean that the consequent is necessarily implied by the antecedent.

In order to pursue this aim, the probability of conditional statements has been manipulated within subjects and the experimental instructions have been manipulated across subjects. According to conditional probability criteria, the conditional statements presented in the study entailed either high or low probability degree that p implied q. The inferential tasks assigned to participants were introduced by three different types of instructions: in the first type, the statements acting as major premise of conditional syllogisms were headed by the clause “Consider the following statement to be true”; in the second type, they were headed by the clause “Read the following statement”; in the third type, only the conditional probability that the event “p” would entail the event “q” was requested to be assessed, without presenting any conditional statement. In our opinion, the first type of instructions should increase the endorsement of inferences, irrespective of the probability degree of conditionals, because of the injunction to consider them to be true; the third type should reflect the subjective probability degree that people attribute to an event, given the occurrence of another event. The second type of instructions should be the crucial factor to establish how conditionals are understood, because the lacking constraint to consider them to be true should allow to test the two competing hypotheses: a) whether the mere presentation of a conditional statement in a conditional syllogism as that described above is a sufficient condition for endorsing the “q” conclusion; b) whether the conclusion of such a syllogism depends on the degree of belief in the occurrence of “q” given “p” occurrence.

We have assumed Modus Ponens as the crucial inference to contrast the two hypotheses, because MP is judged to be the simplest inference by all theories of human reasoning. For example, according to mental logic theories, MP is a natural inference scheme of the human mind; according to mental model theory, MP is the immediate conclusion that one can draw from the representation of the state of affairs - e. g., p and q - involved by a conditional; according to probabilistic theories, the probability of MP derives directly from the probability of q given p:  $P(MP) = P(q|p)$ .

The other inference we have considered has been Modus Tollens, because from a logical point of view it is the contrapositive of MP and the two propositions are equivalent.

The second aim of this study was to test the hypothesis formulated by Oaksford *et al.* [24], [25], according to which when drawing inferences, people show a preference for high probability conclusions and for low probability minor premises. As we have seen, these authors equate the probability of an event with the size of the class to which the event belongs: thus, the high probability conclusions should correspond to the high size of the inferred proposition and the low probability minor premises should correspond to the low size of minor premises. So, the probability to endorse “q” inference from a conditional argument with “if p then q” and “p” premises is as higher as higher is q size and as lower is p size. In fact, Oaksford & Chater ([25], p. 223) posit that “high probability conclusions are associated with higher values of the relevant conditional probabilities”. Contrarily to this hypothesis, we have assumed that, with thematic conditionals, as those presented in this study, the conditional probability of q given p does not depend on the antecedent and consequent sizes but rather derives from inductive processes leading to establish a link of plausible co-occurrence between events experientially associated. To test these contrasting hypotheses, we varied the statement probability and the antecedent and consequent sizes separately.

## III. EXPERIMENT

### Method

#### A. Participants

Ninety undergraduates of an introductory course in Psychology of the Second University of Naples participated in this study as unpaid volunteers. Their age ranged from 18 to 35 ( $M = 23,14$ ;  $SD = 4,93$ ). None of the participants had any prior knowledge of logic or conditional reasoning.

#### B. Design

The  $3 \times 2 \times 2 \times 2$  mixed design involved the manipulation of four variables: type of instructions (“Consider the following statement to be true”, “Read the following statement” and condition with no conditional statement); antecedent size (high/low); consequent size (high/low); statement probability (high/low). The first variable was between-subjects, the others were within-subjects. The participants were randomly assigned to one of the three between-subjects experimental conditions.

#### C. Materials

The eight experimental statements resulting from the manipulation of the three ( $2 \times 2 \times 2$ ) within-subjects variables were selected through a pilot study in which participated eighty undergraduates of the Second University of Naples as unpaid volunteers (age range: 18 to 33;  $M = 22,16$ ;  $SD = 3,92$ ). Preliminarily, twenty-four conditional statements of the form “if p then q” had been built on the basis of the antecedent size, consequent size, and conditional probability -  $P(q|p)$  - variation. Then, forty participants assessed on 100-point scales (none-all) the size of the twenty-four antecedents and the twenty-four consequents, presented in random order; forty participants assessed on 100-point scales (not at all likely-extremely likely) the conditional probability of the twenty-four randomly presented consequents, given their antecedents, i.e.

