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Prediction of Load Capacity of Reinforced Concrete Corbels Strengthened with CFRP Sheets

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***Abstract*—**Analytical procedure was carried out in this paper to calculate the ultimate load capacity of reinforced concrete corbels strengthened or repaired externally with CFRP sheets. Strut and tie method and shear friction method proposed earlier for analyzing reinforced concrete corbels were modified to incorporate the effect of external CFRP sheets bonded to the corbel. The points of weakness of any method that lead to an inaccuracy, especially when overestimating test results were checked and discussed. Comparison of prediction with the test data indicates that the ratio of test / calculated ultimate load is 0.82 and 1.17 using strut and tie method and shear friction method, respectively. If the limits of maximum shear stress is followed, the calculated ultimate load capacity using shear friction method was found to underestimates test data considerably.

***Keywords*—**Corbel, Strengthening, Strut and Tie Model, Shear Friction

NOMENCLATURE

a	shear span
b	corbel width
A_{fs}	total area of CFRP layers provided to the shear zone.
A_{sf}	total area of steel reinforcement provided to the flexural zone.
A_{ff}	total area of CFRP layers provided to the flexural zone.
A_{fi}	total area of inclined CFRP strips.
A_{sv}	area of shear reinforcement provided to direct shear zone
α	angle of inclination of CFRP strip from the horizontal.
α_1	compressive stress distribution parameter.
c	depth of compressive stress zone.
C	compressive force due to compressive stress block.
λ, μ	parameters used in shear friction equation.
Φ	bond reduction factor between CFRP strip and concrete.
d	distance from flexural reinforcement to corbel- column junction point.
d_s	average distance from shear reinforcement to corbel-column junction point.
d_f	average distance from CFRP layers provided for shear to corbel- column junction point.
d_{fi}	average distance from inclined CFRP layers to corbel-column junction point.
h	corbel height.
k	height of the slope portion of the corbel.
ρ_f	ratio of CFRP layers perpendicular to shear plane.
ρ_{fi}	ratio of inclined CFRP layers provided to shear plane.
Δ_{vn}	ratio of change in shear stress due to strengthening.
T_{sf}	horizontal tensile force carried by steel reinforcement and CFRP strip in flexural zone bonded to concrete at both sides of the corbel.
T_s	tensile force resisted by stirrups.
T_{fs}	total tensile force resisted by the horizontal CFRP strip in shear zone.
T_{fi}	total tensile force carried by inclined strips of CFRP, all provided to both sides of the corbel.
f_{yf}	yield stress of steel reinforcement in flexural zone.
f_{yv}	yield stress of steel reinforcement in shear zone.
f_{ff}	Fracture stress of CFRP strips- epoxy composite in flexural zone.
f_f	fracture stress of CFRP strips- epoxy composite in shear zone.
f_{fi}	Fracture stress of CFRP strips- epoxy composite in inclined strips.
V_n	maximum shear force capacity of the corbel.
v_{nc}	shear stress capacity of the corbel.

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I. INTRODUCTION

ADVANCED polymer products were used extensively in concrete structures to elongate their lifetime. FRP layers are successful to control crack extension and propagation in concrete. Indeed FRP application has an important role in the case of those concrete members undergo cracks concentration like in the case of corbels. Test results [1] indicate that strengthening reinforced concrete corbels with CFRP sheets able to enhance a load capacity by 28.3%. Results also showed that the benefit of the provided CFRP layers for strengthening increased by reducing the amount of flexural and/or shear reinforcement and reducing the concrete compressive strength. However the benefit is more important in repairing damaged corbels occurred by preloading especially in the case of corbels of lower concrete strength.

In this paper an attempt was made to calculate the load capacity of RC corbels strengthened externally with CFRP sheets. First, the strut and tie model was used and adjusted to incorporate the action of bonded CFRP layers. Later, the shear friction method was used and equations were adjusted for the case of corbels strengthened with CFRP sheets. The accuracy of each method was checked by making a comparison with the previous test data. The suitability of each method was discussed to use the better one in the case of strengthening and repairing of reinforced concrete corbels.

