Preemptive Possibilistic Linear Programming: Application to Aggregate Production Planning

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Abstract—This research proposes a Preemptive Possibilistic Linear Programming (PPLP) approach for solving multiobjective Aggregate Production Planning (APP) problem with interval demand and imprecise unit price and related operating costs. The proposed approach attempts to maximize profit and minimize changes of workforce. It transforms the total profit objective that has imprecise information to three crisp objective functions, which are maximizing the most possible value of profit, minimizing the risk of obtaining the lower profit and maximizing the opportunity of obtaining the higher profit. The change of workforce level objective is also converted. Then, the problem is solved according to objective priorities. It is easier than simultaneously solve the multiobjective problem as performed in existing approach. Possible range of interval demand is also used to increase flexibility of obtaining the better production plan. A practical application of an electronic company is illustrated to show the effectiveness of the proposed model.

Keywords—Aggregate production planning, Fuzzy sets theory, Possibilistic linear programming, Preemptive priority

I. INTRODUCTION

AGGREGATE Production Planning (APP) is a medium range capacity planning that typically encompasses a time horizon from 3 to 18 months. A production planner must make decisions regarding output rates, employment levels, inventory levels, backordering level as well as subcontracting to optimize the production plan. Among the numerous methods capable of developing mathematical optimization models include APP problems [1]-[4], Linear Programming (LP) is a conventionally used technique. However, LP models assume that all required data input can be uniquely determined. These models cannot be applied to real APP problems since a Decision Maker (DM) frequently has insufficient information on how to specify demand, related operation costs coefficients and unit price by crisp real numbers. These data are typically fuzzy in nature. Moreover, LP can deal with only a single objective problem. In general, an APP problem involves multiple objectives, which are often conflicting in nature [4], such as minimize cost, maximize profit, minimize inventory level or minimize change of workforce level etc.

The input data or parameters in real-world, such as demands, resources, costs and unit price are often imprecise or

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fuzzy because some information is incomplete or unobtainable. Conventional mathematical programming cannot solve all problems those have imprecision. In dealing with imprecise data, some researchers may apply stochastic programming to solve. However, the main problem is the lack of computational efficiency and inflexible probabilistic doctrines in which the real imprecise meaning of the DM might be impossible to model [5]. In 1976, Zimmermann first introduced fuzzy set theory into conventional LP problems [6]. That study considered LP problems with a fuzzy objective and constraints, which multiple objectives problems can be solved. Following the fuzzy decision-making method proposed by [7],[8] many of Fuzzy Linear Programming (FLP) models have been developed for solving industrial problems [9]-[11]. Moreover, Zadeh (1978) presented the prominence of the theory of possibility, which is related to the theory of fuzzy sets by defining the concept of a possibility distribution as a fuzzy restriction, which acts as an elastic constraint on the values that can be assigned to a variable [5]. He demonstrated the significance of the theory of possibility stems from the fact that much of the information on which human decisions is based on is possibilistic rather than probabilistic in nature [12]. In 1992, Lai and Hwang proposed a new approach to some of Possibilistic Linear Programming (PLP) problems. Wang and Lai (2005) proposed a PLP model for solving a single objective APP problem with imprecise demand, parameters and capacity [5]. The fuzzy objective is converted to a Multiple Objective Linear Programming (MOLP) model using the method of Lai and Hwang [15]. Afterward, possibilistic optimization methods have been applied in some of practical applications [5],[13]-[20].FLP is based on the subjective preferred concept for establishing membership functions with fuzzy data, while the PLP is based on the objective degree of event occurrence required to obtain possibilistic distributions with imprecise data. FLP techniques may not be applicable for PLP [5]. PLP provides computational efficiency and flexibility. It also supports possibilistic decision making in an uncertain environment [5],[16],[18]. PLP approach simultaneously minimizes the most possible value of the imprecise total costs, maximizes the possibility of obtaining lower total costs, and minimizes the risk of obtaining higher total costs. Imprecise forecast demands are converted to crisp demands by adopting weight average method. Other studies of PLP problems also use this strategy to solve their applications [5],[16]-[18]. Another way to deal with uncertain information is using of interval numbers [21]-[22]. Recently, Li and Huang (2010) proposed Interval based Possibilistic Programming (IPP) by solving sub-problems to find interval solutions because many of uncertain parameters are expressed as fuzzy sets, interactions among these uncertainties may lead to serious complexities [21].Simultaneously solve MOLP by adjusting their membership functions are difficult and take time because