$P(q|p)$ . For example, as regards the conditional statement "If a person lives in an apartment, then s/he gets electricity", the instructions assessing the antecedent and the consequent size were, respectively: "Out of every 100 people in your country, how many can live in an apartment?", "Out of every 100 people in your country, how many can get electricity?". The instructions to assess the conditional probability of  $q$  given  $p$  were: "Given that a person lives in an apartment, how likely does this imply that s/he gets electricity?"

The selected statements had been those meeting the following criteria: a) mean ratings  $> .75$  for high conditions (high size of the antecedent; high size of the consequent; high statement probability (i.e. high probability of  $q$  given  $p$ ); b) mean ratings  $< .30$  for low conditions (low size of the antecedent; low size of the consequent; low statement probability (i.e. low probability of  $q$  given  $p$ ). The eight statements used for the experiment were the following:

1. If a person lives in an apartment, then s/he gets electricity (High size of the antecedent; High size of the consequent; High statement probability)
2. If a person watches TV, then s/he has a pen (High size of the antecedent; High size of the consequent; Low statement probability)
3. If a Christmas tree is decorated, then it's Christmas time (High size of the antecedent; Low size of the consequent; High statement probability)
4. If a boy/girl sends a text message, then s/he gets bad marks in Physical Education (High size of the antecedent; Low size of the consequent; Low statement probability)
5. If a person is a Formula one pilot, then s/he drives a car into town (Low size of the antecedent; High size of the consequent; High statement probability)
6. If a person drinks warm water, then s/he eats pasta (Low size of the antecedent; High size of the consequent; Low statement probability)
7. If a person plays tennis well, then s/he has a tennis racket (Low size of the antecedent; Low size of the consequent; High statement probability)
8. If a person is Jewish, then s/he is a toreador (Low size of the antecedent; Low size of the consequent; Low statement probability).

The antecedent and consequent size and the statement probability of the eight selected conditionals were re-tested after the main experimental task in order to check if the experiment participants' evaluation conformed to that of the pilot study participants.

The materials consisted of two booklets. One contained the inference task; the other the evaluation task.

#### *D. Procedure*

Participants were tested in group session. In order to avoid mutual influence, they sat far from each other and were requested not to communicate with one another. First, they received the booklet with the inference task and then, after completing it, they received the booklet with the evaluation task. The two tasks were separated in order to avoid any possible influence of the statement probability and the

antecedent and consequent size evaluation on the inference task. In the first booklet, apart from the initial page containing the general instructions, each of the eight remaining pages contained two inferential tasks. In the two conditions in which the conditional statement was presented, it was written at the top of page and was preceded by one of two types of instructions: "Consider the following statement to be true", or "Read the following statement". Then, participants were asked to examine the following cases and choose, for each of them, the most appropriate conclusion. The cases presented were Modus Ponens (MP) and Modus Tollens (MT). For MP, the three proposed conclusions were the affirmation of the consequent, the negation of the consequent, and "no conclusion is certain". For MT, they were the negation of the antecedent, the affirmation of the antecedent, and "no conclusion is certain". In the condition with no conditional statement, participants were directly asked to examine the following cases and choose, for each of them, the most appropriate conclusion, in the same way as described above.

An example of an inference task in the "read the following statement" condition was the following.

- Read the following statement: 'If a person lives in an apartment, then s/he gets electricity'.

Bearing this statement in mind, examine the following cases and, for each of them, choose the conclusion that, in your opinion, is the most appropriate.

(MP) Suppose that a person lives in an apartment: what should one conclude?

1. s/he gets electricity; 2. s/he does not get electricity; 3. no conclusion is certain

(MT) Suppose that a person does not get electricity: what should one conclude?

1. s/he lives in an apartment; 2. s/he does not live in an apartment; 3. no conclusion is certain.

The procedure for the evaluation task was the same as the one used in the pilot study, apart from the fact that the experiment participants evaluated not only the probability of the consequent, given the antecedent – i.e.  $P(q|p)$  – but also the probability of the negation of the antecedent, given the negation of the consequent, i.e.  $P(\neg p|\neg q)$ . This task was included in order to check if participants attributed the same degree of probability to a conditional statement presented in the form "if  $p$  then  $q$ " and in its contrapositive – and logically equivalent – form "if not- $q$  then not- $p$ ". Since both MP and MT inferences had been required, it was necessary to assess the conditional probability of contrapositive statements in order to employ this information in statistical analyses concerning MT conclusions. On the contrary, the negated antecedent and consequent sizes were not assessed because they corresponded to the reversal of their original size in the affirmative statement.

