

# Prediction of Nonlinear Torsional Behavior of High Strength RC Beams

Woo-Young Jung, Minho Kwon

**Abstract**—Seismic design criteria based on performance of structures have recently been adopted by practicing engineers in response to destructive earthquakes. A simple but efficient structural-analysis tool capable of predicting both the strength and ductility is needed to analyze reinforced concrete (RC) structures under such event. A three-dimensional lattice model is developed in this study to analyze torsions in high-strength RC members. Optimization techniques for determining optimal variables in each lattice model are introduced. Pure torsion tests of RC members are performed to validate the proposed model. Correlation studies between the numerical and experimental results confirm that the proposed model is well capable of representing salient features of the experimental results.

**Keywords**—Torsion, non-linear analysis, three-dimensional lattice, high-strength concrete.

## I. INTRODUCTION

A structural design, besides concern of safety in general, is also typically based on cost. Small cross-sectional areas and large spans are usually targeted to achieve optimal spaces. Typical design of a structure subjected to an earthquake-induced load is based on an assumption that the seismic load is applied on the structure only along its main direction. Seismic loads, however, could act on a structure at any skew angle; resulting in unexpected torsional loads. Thus, structures under skew earthquake loads are simultaneously subjected to bending, shear, and torsion loads. Since seismic, and also wind loads, act in random directions, torsional performance of the structure should not be disregarded because unpredicted torsional loads could cause significant damages to structures. So far, torsion in reinforced concrete (RC) structures has received least attention compared to bending and shear in RC structures.

The use of nonlinear analytical technology has recently been attempted to match and rival performance-based design and assessment of RC structures constructed in critical earthquake regions. Modeling strategies to predict global responses of structural systems have been focused on the local failures of a member since inelastic behaviors are concentrated at the end region of members. An early approach to model the behavior had involved the use of nonlinear spring at the ends of member to capture such inelastic behavior. Distributed nonlinearity

models had also used to provide more accurate and detailed descriptions of inelastic behaviors in RC members [1]-[5]. One other approach is the use of a strut-tie model, having a portion of the structure or the entire structure modeled as a system of struts and ties, where struts represent compressive forces in concrete while ties represent tension forces in the reinforcing bars [6]-[8]. The strut-tie model is, nevertheless, suitable for predicting shear strength in deep RC beams and walls but is not capable of evaluating inelastic deformation of members or cyclic response of members under seismic loads, since it is only based on the equilibrium equation. In contrast, a lattice model possesses capability to predict deformation as well as strength in the inelastic analysis of RC structures.

Lattice model can evaluate shear strengths of structural members and inelastic deformations at both global and local levels. It also possesses shear transmission ability, which is achieved by treating a connection like a moment-resisting beam in a frame. Two and three dimensional lattice models reportedly had been employed to investigate influences of material meso-structure of concrete, in which a linearly elastic and purely brittle behavior was adopted at element level [9]-[11]. In this research, a nonlinear lattice model has been introduced to predict nonlinear torsional behaviors of structural members. This lattice model has also been extended to high-strength concrete columns subjected to cyclic load reversals, and constitutive laws of high-strength concrete have been implemented. Confinement effects of high-strength concrete and hysteretic model of steel bars have also been introduced. The effective width of a strut is determined by the principle of minimization of strain energy. Torsional analysis using the lattice model has been attempted by several researchers [12]-[14]; however they did not clearly define the process for determining parameters of cross-sectional areas in the lattice members.

In this study, an optimization technique is used to determine the parameters of cross-sectional area of lattice model. Pure torsion tests on normal and high strength RC beams are carried out. Based on the test results, the proposed lattice model is verified.

### A. Review Stage Fixed-Truss Lattice Model

The lattice model consists of flexural members, diagonal members, arch members, longitudinal members, and transverse members. In the lattice model,  $d$  is determined by the distance between the centers of the longitudinal members. Due to that the diagonal members are placed with angle of  $45^\circ$  or  $135^\circ$ , the lattice model is classified as a fixed-truss model. As results the diagonal crack of concrete in the lattice model will be fixed as

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45°, and dowel action and aggregate interaction cannot be directly considered. The arch member is introduced to consider the flows of internal forces, thus enabling adequate prediction of the shear-resistance mechanism [15]. Analysis model for the torsional analysis of reinforced concrete structures has been developed based on the concept of an eight-node lattice element as shown in Fig. 1. The lattice element composes of concrete and reinforcements such as flexural compressive members, flexural tensile members, diagonal compressive members, diagonal tensile members, and arch members. Members in the directions of longitudinal and transverse axes represent both concrete cover and reinforcement bars. The areas of those members are taken to be equal to the original concrete cover and cross-sectional areas of reinforcements in the structure. Arch members connecting the nodes at opposite diagonal corners between the loading point and the bottom of the column are arranged according to the directions of internal compressive stress flows.