II. Load capacity prediction

A. Strut and Tie Model

Fig.1 shows the force acting on the reinforced concrete corbel externally bonded with CFRP strips. Compressive and tensile forces shown in the figure are given by the following equations

$$C = \alpha_1 f_c' b\, c \tag{1}$$

$$T_{sf} = A_{sf} f_{yf} + \phi A_{ff} f_{ff} \tag{2}$$

$$T_s = A_{sv} f_{yv} \tag{3}$$

$$T_{fs} = \phi A_{fs} f_f \tag{4}$$

$$T_{fi} cos\alpha = \phi A_{fi} f_{fi} cos\alpha \tag{5}$$

In which T_{sf} is the horizontal tensile force carried by steel reinforcement and CFRP strip in flexural zone bonded to concrete at both sides of the corbel. T_s is the tensile force resisted by stirrups. T_{fs} is the total tensile force resisted by the horizontal CFRP strip in shear zone. T_{fi} is the total tensile force carried by inclined strips of CFRP, all provided to both sides of the corbel. ϕ is the bond reduction factor between CFRP strip and concrete, taken as 0.75[2]. α is the angle of inclination of CFRP strip from the horizontal. A_{sf} and A_{ff} are the total area of steel reinforcement and CFRP layers provided to the flexural zone, respectively. A_{sv} and A_{fs} are the total area of steel reinforcement and CFRP strips provided to the shear zone, respectively. A_{fi} is the total area of inclined CFRP strips. f_{yf} and f_{yv} are the yield stress of steel reinforcement in flexural and shear zones, respectively, and f_{ff}, f_f and f_{fi} are the fracture stress of CFRP strips- epoxy composite in flexural zone, shear zone and inclined strips, respectively.

α_1 is the compressive stress distribution parameter given by[3]

$$\alpha_1 = 0.85 - 0.0015 f_c'$$

The horizontal component of the compressive force C is given by

$$C\, sin\beta = T_{sf} + T_s + T_{fs} + T_{fi} cos\alpha \tag{6}$$

The vertical component of C is given by

$$C\, cos\beta + T_{fi} sin\alpha = V_n \tag{7}$$

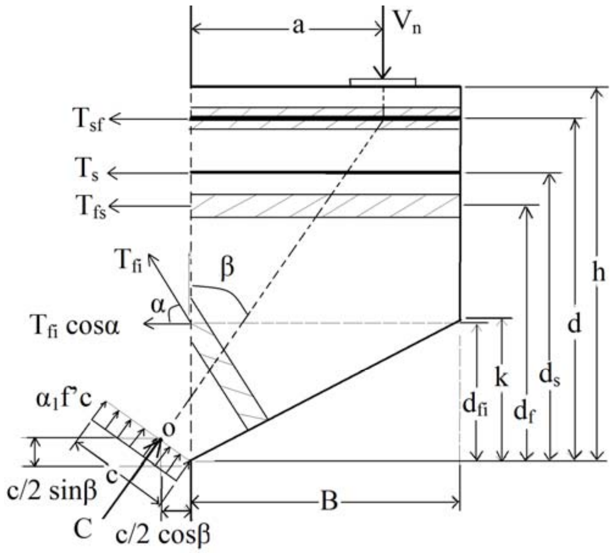

Fig. 1 Forces acting on the corbel

in which V_n is the nominal shear force.

Substituting Eq.(1) into Eq.(6) and rearranging yields

$$c\, sin\beta = \frac{T_{sf} + T_s + T_{fs} + T_{fi} cos\alpha}{\alpha_1 f_c' b} \tag{8}$$

Substituting Eq.(1) into Eq.(7) and rearranging yields

$$V_n = \alpha_1 f_c' \, b\, c\, sin\beta b cot\beta + T_{fi} sin\alpha \tag{9}$$

Combining Eqs.(8) and (9) and simplifying yields

$$V_n = (T_{sf} + T_s + T_{fs} + T_{fi} cos\alpha) cot\beta + T_{fi} sin\alpha \tag{10}$$