each objective has different range or scale to adjust. Moreover, one objective may extremely important than the others. Especially in an APP problem, total profit is extremely important than the others. So, this objective should be set as the first priority. For forecast demand, DM may feel uncomfortable to estimate the demand in each period because opportunity loss of sales and profit will occur if demand is under-estimated or holding cost will increase if demand is over-estimated. Weights which are used to transform fuzzy demand in PLP model may not appropriate because after multiplying weights with their demands, the crisp demand is obtained. If the DM can estimate the possible range of forecast demand, giving this possible range to find the optimal solution of a production plan is better than giving a crisp demand because reducing or increasing a few units of demand may extremely increase or decrease production cost due to change of the number of production line. So, this work proposes a multi-objective APP problem using a Preemptive Possibilistic Linear Programming (PPLP) approach, which can effectively find the compromise solution. The proposed method solves the problem according to objective priorities. Parameters related to the objective coefficients are imprecise. These data are model by triangular possibilistic distributions. Demand in each period is considered as an interval demand from the possible range of demand. Two main objective functions are considered in the model. They are to maximize profit and to minimize change of workforce level. The structure of the paper is as follows: Section 2 describes the notations and formulation of an APP problem. Section 3 proposes the PPLP model and the solution procedure. Section 4 illustrates the APP problem of a case study to show the effectiveness of the proposed model. Finally, section 4 delineates conclusion and scope of future works.

II. NOTATIONS AND FORMULATION

In order to describe the multi-product multi-period APP problem mathematically, notations below are introduced. Assume that a company manufactures N types of products to satisfy the possible market demand over a planning horizon T. Multiple objective functions with imprecise information and possible interval demand is determined in this research.

Indices:

i number of product types, $i = 1,2,…,N$.

t number of periods (month) in the planning horizon,
$t = 1, 2,…,T$.

Parameters:

$\bar{D}_{it}$ upper limit of forecast demand of product i in period t, (units).

$\underline{D}_{it}$ lower limit of forecast demand of product i in period t, (units).

$\tilde{p}_i$ unit price of product i, ($/unit).

$\tilde{r}_i$ regular production cost per unit of product i, ($/unit).

$\tilde{o}_i$ overtime production cost per unit of product i, ($/unit).

$\tilde{l}_t$ laying off cost per worker in period t, ($/worker).

$\tilde{h}_t$ hiring cost per worker in period t, ($/worker).

$\tilde{s}$ holding cost per unit, ($/unit).

$\tilde{b}_i$ backordering cost per unit of product i in period t, ($/unit).

α number of operators per production line, (man/line).

d_i number of days in period t, (days).

a_i processing time per unit of product i, (hrs/unit).

n_i production capacity per production line per period of product i, (units/ line/period)

δ regular working hour per worker per day, (hrs/man-day)

θ overtime working hour per worker per day, (hrs/man-day)

A_t minimum production quantities (regular production and overtime production) in each period t, (units).

$I_{t\max}$ maximum inventory level in period t, (units).

$W_{t\max}$ maximum labor level in period t, (man).

Decision variables:

D_{it} satisfied demand in an interval [$\bar{D}_{it}$, $\underline{D}_{it}$] of product i in period t, (units).

X_{it} regular time production of product i in period t, (units).

Y_{it} overtime production of product i in period t, (units).

B_{it} backordering level of product i in period t, (units).

I_{it} inventory level of product i in period t, (units).

W_t number of workers in period t, (man).

H_t hired worker in period t, (man).

L_t laid off worker in period t, (man).

N_{it} number of production lines of product i in period t, (lines).

~ denotes uncertain information.

A. Objective functions

There are two objective functions; to maximize profit (O_1) and to minimize the changes of workforce level (O_2).

The first objective function is to maximize the total profit

Profit is the main objective function for every company. It comes from revenue minus costs. Revenue generates from quantities of customer demands minus backorder parts multiply by unit price of all products. In this model, backordering is unacceptable because most of customers will not wait for delay parts. Costs compose of total production cost for regular and overtime productions, cost of changing workforce level, inventory cost and penalty of backordering cost. Fuzzy profit function is represented by

$$\text{Max } \tilde{O}_1 = \sum_{i=1}^{N}\sum_{t=1}^{T}\tilde{p}_i*(D_{it}-B_{it})-\sum_{i=1}^{N}\sum_{t=1}^{T}(\tilde{r}_i X_{it}+\tilde{o}_i Y_{it})$$
$$-\sum_{t=1}^{T}(\tilde{l}_t L_t +\tilde{h}_t H_t)-\sum_{i=1}^{N}\sum_{t=1}^{T}(\tilde{s}I_{it})-\sum_{i=1}^{N}\sum_{t=1}^{T}(\tilde{b}_i B_{it}).$$

(1)

The related unit price ($\tilde{p}_i$) and cost coefficients ($\tilde{r}_i, \tilde{o}_i, \tilde{l}_t, \tilde{h}_t, \tilde{s}, \tilde{b}_i$) are imprecise. Triangular possibility distributions are used to represent these data. These are discussed in the next section.