In both tasks, each participant was presented with the items in a different random order.

#### *Results*

##### *A. Evaluation task*

The mean ratings of antecedent size, consequent size, and conditional probabilities –  $P(q|p)$  and  $P(\neg p|\neg q)$  – were reported

in Table III. Four one-way within-subjects ANOVAs were performed on each variable in order to check if the participants' evaluations conformed to the experimental manipulation. Results, reported in Table III, showed that the difference between high conditions - high size of the antecedent; high size of the consequent; high  $P(q|p)$ ; high  $P(-p|-q)$  - vs. low conditions - low size of the antecedent; low size of the consequent; low  $P(q|p)$ ; low  $P(-p|-q)$  - was always highly significant. These findings replicated those of the pilot study, apart from the conditional probability of not-p given not-q, which had not been tested in the preliminary study.

statement probability x types of instructions). As regards the main effects, reasoners drew more MP inferences in the "consider the following statement to be true" ( $M = 0.93$ ) and in the "read the following statement" ( $M = 0.91$ ) conditions than in the "with no conditional statement" ( $M = 0.38$ ) condition. They drew more MP inferences in the high ( $M = 0.88$ ) than in the low ( $M = 0.61$ ) statement probability condition, and in high ( $M = 0.76$ ) than in low ( $M = 0.72$ ) consequent size condition. The interaction effects were interpreted, by means of the simple effects analyses, in the following ways:

TABLE III  
MEAN RATINGS (WITH STANDARD DEVIATIONS) AND ANOVAS RESULTS OF ANTECEDENT SIZE, CONSEQUENT SIZE, AND CONDITIONAL PROBABILITIES.

	Antecedent size		Consequent size		P (q p)		P(-p -q)	
	High	Low	High	Low	High	Low	High	Low
Mean	76.62	29.94	76.62	27.76	84.29	27.22	64.12	23.52
S.D.	12.34	12.97	11.9	12.75	14.95	24.14	16.38	21.49
F	544.347		602.087		191.565		123.425	
d. f.	1,89		1,89		1,89		1,89	
p <	0.001		0.001		0.001		0.001	

Note:  $P(q|p)$  = conditional probability of q given p;  $P(-p|-q)$  = conditional probability of not-p given not-q.

### B. Inference task

For each statement, almost all the answers fell into the two categories: "correct inference" (affirmation of the consequent for MP and negation of the antecedent for MT) or "no conclusion is certain". The third "illogical" option (negation of the consequent for MP and affirmation of the antecedent for MT) was almost never chosen. Percentage frequencies of MP and MT inferences are reported in Tables IV and V. For each type of inference, a  $3 \times 2 \times 2 \times 2$  mixed ANOVA was performed, with types of instructions (read the following statement, consider the following statement to be true, condition with no conditional statement) as between-subjects factor, and antecedent size (high/low), consequent size (high/low), statement probability (high/low) as within-subjects factors. For MT inferences, the negated consequent and the negated antecedent sizes correspond, respectively, to the reversed sizes of consequent and antecedent of the original statements. For example, given a low antecedent and high consequent size original statement, in MT form the size of the negated consequent becomes low and that of the negated antecedent becomes high.

ANOVAs results are shown in Tables VI and VII.

As regards the MP analysis, results showed three main effects (type of instructions, statement probability, and consequent size), three two-way interactions (statement probability x type of instructions, antecedent size x statement probability, and antecedent size x consequent size), two three-way interaction (antecedent size x consequent size x types of Instructions; antecedent size x consequent size x statement probability), and one four-way interaction (antecedent size x consequent size x

#### 1. Statement probability x Types of instructions: the effect of

the statement probability was far more robust in the "with no conditional statement" condition ( $M = 0.68$  vs.  $0.07$ ;  $p < .001$ ) than in the "read the following statement" ( $M = 0.96$  vs.  $0.87$ ;  $p < .05$ ) or in the "consider the following statement to be true" ( $M = 0.98$  vs.  $0.88$ ;  $p < .05$ ) conditions.

2. Antecedent size x Statement probability: in the high statement probability condition, participants drew more inferences with high ( $M = 0.91$ ) than low ( $M = 0.84$ ) antecedent size, whereas in low statement probability this variable did not affect the inferences drawn.