$cot\beta$ must be determined to calculate V_n and can be obtained by equating the external moment caused by the vertical force V_n and the internal moments resisted by the corbel materials. Equilibrium of moment acting on the corbel about point *o* [Fig.1] yields

$$V_n(a + \frac{c\, cos\beta}{2}) = T_{sf}(d - \frac{c\, sin\beta}{2}) + T_s(d_s - \frac{c\, sin\beta}{2}) + T_{fs}(d_f - \frac{c\, sin\beta}{2}) + T_{fi} cos\alpha os_{fi} - \frac{c\, sin\beta}{2}) + T_{fi}\, sin\alpha(\frac{c\, cos\beta}{2}) \tag{11}$$

d_{fi} is the distance between the center of the inclined strip crossing the shear plane and the inclined surface-column junction point. Substituting Eqs.(8) and (10) into (11) and rearranging yields the following equation for calculating $cot\beta$

$$cot\beta = \frac{\alpha_1 f_c' b}{T_{sf} + T_s + T_{fs} + T_{fi} cos\alpha} [\sqrt{(\psi} - (a + \frac{T_{fi} sin\alpha}{2\alpha_1 f_c' b})] \tag{12}$$

In which

$$\Psi = a^2 - \frac{2}{\alpha_1 f'_c b}[\frac{(T_{sf} + T_s + T_{fs} + T_{fi} cos\alpha)^2}{2\alpha_1 f'_c b} + T_{fi} a sin\alpha -$$
$$(T_{sf} d + T_s d_s + T_{fs} d_f + T_{fi} cos\alpha o_{fi})]$$

Later, the value of cot β is substituted into Eq.(10) and vertical shear force, V_n, can be calculated as follows

$$V_n = \alpha_1 f'_c b[\sqrt{\psi} - a] + T_{fi} sin\alpha \tag{13}$$

For those corbels containing no inclined strips of CFRP

$$V_n = \alpha_1 f'_c b[\sqrt{a^2 - \frac{2}{\alpha_1 f'_c b}[\frac{(T_{sf} + T_s + T_{fs})^2}{2\alpha_1 f'_c b} - (T_{sf} d + T_s d_s + T_{fs} d_f)]} - a] \tag{14}$$

Fig. 2 shows the necessary parameter used for calculating the value of $\cos\alpha$ which can be obtained from the geometry of the corbel. If the inclined strips of CFRP are provided in a manner that cover the whole height of the corbel-column junction, the equivalent depth used for calculating V_n in Eq.(13) is equal to h/2. A_{fi} is equal to the distance covered by the direct shear plane multiplied by the CFRP thickness. The thickness of CFRP sheet – epoxy composite in addition to the fracture stress should be taken from tensile measurements obtained from test results.

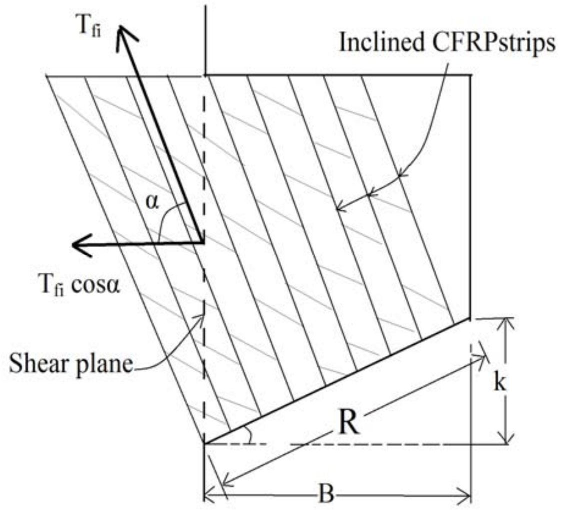

Fig. 2 Parameter of inclined CFRP strips

B. Shear Friction Method

Using the knowledge obtained through many studies carried out on direct shear of concrete, equations of shear friction are adjusted to include the effect of CFRP strips. The method is kept to be simple and applicable to the wide range of concrete grades and clamping stress represented by value of $\rho_{sv} f_{yv}$ due to shear reinforcement in conventionally reinforced concrete.