The second objective function is to minimize the changes

of workforce level

Changing workforce level effects morale and stability for workers. This level should be minimized. The total changes of workforce level are the numbers of laid off and hired workers. This objective can be shown as

$$\text{Min } O_2 = \sum_{t=1}^{T} (L_t + H_t). \qquad (2)$$

B. Constraints

- The labor level constraints:

For each period, the following constraints are applied:

$$W_t = W_{t-1} + H_t - L_t, \qquad \forall t . \qquad (3)$$

$$W_t \leq W_{t\max} , \qquad \forall t. \qquad (4)$$

$$\sum_{i=1}^{N} \alpha * N_{it} = d_t * W_t, \qquad \forall t. \qquad (5)$$

The workforce level in each period (W_t) should equal the workforce level in the previous period (W_{t-1}) plus the new hired workers (H_t) minus the laid off workers (L_t) as shown in (3). The workforce level in each period should not be greater than the maximum available workforce level as shown in (4). Each production line needs a specific number of workers. Number of workers in all production lines in each period should be equal to the available workforce level in period t.

- The inventory level constraint:

The summation of inventory level of all product types should not greater than the maximum available inventory capacity in each period.

$$\sum_{i=1}^{N} I_{it} \leq I_{t\max}, \qquad \forall t. \qquad (6)$$

- The production constraints:

$$X_{it} + Y_{it} + I_{it-1} + B_{it} - I_{it} = D_{it}, \qquad \forall i \forall t. \qquad (7)$$

$$\underline{D}_{it} \leq D_{it} \leq \overline{D}_{it}, \qquad \forall i \forall t. \qquad (8)$$

$$\sum_{i=1}^{N} X_{it} + \sum_{i=1}^{N} Y_{it} \geq A_t, \qquad \forall t . \qquad (9)$$

Regular production, overtime production, inventory level in previous period, backordered units of the current period minus inventory level of the current period of product i equal to demand of the current period of product i as shown in (7). Conventionally, crisp demand is assumed in the APP model. In this proposed model, a possible interval demand is used due to uncertainty in demand. It can increase flexibility in production because the appropriate level of demand can be selected from the possible range of demand for production. Equation (8) shown that satisfied demand is in an interval of a possible interval demand range. The summation of regular production and overtime production quantities for all products in each period should not lower than the minimum requirement in each period as shown in (9).

- The production capacity constraints:

$$\sum_{i=1}^{N} a_i X_{it} \leq \delta \, d_t W_t, \qquad \forall t. \qquad (10)$$

$$\sum_{i=1}^{N} a_i Y_{it} \leq \theta \, d_t W_t, \qquad \forall t. \qquad (11)$$

$$\frac{\delta}{\delta + \theta} n_i N_{it} = X_{it}, \qquad \forall i \forall t. \qquad (12)$$

$$\frac{\theta}{\delta + \theta} n_i N_{it} = Y_{it}, \qquad \forall i \forall t. \qquad (13)$$

The regular and overtime production hours should not be greater than the available labor hour in each period as shown in (10),(11). Equations (12) and (13) represent the number of product i produce in period t for regular time and overtime.

III. MODEL DEVELOPMENT

A. Modeling the imprecise data with triangular possibility distribution

This work assumes that a triangular possibility distribution can be stated as the degree of occurrence of an event with imprecise data [5]. Fig. 1 presents the triangular possibility distributions of imprecise unit price, $\tilde{p}_i = (p_i^p, p_i^m, p_i^o)$ and cost coefficients, $\tilde{A}_i = (A_i^p, A_i^m, A_i^o)$. These kinds of imprecise information exist in the first objective function ($\tilde{O}_1$). In practice, a DM can make triangular possibility distributions of $\tilde{p}_i$ and $\tilde{A}_i$ based on the three prominent data, as follows.

The most pessimistic values (p_i^p, A_i^p) that definitely belongs to the set of available values (possibility degree = 0 if normalized). The most possible values (p_i^m, A_i^m) that definitely belongs to the set of available values (possibility degree = 1 if normalized).
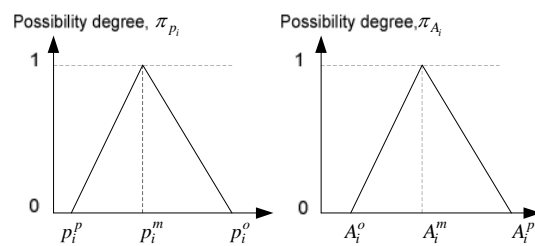
a) Possibility distribution of $\tilde{p}_i$ b) Possibility distribution of $\tilde{A}_i$

Fig.1 The triangular possibility distribution of unit price, $\tilde{p}_i$ and cost coefficients, $\tilde{A}_i$

The most optimistic values (p_i^o, A_i^o) that has a very low likelihood of belonging to the set of available values (possibility degree = 0 if normalized).