3. Antecedent size x Consequent size: in the low antecedent size condition, participants drew more inferences with high ( $0.80$ ) than low ( $M = 0.66$ ) consequent size, whereas in the high antecedent size condition no significant difference was found between the inferences drawn in high and low consequent size conditions.

4. Antecedent size x Consequent size x Types of instructions: in the "with no conditional statement" and high antecedent size conditions, reasoners drew more inferences with low consequent size ( $0.47$ ) than with high consequent size ( $0.32$ ), whereas the opposite occurred in the "with no conditional statement" and low antecedent size conditions ( $0.22$  vs.  $0.50$ ). In the "consider the following statement to be true" and "read the following statement" conditions, antecedent and consequent sizes did not affect results.

5. Antecedent size x Consequent size x Statement probability: in the high statement probability condition, when presented with high antecedent size, participants drew more inferences with low ( $0.97$ ) than high ( $0.84$ ) consequent size; the opposite

occurred with the low antecedent size condition (0.97 vs. 0.72). In the low statement probability condition, the true” (M = 0.61) and in “read the following statement” (M = 0.48) conditions than in “with no conditional statement” (M =

TABLE IV  
MEAN PERCENTAGE FREQUENCIES AND STANDARD DEVIATIONS OF MP INFERENCES

Instruction condition	HHH		HHL		HLH		HLL		LHH		LHL		LLH		LLL	
	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
Read the following statement	0.97	0.18	0.87	0.35	0.97	0.18	0.8	0.41	1	0	0.9	0.31	0.87	0.35	0.9	0.31
Consider the following statement to be true	1	0	0.9	0.31	1	0	0.9	0.31	1	0	0.9	0.31	0.93	0.25	0.8	0.38
Without conditional statement	0.57	0.5	0.7	0.25	0.9	0.31	0.3	0.18	0.9	0.31	0.1	0.31	0.37	0.49	0.07	0.25

Note. The eight statements are represented by three letters: the first indicates the antecedent size (High/Low), the second the consequent size (High/Low) and the third the statement probability (High/Low).

TABLE V  
MEAN PERCENTAGE FREQUENCIES AND STANDARD DEVIATIONS OF MT INFERENCES

Instruction condition	HHH		HHL		HLH		HLL		LHH		LHL		LLH		LLL	
	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
Read the following statement	0.43	0.5	0.5	0.51	0.5	0.51	0.47	0.51	0.63	0.49	0.43	0.5	0.37	0.49	0.47	0.51
Consider the following statement to be true	0.53	0.51	0.6	0.49	0.77	0.43	0.6	0.49	0.57	0.5	0.67	0.48	0.53	0.51	0.63	0.49
Without conditional statement	0.1	0.31	0	0	0.57	0.51	0.03	0.18	0.7	0.47	0	0	0.23	0.43	0	0

Note. The eight statements are represented by three letters: the first indicates the negated consequent size (High/Low), the second the negated antecedent size (High/Low) and the third the statement probability (High/Low). The negated consequent and the negated antecedent sizes correspond, respectively, to the reversed sizes of consequent and antecedent of the original statements.

antecedent and consequent sizes did not affect results.

0.20) condition. They drew more MT inferences in high (M = 0.49)

#### 6. Antecedent size x Consequent size x Statement probability

Types of instructions: the above-described effects were found only in the “with no conditional statement” condition. In the “consider the following statement to be true” and “read the following statement” conditions there was no significant difference between results.

With reference to the MT analysis, results showed two main effects (type of instructions and statement probability), two two-way interactions (statement probability x type of instructions, antecedent size x consequent size), three three-way interactions (antecedent size x statement probability x types of instructions, antecedent size x consequent size x types of instructions, antecedent size x consequent size x statement probability), and one four-way interaction (antecedent size x consequent size x statement probability x types of instructions). As regards the main effects, reasoners drew more MT inferences in “consider the following statement to be

than in low (M = 0.37) statement probability condition. The interaction effects were interpreted, by means of the simple effects analyses, in the following ways:

1. Statement probability x Types of instructions: in “with no conditional statement” condition, participants drew more inferences with high (M = 0.40) than low (M=0.01) statement probability. In the “consider the following statement to be true” and “read the following statement” conditions, the statement probability did not affect results.