The cross-sectional areas of the diagonal and arch concrete members are determined by the principle of minimum strain energy. Fig. 2 illustrates the concept of the proposed lattice model.

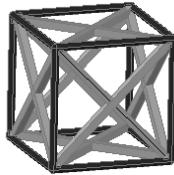
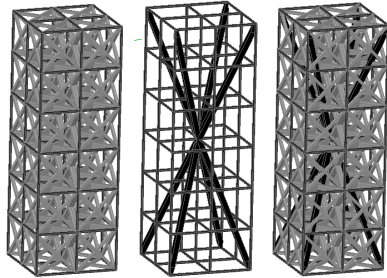


Fig. 1 Concept of 8-node lattice element model



(a) Concrete (b) Rebar, Arch (c) RC column

Fig. 2 Concept of 3D lattice members

### B. Lattice Modeling for Steel and Concrete

The three-dimensional lattice element is consisted of twenty-four vertical and horizontal concrete members, twelve diagonal concrete members, and twelve reinforcement members. The three-dimensional lattice element is formulated within the finite element technique framework. The stiffness matrix of an 8-node lattice element is given in (1) and is derived based on the virtual displacement principle as:

$$K_{Lattice} = \int_V B_c^T D_c B_c dV + \int_V B_{cd}^T D_{cd} B_{cd} dV + \int_V B_s^T D_s B_s dV \quad (1)$$

where  $B_c$ ,  $B_{cd}$ , and  $B_s$  are the compatibility matrix of the concrete, the diagonal truss (concrete), and reinforcement bars, respectively.  $D_c$ ,  $D_{cd}$ , and  $D_s$  is the material matrix of the concrete, the diagonal truss and reinforcement bars, respectively. A representation of the cross-sectional division of the three-dimensional lattice model of RC beam is shown in Fig. 3. The values of  $t_b$  and  $t_d$  in the three-dimensional lattice models are determined by minimizing the total potential energy in (2). This function is employed as the objective function of the optimization problem as:

$$\sum F_i \cdot l_i \cdot \varepsilon_i \quad (2)$$

where  $F_i$ ,  $l_i$ , and  $\varepsilon_i$  are the member force, member length, and member strain, respectively.

The cross-sectional area of concrete in the lattice model is divided into the arch part and the truss part, as shown in Fig. 3. The area of the arch member is determined by  $t_b$  and  $t_d$ . The total area of the longitudinal members and the vertical reinforcement area in the lattice model are equal to the original reinforcement in the RC structure. Therefore, the longitudinal reinforcement ratio and the transverse reinforcement ratio in the lattice model are equal to the reinforcement ratios of the RC structure.

Arch members were placed inside the model to trace shear and torsion resistance mechanisms. The position and orientation of the arch member is an important factor of shear resistance and they are determined by the linear elastic analysis results. Details of the cross-sectional area of the lattice element are shown in Table I.

TABLE I  
DETERMINATION OF SECTION AREA IN LATTICE MODEL

Member	Section Area	Note
Arch	$A_{arch2D} = \frac{1}{2} \cdot b \cdot t_b \cdot d \cdot t_d \cdot \sin\theta$	$b, d$ : size of RC section
	$A_{arch3D} = \left( \frac{1+m^2+n^2}{1+m^2} \right)^{\frac{3}{2}} \cdot A_{arch2D}$	$\theta$ : angle between RC section and arch member
	$A_{truss-xy} = \frac{b(1-t_b)}{2m} \cdot \frac{d}{2} \cdot \sin 45^\circ$	$m = (b-c)/d$
Truss	$A_{truss-yz} = \frac{h(1-t_b)}{2} \cdot \frac{d}{2} \cdot \sin 45^\circ$	$n = h/d$
	$A_{truss-zx} = \frac{a}{2n} \cdot \frac{d}{2} \cdot \sin 45^\circ$	$h$ : height of RC structure
Concrete Vertical	$A_c = \frac{1}{4} \cdot c \cdot (1-t_b) \cdot b \cdot (1-t_d) \cdot d$	$a$ : length of shear span
	$A_{s1} = \left( \frac{n_t}{4} + \frac{n_s}{4} - 1 \right) \cdot A_t$	$c$ : cover of RC member
Bar Vertical	$A_{s2} = \frac{n_t}{2} \cdot A_t$	$A_t$ : vertical rebar area
	$A_{s3} = \frac{n_s}{2} \cdot A_t$	$A_w$ : horizontal rebar area
Bar Horizontal	$A_{w-lattice} = \frac{0.5d}{s} \cdot A_w$	$n_b, n_s$ : number of rebar each direction of section