Cost coefficients, $\tilde{A}_i$ are considered as imprecise data ($\tilde{r}_i, \tilde{o}_i, \tilde{l}_t, \tilde{h}_t, \tilde{s}, \tilde{b}_i$). Triangular distributions of these data can be written as follows:

$$\tilde{r}_i = (r_i^p, r_i^m, r_i^o), \quad \forall i.$$

$$\tilde{o}_i = (o_i^p, o_i^m, o_i^o), \quad \forall i.$$

$$\tilde{l}_t = (l_t^p, l_t^m, l_t^o), \quad \forall t.$$

$$\tilde{h}_t = (h_t^p, h_t^m, h_t^o), \quad \forall t.$$

$$\tilde{s} = (s^p, s^m, s^o).$$

$$\tilde{b}_i = (b_i^p, b_i^m, b_i^o), \quad \forall i.$$

B. An additional MOLP model

Lai and Hwang (1992) referred to portfolio theory and converted the fuzzy objective with a triangular possibility distribution into three crisp objectives. According to their method, the first objective function ($\tilde{O}_1$) can be fully defined by three prominent points (Z^p, 0), (Z^m, 1) and (Z^o, 0) as shown in Fig. 2. The imprecise objective can be maximized by pushing the three prominent points towards the right. Because of the vertical coordinates of the prominent points being fixed at either 1 or 0, the three horizontal coordinates are the only considerations [5]. Consequently, solving the imprecise objective requires maximize Z^m, maximize Z^o-Z^m but minimize Z^o-Z^m, simultaneously. These involve maximize the most possible value of the imprecise profit, Z^m, minimize the risk of obtaining lower profit, (Z^m-Z^p), and maximize the possible of obtaining higher profit, (Z^o-Z^m). Three new crisp objective functions, (Z_1, Z_2, Z_3) are presented as follows.
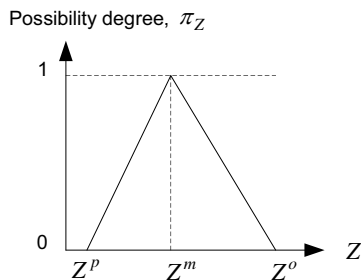
Possibility degree, π_Z

Fig. 2 The triangular possibility distribution of fuzzy objective, $\tilde{O}_1$.

$$\max Z_1 = Z^m$$

$$= \sum_{i=1}^{I}\sum_{t=1}^{T} p_i^m * (D_{it} - B_{it}) - \sum_{i=1}^{I}\sum_{t=1}^{T}(r_i^m X_{it} + o_i^m Y_{it})$$

$$- \sum_{t=1}^{T}(l_t^m L_t + h_t^m H_t) - \sum_{i=1}^{I}\sum_{t=1}^{T} s^m I_{it} - \sum_{i=1}^{I}\sum_{t=1}^{T} b_i^m B_{it}. \qquad (14)$$

$$\min Z_2 = Z^m - Z^p$$

$$= \sum_{i=1}^{I}\sum_{t=1}^{T}(p_i^m - p_i^p) * (D_{it} - B_{it})$$

$$- \sum_{i=1}^{I}\sum_{t=1}^{T}[(r_i^m - r_i^p) X_{it} + (o_i^m - o_i^p) Y_{it}] \qquad (15)$$

$$- \sum_{t=1}^{T}[(l_t^m - l_t^p)L_t + (h_t^m - h_t^p)H_t]$$

$$- \sum_{i=1}^{I}\sum_{t=1}^{T}(s^m - s^p)I_{it} - \sum_{i=1}^{I}\sum_{t=1}^{T}(b_i^m - b_i^p)B_{it}.$$

$$\max Z_3 = Z^o - Z^m$$

$$= \sum_{i=1}^{I}\sum_{t=1}^{T}(p_i^o - p_i^m) * (D_{it} - B_{it})$$

$$- \sum_{i=1}^{I}\sum_{t=1}^{T}[(r_i^o - r_i^m) X_{it} + (o_i^o - o_i^m) Y_{it}] \qquad (16)$$

$$- \sum_{t=1}^{T}[(l^o - l^m)L_t + (h^o - h^m)H_t]$$

$$- \sum_{i=1}^{I}\sum_{t=1}^{T}(s^o - s^m)I_{it} - \sum_{i=1}^{I}\sum_{t=1}^{T}(b_i^o - b_i^m)B_{it}.$$

The remaining objective function (O_2) can be rewritten as Z_4 as the following equation.

$$\min Z_4 = \sum_{t=1}^{T}(L_t + H_t). \qquad (17)$$

C. Solving the additional MOLP problem

The additional MOLP problem can be changed into an equivalent single goal LP problem using the fuzzy decision–making of Bellman and Zadeh [8] and Zimmermann's fuzzy programming method [6],[7]. The positive ideal solution (PIS) and negative ideal solution (NIS) [23] of these objective functions can be used to define a membership function of each objective as follows.