The method, in general, is similar to that provided by ACI 318 Code[5]. The parameters of area and strength of corbel materials can be used here for deriving the shear friction equations. The proposed model is essentially based on calculating the shear stress and comparing with the limits of maximum permissible shear stress of the corbel in the critical section. The smaller value is taken and the shear force capacity is calculated based on the calculated shear stress and corbel dimensions.

1. Shear Stress of the Strengthened Concrete

Based on the fact that the strip of CFRP could carry only the uniaxial stress, the equation of ACI 318 Code[5] can be written as follows

$$V_n = \mu A_{sv} f_{yv} + \phi A_{fs} f_f + \phi A_{fi} f_{fi} cos\alpha \tag{15}$$

For concrete placed monolithically like the case of the tested specimens the value of μ is 1.4λ and λ is equal to 1.0 for normal weight concrete, as recommended by the ACI 318 Code. f_{yv} is the yield stress of the stirrups. □ is the bond factor between CFRP and concrete surface and is taken as 0.75[2]. A_{fs} and f_f is the area and fracture stress of the horizontal CFRP strip, respectively, provided in the shear zone. A_{fi} is the area for the inclined strips, and f_{fi} is the fracture stress for the inclined strips. f_f is not necessary be equal to f_{fi}, because more than one layer of CFRP strip can be provided in each direction.

2. Maximum Shear Capacity of the Strengthened Section

According to the ACI 318 Code, the direct shear capacity of the concrete section should be taken as smaller than *0.2f'c* and 5.5 MPa. For compressive strength larger than 27.6 MPa, the 5.5 MPa governs the shear strength of the section and the use of high strength concrete instead of lower strength concrete in corbel design becomes useless. For this purpose, some attempts were made for deriving equations for calculating the shear strength capacity of reinforced concrete section made from high strength concrete. The following equation was obtained by Hassan and Mohammed[6] and used here beside the limits of ACI 318 Code for calculating the maximum shear strength of the section

$$v_n = 5.77 + 0.88(f_c')^{0.2} \sqrt{\rho_{sv} f_{yv}} \tag{16}$$

v_n is the nominal shear stress, *f □c* is the compressive strength of concrete, and $\rho_{sv} f_{yv}$ is the clamping stress or shear reinforcement index.

Test results obtained by Zangana[4] are used here for making a justification on the above limits of shear capacity. Such justification is necessary for calculating shear capacity of concrete strengthened with CFRP limits. Table (1) shows the results of the direct shear strength of concrete strengthened with CFRP strips obtained from Reference[4]. The ultimate shear capacity is represented by the percentage increase over that of plain concrete. It is assumed here that the percentage increase in direct shear is not affected by the existence of reinforcement in the section. Value of α usually taken from test results. The nonlinear equation of the following form was proposed for the percentage increase in the nominal shear stress

$$\Delta v_n = a(\rho_f f_f + \rho_{fi} f_{fi} cos\alpha)^b \quad (17)$$

ρ_f is the ratio of CFRP layer in the concrete section perpendicular to the shear plane. ρ_{fi} is the CFRP ratio of the inclined strips provided to the section and α is the angle of inclined strips measured from shear plane. Regression analysis carried out on the data of Table 1 shows that the constant a is equal to 0.069, and the constant b is equal to 1.177. Accordingly, the value of Δv_n becomes

TABLE I
PARAMETERS FOR CALCULATING MAXIMUM SHEAR STRESS OF STRENGTHENED CONCRETE [4]