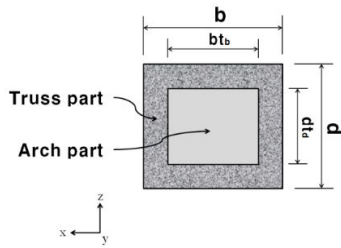


Fig. 3 Partition of cross section of 3D lattice model

### C. Lattice Modeling for Steel and Concrete

The lattice model of the structure requires optimal values of  $t_b$  and  $t_d$  for minimum strain energy. These variables, varying between 0 and 1, consist of the ratio of cross-sectional area of the arch member to the cross-sectional area of the entire concrete member. As mentioned earlier, strain energy is the objective function in the optimization. The Sequential Unconstrained Minimization Technique (SUMT) method is employed for the optimization technique to determine the minimum value of the objective function using penalty parameters within the range of possible values from the boundary area of dominant constraints. The SUMT method embraces three techniques: internal penalty function method, external penalty function method, and a combination of the two aforementioned methods. The proposed lattice model, however, employs just the external penalty function method. In the objective function, defined in (3), the design variables are  $t_b$  and  $t_d$ . The four constraints are expressed in (4), (5), (6) and (7).

The objective function is:

$$\text{Minimum } \Pi(t_b, t_d) = \sum_{i=1}^{\text{Num-}ele} \sigma_i A_i \delta_i / l_i \quad (3)$$

and is subjected to

$$G(1) = 1 - t_b \geq 0 \quad (4)$$

$$G(2) = t_b \geq 0 \quad (5)$$

$$G(3) = 1 - t_d \geq 0 \quad (6)$$

$$G(4) = t_d \geq 0 \quad (7)$$

where  $\Pi$ ,  $\sigma_i$ ,  $A_i$ , and  $\delta_i$  are the strain energy, stress, cross-sectional area, and relative deformation of each member, respectively.

## II. CONSTITUTIVE LAWS OF MATERIALS

### A. Concrete

The Mohd Yassin model was employed for the concrete non-linear constitutive model as shown in Fig. 4. It uses the Kent-Park concrete model as the monotonic envelope and the hysteretic rule proposed by Karsan and Jirsa for cyclic reversals [16]-[18]. The monotonic curve, consisting of three sections, is expressed by (8), (9) and (10) as:

$$f_c = f'_c \left[ \frac{2\varepsilon_c}{\varepsilon_o} - \left( \frac{\varepsilon_c}{\varepsilon_o} \right)^2 \right] \quad (8)$$

AB section:

$$f_c = f'_c [1 - Z(\varepsilon_c - \varepsilon_o)] \quad (9)$$

BC section:

$$f_c = 0.2 f'_c \quad (10)$$

$$Z = \frac{0.5}{\varepsilon_{s0} - \varepsilon_o} \quad (11)$$

$$\varepsilon_{s0} = \frac{3 + 0.002 f'_c}{f'_c - 1000} + \frac{3}{4} \rho'' \sqrt{\frac{b''}{s}} \quad (12)$$

where  $f_c$  is the concrete stress in concrete,  $f'_c$  is the compressive strength of concrete,  $Z$  is expressed by (11) and (12),  $\rho''$  is the transverse rebar ratio,  $b''$  is the width of the concrete core and  $s$  is the interval of the transverse rebar. The OA section of the Mohd Yassin model is expressed as a quadratic function, the AB section is expressed as a linear function, and the BC section is expressed as a constant function.