$$Z_1^{PIS} = \max Z^m, Z_1^{NIS} = \min Z^m. \qquad (18)$$

$$Z_2^{PIS} = \min(Z^m - Z^p), Z_2^{NIS} = \max(Z^m - Z^p). \qquad (19)$$

$$Z_3^{PIS} = \max(Z^o - Z^m), Z_3^{NIS} = \min(Z^o - Z^m). \qquad (20)$$

$$Z_4^{PIS} = \min Z^m, Z_1^{NIS} = \max Z^m. \qquad (21)$$

For each crisp objective function, the corresponding linear membership function is defined by

$$\mu(Z_1) = \begin{cases} 1 & if \ Z^m > Z_1^{PIS} \\ \dfrac{Z^m - Z_1^{NIS}}{Z_1^{PIS} - Z_1^{NIS}} & if \ Z_1^{NIS} \le Z^m \le Z_1^{PIS}, \\ 0 & if \ Z^m < Z_1^{NIS} \end{cases} \qquad (22)$$

$$\mu(Z_2) = \begin{cases} 1 & if\ (Z^m - Z^p) < Z_2^{PIS} \\ \dfrac{Z_2^{NIS} - (Z^m - Z^p)}{Z_2^{NIS} - Z_2^{PIS}} & if\ Z_2^{PIS} \le (Z^m - Z^p) \le Z_2^{NIS}, \quad (23) \\ 0 & if\ (Z^m - Z^p) > Z_2^{NIS} \end{cases}$$

$$\mu(Z_3) = \begin{cases} 1 & if\ (Z^o - Z^m) > Z_3^{PIS} \\ \dfrac{(Z^o - Z^m) - Z_3^{NIS}}{Z_3^{PIS} - Z_3^{NIS}} & if\ Z_3^{NIS} \le (Z^o - Z^m) \le Z_3^{PIS}, \quad (24) \\ 0 & if\ (Z^o - Z^m) < Z_3^{NIS} \end{cases}$$

$$\mu(Z_4) = \begin{cases} 1 & if\ L_t + H_t < Z_4 \\ \dfrac{Z_4^{NIS} - (L_t + H_t)}{Z_4^{NIS} - Z_4^{PIS}} & if\ Z_4^{PIS} \le L_t + H_t \le Z_4^{NIS}, \quad (25) \\ 0 & if\ L_t + H_t > Z_4^{NIS} \end{cases}$$

Then, the complete equivalent multi-objective model for solving the APP problem can be formulated. Normally Zimmerman's equivalent single-objective linear programming model is used to obtain the overall satisfaction compromise solution [5][16]-[18]. However, in order to obtain the satisfactory solution, DM needs to modify membership function of each objective until the compromise solution is found. It is difficult to adjust these membership functions due to the difference in range or scale. Moreover, DM may need higher degree of satisfaction level of one objective function than the others. The better way to adjust the level of satisfaction for different priority of objectives can be done by preemptive priority, which is usually apply to goal programming [23]. This type of weight can also be applied to PLP model. It is easier to set the desire level according to their priorities and solve them orderly than adjusting membership functions of all objectives simultaneously. In the preemptive priority fuzzy goal programming, the kth priority, P_k is preferred to the next priority, P_k+1. The relationships among the priorities are

$$P_1 >>> P_2 >>> \ldots >>> P_k >>> \ldots >>> P_K. \qquad (26)$$

Expression (26) indicates that the goals at the highest priority level (P_1) have been achieved to the minimum extent possible, before the set of goals at the second priority level (P_2) are taken into consideration and the process goes on until the last priority level P_K is considered [24].

After applying the preemptive priority to the PLP for APP, the Preemptive Possibilistic Linear Programming (PPLP) for APP with possible interval demand can be transformed to the equivalent preemptive LP model as follows:

$$lex\ max[\mu_1, \ldots, \mu_K] \qquad (27)$$

Subject to $\mu_k \ge \mu_k^*, \quad k = 1, \ldots, K-1.$

$\qquad$ (3)-(13), (22)-(25).

μ_k are the membership functions of the objectives that are rank to be the kth priority. μ_k^* is the desirable achievement degrees for the membership functions of kth piority. The DM cannot only formulate preemptive priority structure for the problem, but also can require minimum achievement degrees for some objective belonging to the same priority level.

D. Algorithm

The algorithm of the proposed PPLP approach for solving an APP problem is as follows.

1. Formulate the PPLP model for the APP problem.

2. Determine possible range of interval demand. Model the imprecise coefficients ($\tilde{p}_i, \tilde{r}_i, \tilde{o}_i, \tilde{l}_t, \tilde{h}_t, \tilde{s}, \tilde{b}_i$) using triangular possibility distributions.

3. Develop crisp objective functions of the auxiliary MOLP problem that are maximizing the most possible total profit, minimizing the risk of obtaining lower total profit, maximizing the opportunity to obtain the higher total profit and minimizing the change of workforce level.