Angle between strip and shear planes	No. of strips (at both sides)	Width of strips (mm)	Thickness of strips (mm)	A_{fs} (or A_{fi})	$\rho_f = \frac{A_{fs}}{bd}$	$\rho_{fi} = \frac{A_{fi}}{bd}$	f_f (MPa)	$\rho_f f_f$	Δv_n
90^0	8	20	1.4 (one layer)	224	0.00644	-	686	4.421	0.32
90^0	8	20	2 (two layers)	320	0.00921	-	590	5.432	0.52
90^0	8	20	2.7 (three layers)	432	0.01243	-	520	6.463	0.6
45^0	8	20	1.4 (one layer)	224	-	0.00644	686	3.125	0.25
45^0	8	20	2 (two layers)	320	-	0.00921	590	3.84	0.38
45^0	8	20	2.7 (three layers)	432	-	0.01243	520	4.569	0.50

TABLE II
RESULTS OF TEST AND CALCULATED ULTIMATE LOAD USING DIFFERENT METHODS

Corbel	*Vn,test* (kN)	*Vn.ST* (kN)	*Vn,test* / *Vn,ST*	*Vn,SF Eq.(15)* (kN)	*Vn,test* / *Vn,SF* Eq.(15)	*Vn,Eq.(20)* (kN)	*Vn,test* / *Vn,Eq.(20)*	*Vn.Eq.(21)* (kN)	*Vn,test* / *Vn,Eq.(21)*	*Vn,Eq.(22)* (kN)	*Vn,test* / *Vn,Eq.(22)*
C_1	478.0	510.19	0.937	342.05	1.39	527.54	0.91	287.1	1.66	639.76	0.75
C_2	462.8	506.33	0.914	342.05	1.35	526.16	0.88	287.1	1.61	620.55	0.75
C_3	408.65	450.96	0.906	342.05	1.19	509.24	0.8	287.1	1.42	419.69	0.97
C_4	494.55	372.59	1.327	342.05	1.44	526.83	0.94	287.1	1.72	629.74	0.78
C_5	520.0	592.14	0.878	342.05	1.52	527.34	0.99	287.1	1.81	639.42	0.81
C_6	410.0	523.50	0.783	342.05	1.19	510.47	0.80	287.1	1.43	432.22	0.95
C_7	548.15	671.17	0.817	486.13	1.13	696.77	0.79	378.9	1.45	852.04	0.64
C_8	553.1	687.75	0.804	486.13	1.14	695.99	0.79	378.9	1.46	841.02	0.66
C_9	491.8	645.47	0.762	372.11	1.32	641.34	0.77	378.9	1.29	843.22	0.58
C_{10}	469.3	681.63	0.688	486.13	0.96	694.57	0.68	378.9	1.24	821.18	0.57
C_{11}	516.7	683.63	0.756	486.13	1.06	695.44	0.74	378.9	1.36	833.3	0.62
C_{12}	583.85	827.74	0.751	551.45	1.06	788.0	0.74	429.65	1.36	937.43	0.62
C_{13}	603.35	865.13	0.719	551.45	1.09	790.3	0.76	429.65	1.40	969.9	0.62
C_{15}	511.3	703.92	0.768	551.45	0.93	787.42	0.65	429.65	1.19	928.8	0.55
C_{16}	430.0	706.15	0.646	551.45	0.78	762.09	0.59	429.65	1.00	628.07	0.68
Mean	-	-	0.82	-	1.17	-	0.79	-	1.426	-	0.703

$$\Delta v_n = 0.069(\rho_f f_f + \rho_{fi} f_{fi} cos\alpha)^{1.177} \quad (18)$$

The value of correlation coefficient (r) for the above equation is equal to 0.90.

The ultimate shear capacity for the composite section, v_{nc}, can be written as follows

$$v_{nc} = v_n(\Delta v_n + 1) \tag{19}$$

Therefore, the final form of the shear strength of the composite section using equations (16) and (19) becomes

$$v_{nc} = [5.77 + 0.88(f_c^{'})^{0.2} \sqrt{\rho_{sv} f_{yv}}][1 + 0.069(\rho_f f_f + \rho_{fi} f_{fi})^{1.177}] \tag{20}$$