The confinement effect of normal strength concrete is computed by the equation suggested by Mander et al. [8], which are formulated in (13) and (14) as:

$$f'_c = f'_c \left[ 2.254 \sqrt{1 + 7.94(f'_i / f'_c)} - 2(f'_i / f'_c) - 1.254 \right] \quad (13)$$

$$f'_i = 0.75 \rho_w f_{wy} \quad (14)$$

where  $f'_c$  is the compressive strength of concrete prior to confinement,  $\rho_w$  is the transverse reinforcement ratio and  $f_{wy}$  is the yield strength of the transverse rebar.

In high-strength concrete, strength enhancement due to the confining effect of transverse reinforcement, as defined by Sheikh and Uzumeri [19], is expressed as:

$$f'_{cc} = 0.86 f'_c + 10.3 (\alpha \rho_w f_{wy})^{0.4} \quad (15)$$

$$\alpha = \left( 1 - \frac{\sum b_i^2}{6b_c d_c} \right) \left( 1 - \frac{s}{2b_c} \right) \left( 1 - \frac{s}{2d_c} \right) \quad (16)$$

where  $b_i$  is the spacing of the longitudinal reinforcements,  $b_c$  is the center-to-center spacing of the stirrups or hoop sets,  $d_c$  is the center-to-center width of the stirrups or hoop sets,  $\rho_w$  is the ratio of transverse reinforcement, and  $f_{wy}$  is the yield strength of the transverse reinforcement.

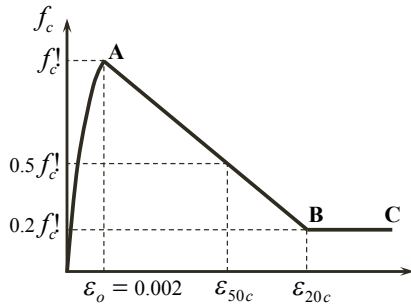


Fig. 4 Stress-strain relationship by monotonic loads

### B. Steel Reinforcements

The reinforcing steel stress-strain behavior is determined by Monti and Nuti model [20]. This model can predict the local buckling behavior of the reinforcement bar under compressive loads. The representative plot of the model is depicted in Fig. 5.

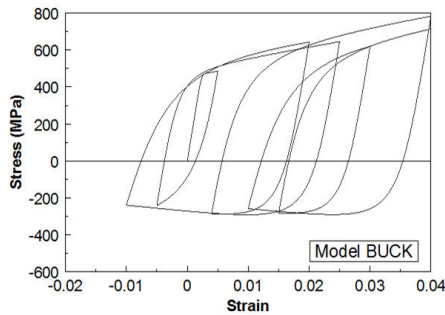


Fig. 5 Stress-strain relationship of steel bars

## III. VERIFICATION OF THE PROPOSED LATTICE MODEL

### A. Experimental Program

Torsional analysis was performed to validate the proposed lattice model for RC structures. Two RC beams were designed according to ACI 318 [21]. The first beam was constructed with a concrete compressive strength of 21 MPa and the second beam was constructed with a concrete compressive strength of 40 MPa. The span length of the beam was 3,000 mm, the cross-sectional area of the beam was 300 mm × 300 mm, and the thickness of the cover concrete was 50 mm. The longitudinal rebar was composed of 4-D16 rebar and the transverse rebar was composed of D10 rebar with a general spacing of 150 mm. The yield stress of the rebar was 400 MPa. Details of the specimen are shown in Fig. 6.

Details of the pure torsion beam test setup are shown in Fig. 7. The first boundary was designed with upper and lower rollers to enable elongation of the specimen; the second boundary was designed for pure torque. A 300 kN-capacity actuator is used to applied torsion load to the specimen. The circumferential direction of loading, which is the center of the torsion, coincides with the center of the cross-section. The loading speed was set at 1 mm/sec.

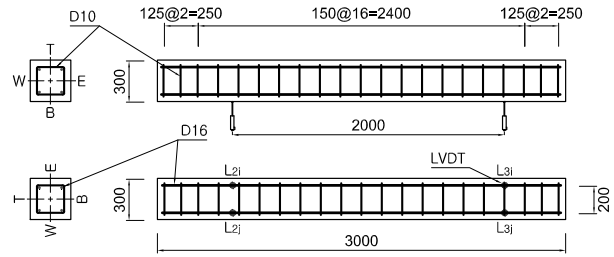


Fig. 6 Detail of specimen

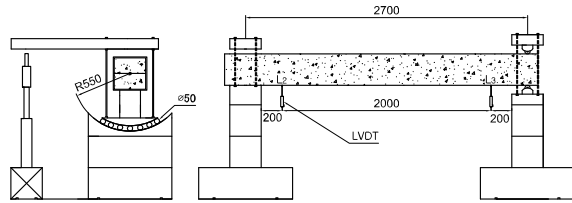


Fig. 7 Test setup

To evaluate behaviors under pure torsion, a twist angle was calculated by measuring displacement of each of the four measurement points on one side of the specimen. The distance between the measuring points in the longitudinal direction is 2,000 mm and the distance between the measuring points in the transverse direction is 200 mm. The twist angle was calculated by an equation proposed by Peng and Wong [22] as:

$$\theta = \left[ \arctan\left(\frac{L_{3i} - L_{3j}}{b}\right) - \arctan\left(\frac{L_{2i} - L_{2j}}{b}\right) \right] / h \quad (17)$$

where  $b$  is the distance between measuring points in the transverse direction,  $h$  is the distance between measuring points in the longitudinal direction and  $L_{2i}$ ,  $L_{2j}$ ,  $L_{3i}$ , and  $L_{3j}$  are the displacements of the measuring points.

A plot of the relation between twist angle and torque is shown in Fig. 8. Observed ultimate torques were 12.052 kN-m for C21, and 14.515 kN-m for C40. Obtained ultimate twist angles were 1.267 degree/m for C21, and 1.504 degree/m for C40.

### B. 3-Dimensional Lattice Analysis

Torsional strength of the beam was evaluated using the proposed lattice model. The length of one side of the lattice element is 100 mm. The lattice model for torsional analysis is shown in Fig. 9. The values of  $t_b$  and  $t_d$  were determined using nonlinear optimization technique, SUMT. The value of  $t_b$  is identical to the value of  $t_d$  because the specimens have square cross-sections. The optimal value of  $t_b$  is 0.44 for specimen C21 and 0.3 for specimen C40.

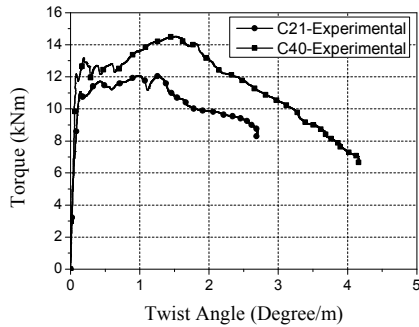


Fig. 8 Torque-twist angle relation in experimental results

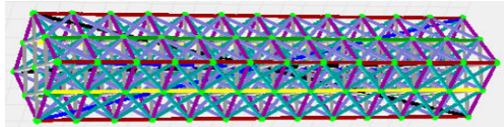
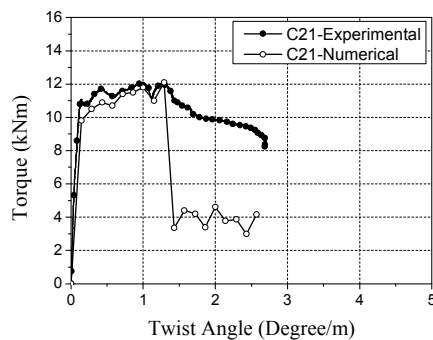
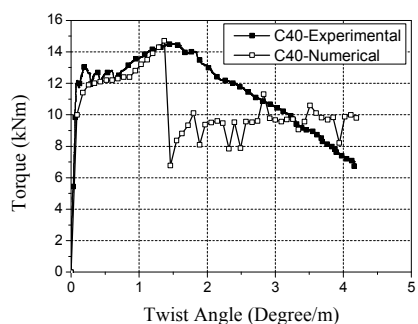


Fig. 9 3-D lattice modeling of beam

The obtained numerical results were compared with experimental results in terms of torque-twist angle responses as plotted in Fig. 10. The proposed model well predicts torsional strengths of both RC members. However, post-peak behaviors do not correlate well with experimental results.



(a) C21



(b) C40

Fig. 10 Torque-Twist angle relation of analytical results

#### IV. CONCLUSION

In this study, a lattice model capable of predicting torsional behavior of reinforced concrete members is proposed. The

essential model parameters for the lattice element are determined using the optimization technique minimizing the system potential energy. Two cases of correlation studies between experimental and numerical results are performed. The correlation study involves two pure torsion tests of normal and high strength concrete beams with square cross-sections. Discrepancies in post-peak response between numerical and experimental results are observed; however, the proposed lattice model is capable of representing the general responses of the tested specimen.

#### ACKNOWLEDGMENT

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