4. Order priority of all crisp objective functions.

5. Specify linear membership functions of crisp objective functions, and then convert the auxiliary MOLP problem into an equivalent preemptive LP model.

6. Solve and modify the model interactively according the objective priority by setting the desirable achievement degrees for the membership function of the kth objective priority (μ_k^*) until a satisfactory solution is found.

IV. A CASE STUDY

A. Case description

A real case study of a company who produces electronic component is illustrated. The planning horizontal is 6 months. There are 16 models. Possible range of a forecast demand is estimated from a crisp forecast demand and its error from the actual demand of historical data. The unit price, regular production costs, overtime production costs and backorder cost are imprecise numbers with triangular possibility distributions.

Initial labor level is 84 workers with 16 production lines (6 workers per line). Hiring and firing costs are $64.84 and $78.67 per worker. Labor hour per day is 16 hours for regular time and 5.5 hours for overtime production. Inventory cost per unit is $0.0011. Maximize inventory level for each period is 304,050 units. Capacity per production line is 5,500 units for all products except product 5 and 6 capacity are 2,750 units. Processing time is 0.11 hour per unit for all products except product 5 and 6 are 0.22 hour per unit.

Working day from period 1 to 6 are 25, 24, 26, 25, 24, respectively and the maximize labor level for each period are 96, 97, 98, 120, 120 and 120, respectively. Beginning inventory for product 1-16 are 750, 12,010, 0, 0, 3,657, 0, 413, 2,391, 0, 0, 0, 1,384, 3,358, 7,360, 1,325 and 680 units.

B. Solving procedures

The solution procedures using the PPLP model for the case study is described as follows:

1. Formulate the PPLP model for the APP problem using (1)-(13).

2. Determine possible range of interval demand. Model the imprecise coefficients ($\tilde{p}_i, \tilde{r}_i, \tilde{o}_i, \tilde{b}_i$) using triangular possibility distribution. Possible range of forecast demand data is ±2% of crisp forecast demand as shown in Table I. The unit price, regular production cost, overtime production cost and backorder cost are imprecise numbers with triangular possibility distributions shown in Table II-III.

3. Develop crisp objective functions of the auxiliary MOLP problem according to (14)-(17).

4. Order priority of objective functions. From the DM's viewpoint for APP problem of the case study company, the objective functions can be ranked as follows:

The most important objective is the first objective (total profit) that needs to be maximized. It should be satisfied at one level of satisfaction.

Secondly, the risk to obtain the lower level of profit should be minimized. Most of companies try to reduce their risks as much as possible. They may allow dropping some of their profit.

TABLE I
CRISP FORECAST DEMAND DATA

Prod.	Crisp forecast demand data in each period (units)					
	1	2	3	4	5	6
1	195,000	170,000	264,396	65,000	12,000	60,000
2	170,000	120,000	362,870	450,000	400,000	155,000
3	10,000	29,063	30,000	35,000	43,000	45,000
4	0	0	12,000	5,124	33,000	8,000
5	65,000	70,000	60,000	70,000	35,000	26,000
6	250,000	246,000	280,000	191,000	0	7,939
7	170,000	140,000	44,457	0	100,000	140,000
8	564	5,000	0	0	0	0
9	0	2,551	8,000	5,640	0	0
10	40,000	40,000	30,000	19,006	27,198	40,000
11	20,000	0	30,000	43,000	35,000	28,011
12	8,000	10,000	1,640	3,789	18,000	26,167
13	0	0	0	0	0	0
14	100,000	58,356	14,600	80,000	70,000	200,000
15	615,000	600,000	535,400	410,000	660,000	800,000
16	60,000	35,385	0	60,000	20,000	110,000
Total	1,703,564	1,526,355	1,673,363	1,437,559	1,453,198	1,646,117

TABLE II
IMPRECISE UNIT PRICE

Product	Price ($)	Product	Price ($)
1	1.420,1.440,1.444	9	2.338,2.475,2.528
2	1.420,1.440,1.444	10	1.777,1.843,1.967
3	2.268,2.360,2.363	11	2.167,2.289,2.360
4	1.926,1.989,1.994	12	2.400.2.450,2.550
5	1.520,1.680,1.688	13	2.412,2.480,2.510
6	1.500,1.520,1.532	14	1.403,1.424,1.473
7	1.806,1.815,1.856	15	1.423,1.444,1.473
8	2.405,2.450.2.550	16	1.511,1.523,1.543