The shear stress capacity of the composite section using the limits of ACI 318 Code is the smaller value of the following

$$v_{nc} = 5.5[1 + 0.069(\rho_f f_f + \rho_{fi} f_{fi} cos\alpha)^{1.177}] \tag{21}$$

$$v_{nc} = 0.2f_c^{'}[1 + 0.069(\rho_f f_f + \rho_{fi} f_{fi} cos\alpha)^{1.177}] \tag{22}$$

III. Validity of the Predictions

Table II contain results of calculated ultimate load capacity of corbels using different methods: strut and tie model, shear friction method and the limits of maximum shear force capacity in addition to the test ultimate load (taking from Reference[1]) for the comparison sake. For obtaining the best view of comparison between the test and calculated values, Figs.3 to 7 were drawn. The values of calculated ultimate load using strut and tie model are larger than the test ultimate load and accordingly the ratio of (tested/ calculated) is smaller than unity for all corbels with a mean value equal to the 0.82. Therefore, the strut and tie model is not accurate and not safe for calculating the ultimate load capacity of corbels strengthened with CFRP sheets. The reason of overestimating the ultimate load using strut and tie model is due to neglecting the effect of local stress concentration at the critical nodes, especially in that node near the corbel column junction point. Such effect was not included in the analysis. The effect of compressive stress concentration has a particular importance because other zones are far from failure as a result of strengthening with CFRP sheets. It was observed from test results[1] that the compression zone is the source of failure nearly for all the corbels due to crushing as a result of high stress concentration.

On the contrary, the shear friction method offers the calculated ultimate load smaller than the test ultimate load for all corbels except for corbel C_{10}, C_{15}, C_{16} but the mean value was found to be 1.17 as shown in Table 2. Therefore using shear friction method for analyzing reinforced concrete corbels strengthened with CFRP is safe and accurate. According to Eq.(15), the shear force capacity not depends on the concrete compressive strength and accordingly the shear force depends on the shear reinforcement properties, because the constant value of μ was used which is 1.4.

This approximation indicates that for the same reinforcement and CFRP properties of the corbel the ultimate load capacity of both HSC and NSC is identical. From the results of the tested corbels, one can find that such simplification is not correct because the ultimate load capacity for all corbels made from HSC was higher than that of NSC provided that the corbels reinforced with the same steel reinforced and CFRP configuration. Hence, the larger mean value takes place due to treating of the NSC like the HSC corbel and neglecting the effect of compressive strength on shear friction capacity. Fig.4 indicates that the prediction of shear friction method [Eq.(15)] is conservative and of good degree of accuracy. Now, it is necessary to compare the shear friction capacity with the limits of ACI 318 Code which were adjusted to include the effect of strengthening with CFRP. The limit of maximum shear force calculated from Eq.(22) which is based on $0.2f'c$ is not important and neglected in this discussion because it is larger than the limit 5.5 MPa, because all the tested corbels has a compressive strength larger than 27.6 MPa. It should be noted that the mean value for prediction of Eq.(22) is 0.703. The mean value of the prediction of Eq.(20) is 0.79 which is better than the Eq.(22). According to the recommendation of ACI 318 (in its basic form), the calculated ultimate load using Eq.(15) should not be larger than that of Eq.(21) and accordingly the limit of Eq.(21) governs the ultimate load capacity of all the tested corbels except corbel(9). If the limits of ACI 318 Code is not followed, and instead, other limits suitable for a concrete of higher compressive strength as given by Eq.(20) is used, Eq.(15) governs and the prediction will be more accurate compared with the test data of test / calculated ultimate load equal to 1.17.

IV. Conclusions

From the theoretical work presented in this paper the following conclusions can be drawn

1- Strut and tie model of its basic form is not accurate for calculating the load capacity of strengthened or repaired RC corbel with CFRP sheets due to neglecting the effect of stress concentration in critical zones. Better prediction can be obtained using modified shear friction theory of average test / calculated ultimate load equal to 1.17.
2- Using maximum shear stress limits suggested by ACI 318 in strengthened corbel considerably underestimates the predicted ultimate load capacity, especially for those corbels made of high strength concrete. If other limits suitable for the case of higher concrete strength is used the shear friction prediction will be accurate.