2. Antecedent size x Consequent size: in the high antecedent size condition, participants drew more inferences with low (M = 0.49) than high (M = 0.36) consequent size, whereas in the low antecedent size condition, participants drew more

inferences with high ( $M = 0.50$ ) than low ( $M = 0.37$ ) consequent size.

3. Antecedent size x Statement probability x Types of instructions: in the “with no conditional statement” condition and high statement probability, participants drew more

5. Antecedent size x Consequent size x Statement probability: in the high statement probability condition, when presented with high antecedent size, participants drew more inferences with low ( $M = 0.61$ ) than high ( $M = 0.35$ ) consequent size; the opposite occurred in the low antecedent size condition ( $M =$

TABLE VI  
MP INFERENCES: MIXED ANOVA RESULTS

	F	d. f.	P<	$\eta^2$
Types of Instructions	127	2,87	0.001	0.75
Statement Probability	117.16	1,87	0.001	0.57
Consequent size	6.14	1,87	0.01	0.07
Statement Probability x Types of Instructions	48.65	2,87	0.001	0.53
Antecedent size x Statement Probability	5.28	1,87	0.05	0.06
Antecedent size x Consequent size	25.29	1,87	0.001	0.23
Antecedent size x Consequent size x Types of Instructions	11.77	2,87	0.001	0.21
Antecedent size x Consequent size x Statement Probability	23.48	1,87	0.001	0.21
Antecedent size x Consequent size x Statement Probability x Types of Instructions	11.71	2,87	0.001	0.21

TABLE VII  
MT INFERENCES: MIXED ANOVA RESULTS

	F	d. f.	P<	$\eta^2$
Types of Instructions	13.91	2,87	0.001	0.20
Statement Probability	22.74	1,87	0.001	0.21
Statement Probability x Types of Instructions	24.45	2,87	0.001	0.36
Antecedent size x Consequent size	23.48	1,87	0.001	0.21
Antecedent size x Statement Probability x Types of Instructions	3.57	2,87	0.05	0.08
Antecedent size x Consequent size x Types of Instructions	4.67	2,87	0.01	0.1
Antecedent size x Consequent size x Statement Probability	21.93	1,87	0.001	0.2
Antecedent size x Consequent size x Statement Probability x Types of Instructions	3.37	2,87	0.05	0.07

inferences with low ( $M = 0.47$ ) than high ( $M = 0.00$ ) antecedent size. In the “consider the following statement to be

true” and “read the following statement” conditions, antecedent size and statement probability did not affect results.

4. Antecedent size x Consequent size x Types of instructions: the above-depicted effects occurred only in the “with no conditional statement” condition (in the high antecedent size condition,  $M = 0.05$  with high consequent size vs.  $0.30$  with low consequent size; in the low antecedent size condition,  $M = 0.35$  with high consequent size vs.  $0.18$  with low consequent size). In the “consider the following statement to be true” and “read the following statement” conditions there was no significant difference between results as a function of the antecedent and consequent sizes.

$0.63$  vs.  $0.39$ ). In low statement probability condition, antecedent and consequent sizes did not affect results.

6. Antecedent size x Consequent size x Statement probability x Types of instructions: the above-described effects were found only in the “with no conditional statement” condition. In the “consider the following statement to be true” and “read the following statement” conditions there was no significant difference between results.

#### IV. CONCLUSION

In this study we have tested: a) whether people understand indicative conditionals on the basis of syntactic factors or on the basis of subjective conditional probability; b) whether the conditional probability of a conditional statement depends on the antecedent and consequent sizes or derives from semantic/experiential association between antecedent and

consequent. The inferences through which these hypotheses had been tested were MP and MT.

The manipulation of statements' probability (both in MP and in MT form), antecedent and consequent sizes was successful: the difference between high and low conditions was greatly significant. Thus, the competing hypotheses have been examined in an adequate way.

Results showed a robust effect of the type of instructions on conditional reasoning, with a polarization between statement/no-statement conditions: in presence of a conditional statement, participants showed the same response pattern. Irrespective of the request to merely read conditional or consider it to be true, they drew a great deal of MP inferences and a fairly high number of MT inferences. In absence of conditional statement, the amount of inference tended to decrease dramatically and to depend on the conditional probabilities (high vs. low) of  $q$  given  $p$  (for MP inferences) and of not- $p$  given not- $q$  (for MT inferences). This finding suggests that people tend to spontaneously understand conditionals in a syntactic way, i.e. as the necessary implication of  $q$ , given  $p$ . Nevertheless, it is worthy to note that when drawing MP inferences, participants were at least partially sensible to conditional probability of  $q$  given  $p$ , as the main effect of the statement probability revealed, even through this effect was far less robust in the two statement conditions than in the condition with no statement. On the contrary, when drawing MT inferences, participants in statement conditions were not affected by the statement probability and this result further corroborates the conjecture of the syntactic comprehension of conditionals.