TABLE III
IMPRECISE REGULAR PRODUCTION COST, OVERTIME PRODUCTION COST AND BACKORDER COST

Product	Regular production cost ($)	Overtime production cost ($)	Backordering cost ($)
1	1.047,1.044,1.036	1.051,1.048,1.040	0.217,0.216,0.213
2	1.047,1.044,1.036	1.051,1.048,1.041	0.217,0.216,0.214
3	1.753,1.75,1.742	1.757,1.754,1.746	0.355,0.354,0.340
4	1.354,1.35,1.349	1.362,1.358,1.357	0.299,0.298,0.289
5	0.666,0.660,0.656	0.675,0.669,0.665	0.253,0.252,0.228
6	0.666,0.660,0.657	0.670,0.664,0.660	0.230,0.227,0.228
7	1.762,1.752,1.730	1.766,1.756,1.734	0.278,0.272,0.271
8	2.036,2.030.2.027	2.040,2.034,2.031	0.383,0.368,0.361
9	2.156,2.122,2.120	2.160,2.126,2.124	0.379,0.371,0.351
10	1.439,1.432,1.430	1.443,1.436,1.434	0.296,0.276,0.266
11	1.752,1.750,1.749	1.756,1.754,1.753	0.354,0.343,0.325
12	1.790,1.700,1.660	1.794,1.704,1.664	0.383,0.368,0.360
13	1.758,1.750,1.700	1.762,1.754,1.704	0.377,0.372.0.362
14	1.329,1.326,1.300	1.333,1.330,1.304	0.221,0.218,0.210
15	1.329,1.326,1.300	1.333,1.330,1.304	0.221,0.217,0.213
16	1.755,1.749,1.742	1.746,1.753,1.759	0.231,0.228,0.226

Next, the change of workforce level should be minimized, if it is possible.

Finally, the opportunity to obtain the higher profit should be increased, if there are some chances to improve the plan.

So, objective priority of this case study is Z_1, Z_2, Z_4 and Z_3.

5. Specify linear membership functions of these crisp objective functions using PIS and NIS. The PIS of Z_1-Z_4 are ($3,211,910.33, $217,754.86, $478,550.87, 0) and the NIS of Z_1-Z_4 are ($-2,550,249.26, $480,740.55, $246,908.84, 48). The corresponding linear membership functions of the four objective functions can be defined according to (22)-(25). Next, convert the auxiliary MOLP problem into equivalent preemptive LP model (27).

6. Solve and modify the model according the objective priority by setting the desirable achievement degrees for the membership function of the kth priority (μ_k^*). Satisfaction levels of selected compromise solution from objective 1 to 4 are 1.00, 0.6012, 0.59 and 0.625, respectively.

The results of the single objective optimization with crisp demand and interval demand, the compromise solution by PPLP model are shown in Table IV. It can be seen that interval demand can obtain the better profit than crisp demand due to the flexibility of selecting the appropriate demand for production. In this case study, range of Z_1 is 5,762,159.56, ($Z_1^{PIS} - Z_1^{NIS}$), but range of Z_4 is 48, ($Z_4^{NIS} - Z_4^{PIS}$). These range are extremely different, it is difficult to adjust if PLP approach is applied. PPLP method does not require adjusting of membership functions. DM can select the appropriate level of satisfaction of each objective according to its priority that DM desires. PPLP can obtain the compromise solution, which has the highest profit and has additional information about

pessimistic case and optimistic case of the total profit. It is better than the single objective optimization. Moreover, uncertainty is also considered. DM can adjust the desire level of satisfaction of each objective to get the other satisfactory solutions.

TABLE IV
OPTIMIZATION RESULTS

Maximize/Minimize	Profit ($)	Change of workforce (Man)
Maximize total profit with crisp demand and precise information	3,150,304.48	-
Minimize change of workforce level with crisp demand and precise information	-	0
Maximize total profit with interval demand and precise information	3,211,910.30	-
Compromise solution with interval demand and imprecise solution	2,889,273.30 3,211,910.30 3,595,490.77	18

TABLE V
SATISFIED DEMANDS

Product	Satisfied demand in each period (units)					
	1	2	3	4	5	6
1	198,900	173,400	269,684	66,300	12,240	61,200
2	173,400	122,400	370,127	459,000	408,000	158,100
3	10,200	29,644	30,600	35,700	43,860	45,900
4	0	0	12,240	5,226	33,660	8,160
5	66,300	71,400	61,200	71,400	35,700	26,520
6	255,000	250,920	285,600	194,820	0	8,098
7	173,400	142,800	45,346	0	102,000	142,800
8	575	5,100	0	0	0	0
9	0	2,602	8,160	5,753	0	0
10	40,800	40,800	30,600	19,386	27,742	41,600
11	20,400	0	30,600	43,860	35,700	28,571
12	8,160	10,200	1,673	3,865	18,360	26,690
13	0	0	0	0	0	0
14	102,000	59,523	14,892	81,600	71,400	204,000
15	627,300	612,000	546,108	418,200	673,200	816,000
16	58,800	34,677	0	58,800	19,600	107,800
TTL	1,735,235	1,555,467	1,706,830	1,463,910	1,481,462	1,675,439