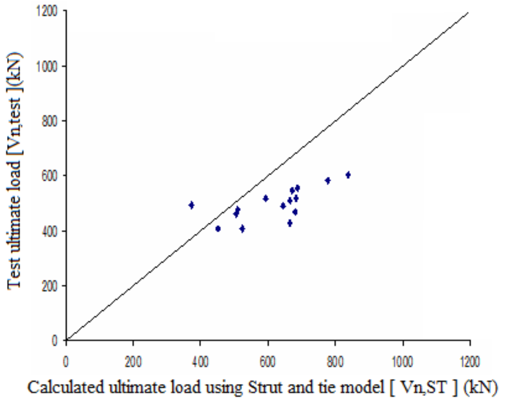

Fig. 3 Test versus calculated ultimate load using Strut and tie model

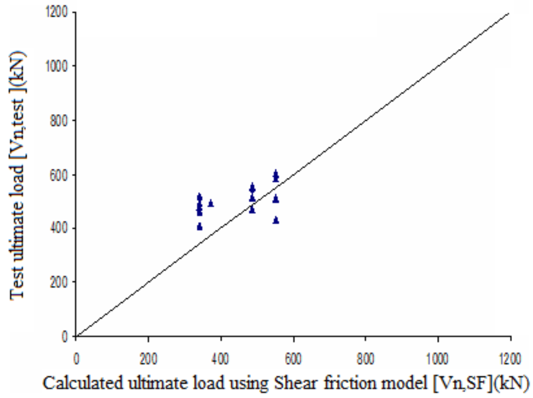

Fig. 4 Test versus calculated ultimate load using Shear friction model

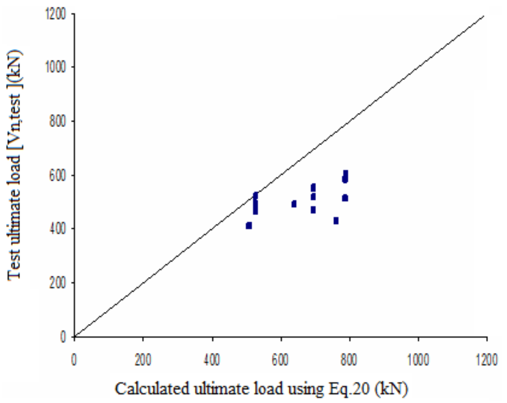

Fig. 5 Test versus calculated ultimate load using Eq.20

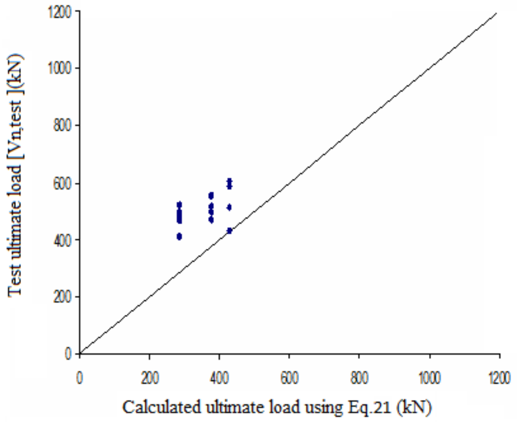

Fig. 6 Test versus calculated ultimate load using Eq.21

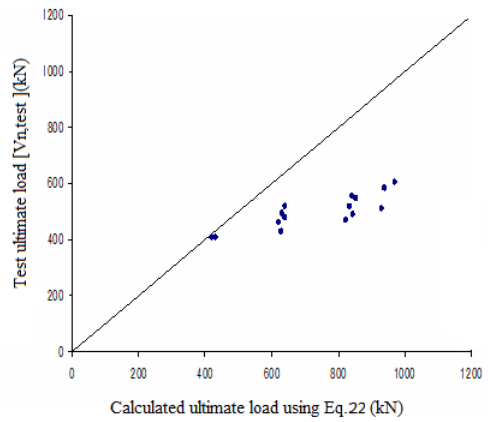

Fig. 7 Test versus calculated ultimate load using Eq.22

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