The widely documented difficulty to endorse MT inferences, compared to frequency of MP inferences (for review, see Evans [3]), is confirmed by this study. Nevertheless, the finding that MT inferences were less frequent than MP inferences even in condition with no statement and high conditional probability of not- $p$  given not- $q$  suggests that such a difficulty is not due to the difficult to draw a backward inference in a conditional argument - from the negation of the consequent to the negation of the antecedent - but rather to the difficulty of linguistic negation processing. This hypothesis conforms to the finding that in the evaluation task the values of the conditional probability of not- $p$  given not- $q$  are lower than those of the conditional probability of  $q$  given  $p$ .

As regards the other contrasting hypotheses tested in this study - i.e. whether the conditional probability of a conditional statement depends on the antecedent and consequent sizes or derives from pragmatic association between antecedent and consequent - results showed that in statement conditions the antecedent and consequent sizes did not affect MP and MT inferences. For both these inferences, only in the condition with no statement and high  $P(q|p)$  or high  $P(\neg p|\neg q)$ , interaction effects of antecedent and consequent sizes were found: in high antecedent size condition, inferences increased with low consequent size; in low antecedent size condition, inferences increased with high consequent size. It is worthy to repeat that, for MT form, the "antecedent" corresponds to the negated consequent and the "consequent" corresponds to the negated antecedent of the original conditional statement. These findings seem to clearly show that the statement components'

size did not affect the frequency of the endorsed conclusions in the conditional reasoning task. Only when requested to draw a conclusion about the (non-)occurrence of an event on the basis of the (non-)occurrence of another event - as well as in our experimental no-statement condition - people seem to make use of a heuristic processing strategy, according to which the events of different sizes tend to be seen as more frequently co-implicated than the events of equal size. Note that in our study such a strategy has been employed only with the events involved in highly probable statements, i.e. with events that were judged to correspond to high  $P(q|p)$  or high  $P(\neg p|\neg q)$ . When the events did not correspond to these criteria, very few conclusions were drawn from their co-presentation: people have simply seen them as unrelated.

In sum, with reference to the main aim of this study, our results suggest that people understand conditionals in a syntactic way rather than in a probabilistic way, even though the perception of the conditional probability of  $q$  given  $p$  is at least partially involved in the conditionals' comprehension. As regards the other competing hypotheses tested in this study, the assumptions of Oaksford *et al.* [24]-[27], according to which people prefer high probability conclusions - which, in turn, are associated with higher values of the relevant conditional probabilities - have been disconfirmed, as well as their supposed equivalence between high probability conclusions and high size of the consequent (or of the negated antecedent, in MT form). In presence of a conditional syllogism, MP and MT inferences are not affected by the antecedent or consequent sizes. Thus, also Oaksford and colleagues' assumption about people's preference for low probable minor premises is not corroborated by our results.

In conditions with no conditional statement, the frequency of the conclusions is affected by the subjective perception of the conditional probability linking the events but the probability to endorse a conclusion does not correspond to the size of the inferred proposition. When events are perceived as unrelated, almost no conclusion is drawn from their co-presentation. On the contrary, when they are perceived as to be connected by a conditional probability relationship, their respective size seems to elicit the above mentioned heuristic processing strategy, according to which the events of different sizes tend to be seen as more frequently co-implicated than the events with equal size. Such a heuristics has nothing to do with the preference for the highly probable conclusions and low probable minor premises advocated by the probabilistic approach to conditional reasoning of Oaksford *et al.* [24]-[27].

On the whole, our results suggest that it would be inappropriate to abandon the idea that conditionals are naturally understood in a syntactic way for the idea that they are understood in a probabilistic way. Our findings, rather, suggest that the conditional probability is an additional manner of conceiving conditionals, after syntactic mode: it is reasonable to assume that the human mind does not work by using only one type of processes at a time but rather deductive and inductive processes coexist and interfere in similar or in the same tasks.



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