TABLE VI
REGULAR PRODUCTION

Product	Regular production in each period (units)					
	1	2	3	4	5	6
1	192,372	171,907	266,047	65,550	11,490	60,450
2	159,628	106,419	270,140	446,990	395,990	146,090
3	8,186	28,651	28,651	35,700	43,860	45,023
4	0	0	12,240	5,226	33,660	8,160
5	61,395	67,535	57,302	67,743	32,043	22,512
6	190,326	188,279	212,837	194,820	0	8,098
7	171,907	139,163	40,930	0	101,587	142,387
8	0	2,709	0	0	0	0
9	0	2,602	8,160	5,753	0	0
10	36,837	36,837	28,651	19,386	27,742	40,930
11	16,372	0	28,651	43,860	35,700	28,571
12	6,776	8,186	289	2,481	16,976	24,558
13	0	0	0	0	0	0
14	73,674	49,116	7,532	74,240	64,040	196,465
15	466,605	511,628	466,605	416,875	671,875	814,512
16	0	0	0	58,120	18,920	98,233

Table V shows the satisfied demands that were obtained from the proposed PPLP model, these demands are a little bit greater than the crisp forecast demand presented in Table I but it is in the possible range of forecast demand. Using possible range of demand has advantages over crisp demand because the model can select the best solution for DM from the possible range of demand that can satisfy the maximum utilization of capacity. It is more flexible than existing models. The number of production quantities by regular and overtime production are shown in Table VI-VII.

TABLE VII
OVERTIME PRODUCTION

Product	Overtime production in each period (units)					
	1	2	3	4	5	6
1	5,778	743	2,887	0	0	0
2	1,762	3,971	87,978	0	0	0
3	2,014	993	1,949	0	0	877
4	0	0	0	0	0	0
5	1,248	208	241	0	0	351
6	64,674	62,641	72,763	0	0	0
7	1,080	3,224	4,003	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	3,963	3,963	1,949	0	0	670
11	4,028	0	1,949	0	0	0
12	0	630	0	0	0	748
13	0	0	0	0	0	0
14	20,966	3,047	0	0	0	175
15	159,370	99,047	78,178	0	0	163
16	0	0	0	0	0	0

TABLE VIII
NUMBER OF PRODUCTION LINE

Product	Number of production line in each period (lines)					
	1	2	3	4	5	6
1	47	42	65	19	3	15
2	39	26	66	110	97	36
3	2	7	7	9	11	11
4	0	0	3	2	9	2
5	30	33	28	34	16	11
6	93	92	104	96	74	4
7	42	34	10	0	25	35
8	0	1	0	0	0	0
9	0	1	2	2	0	0
10	9	9	7	5	7	10
11	4	0	7	11	9	7
12	2	2	1	1	5	6
13	0	0	0	0	0	0
14	18	12	2	19	16	48
15	114	125	114	102	165	199
16	0	0	0	15	5	24
TTL	400	384	416	425	442	408

TABLE IX
LABOR LEVEL, HIRING LEVEL, AND FIRING LEVEL

Item	Period					
	1	2	3	4	5	6
No. of worker (men)	96	96	96	102	102	102
Hired worker (men)	12	0	0	6	0	0

TABLE X
INVENTORY LEVEL

Product	Inventory level in each period (units)					
	1	2	3	4	5	6
7	0	0	0	413	0	0
8	1,816	0	2,391	2,391	2,391	2,391
13	3,358	3,358	3,358	3,358	3,358	3,358

Table VIII shows the number of production line in each month. The number of production line per day from period 1 to period 3 is 16 production lines and from period 4 to period 6 is 17 production lines. Currently the number of production line per day is 14 lines. It will be increased 2 lines at the beginning of period 1 with additional 12 workers. At the end of period 3, 6 additional workers will be hired for one more production line a day. Total number of worker in each period is shown in Table IX.

Backordering units from period 1 to period 6 occur only for product 16, they are 58,120, 33,997, 0, 0, 0, 8,887 units. Inventory levels are not necessary for almost all products in each period except product 7, 8 and 13 as shown in Table X.

V. CONCLUSION

This research presents a Preemptive Possibilistic Linear Programming (PPLP) method for solving multiple objectives of an APP problem with two objective functions; to maximize profit and minimize change of workforce level. The proposed PPLP approach attempts to maximize the most possible total profit, minimize the risk of obtaining the lower total profit, minimize the change of workforce level and maximize the opportunity to obtain the higher profit respectively by setting the satisfaction level of each objective. This method can reduce the problem of adjusting the membership functions of existing PLP approach. DM can manipulate the compromise solution based on own preferences to get the satisfactory solution. Possible forecast demand interval is used in the model. It can increase flexibility to the model to obtain the better solution. Moreover, utilization of production lines can be appropriately planed for each period.

This method can be applied to others case studies in industrial applications. Imprecise capacity can also be included.

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