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Prediction of Compressive Strength of Concrete from Early Age Test Result Using Design of Experiments (RSM)

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***Abstract*—**Response Surface Methods (RSM) provide statistically validated predictive models that can then be manipulated for finding optimal process configurations. Variation transmitted to responses from poorly controlled process factors can be accounted for by the mathematical technique of propagation of error (POE), which facilitates 'finding the flats' on the surfaces generated by RSM. The dual response approach to RSM captures the standard deviation of the output as well as the average. It accounts for unknown sources of variation. Dual response plus propagation of error (POE) provides a more useful model of overall response variation. In our case, we implemented this technique in predicting compressive strength of concrete of 28 days in age. Since 28 days is quite time consuming, while it is important to ensure the quality control process. This paper investigates the potential of using design of experiments (DOE-RSM) to predict the compressive strength of concrete at 28th day. Data used for this study was carried out from experiment schemes at university of Benghazi, civil engineering department. A total of 114 sets of data were implemented. ACI mix design method was utilized for the mix design. No admixtures were used, only the main concrete mix constituents such as cement, coarse-aggregate, fine aggregate and water were utilized in all mixes. Different mix proportions of the ingredients and different water cement ratio were used. The proposed mathematical models are capable of predicting the required concrete compressive strength of concrete from early ages.

***Keywords*—**Mix proportioning, response surface methodology, compressive strength, optimal design.

I. Introduction

CONCRETE is the most widely used structural material for construction today. Traditionally, concrete has been fabricated from a few well-defined components: Cement, water, fine aggregate, coarse aggregate, etc. In concrete mix design and quality control, the strength of concrete is regarded as the most important property. The design strength of the concrete normally represents its 28^{th} day strength .In case of construction work 28 days is considerable time to wait for the test results of concrete strength, while it also represents the quality control process of concrete mixing, placing, proper curing etc. If due to some experimental error in mix design, the test results fail to achieve the designed strength, then repetition of the entire process becomes necessary, which can be costly and time consuming. It is necessary to wait at least 28 days, thus the need for an easy and suitable method. Data used for this study was carried out from experiment schemes at university of Benghazi, civil engineering department. A total of 114 sets of data were implemented. ACI mix design method was utilized for the mix design. No admixtures were used, only the main concrete mixes constituents such as cement, coarse-aggregate, fine aggregate and water were utilized in all mixes. Different mix proportions of the ingredients and different water cement ratio were used. The proposed mathematical models are capable of predicting the required concrete compressive strength of concrete from early ages.

Proportions of the ingredients and different w/c. estimating the strength at an early age of concrete were investigated. A rapid and reliable concrete strength prediction would be of great significance. There are many industrial problems where the response variables of interest in the product are a function of the proportions of the different ingredients used in its formulation. This is a special type of response surface problem called a (RSM). Using polynomial regressions, the DOE approach permits calculation of the response surfaces for the parameters under study over the experimental domain. Because of the high complexity of the relation between the compressive strength and component composition of concrete, conventional regression analysis could be insufficient to build an accurate model. This paper investigates the potential of using design of experiments (DOE) to predict the compressive strength of concrete at 28th day as the response. Response surface method (RSM) opens new possibilities in the classification and generalization of available experimental results to estimate concrete strength from the mix components. This work presents and examines the mix proportioning of ordinary concrete using RSM technique to optimize the mixture proportions and development of mathematical model for desired properties of concrete compressive strength in terms predictable and increase the efficiency of the prediction [1].

II. Experimental Program

Locally produced Ordinary Portland Cement (OPC) was used. The coarse aggregate was 19 to 25 mm maximum size crushed stone. No admixtures or additives was used in this study only the ordinary constituents of concrete (cement, sand, gravel, water) to study the effect of the ordinary mix proportion on the compressive strength of concrete. Specimens were immersed in water until the day of testing at

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3, 7, 28 days. The experiment work was carried out at university of Benghazi – civil engineering department. Total 114 sets of data were used to analyze the behavior of the concrete with time (age). ACI mix design method (ACI 211.1-91) was used for the mix design process. The concrete compressive strength after 28 day was defined as the strength obtained from standard cubes 150mm. The statistical analysis was performed using the computer software (Minitab). Computing the statistical information of the data like the mean, standard deviation and the correlation coefficient between the variable was present at Tables I and II. General constituents of concrete [cement(C), coarse-aggregate (CA), fine aggregate (FA) and water (W)] were used to evaluate the concrete compressive strength [2].

TABLE I
DESCRIPTIVE STATISTICS INFORMATION'S

Variable	Quantity	Mean	StDev	Min.	Max.
W/C	Water cement ratio	0.60	0.10	0.40	0.90
W	Wight of water, Kg	209.56	15.3	185.0	230.0
C	Wight of cement, Kg	356.28	55.0	250.0	513.0
CA	Wight of coarse aggregate Kg	1109.90	240.1	480.0	1401.0
FA	Wight of fine aggregate Kg	697.50	175.2	470.0	1119.0
fc`7	concrete compressive strength at 7 day Mpa	25.27	6.20	10.40	39.60
fc`28	concrete compressive strength at 28 day Mpa	34.81	7.20	16.40	49.20

TABLE II
CORRELATION BETWEEN THE VARIABLES USED FOR ANALYSIS

	W/C	W	C	CA	FA	fc`7
W	0.52					
C	-0.85	-0.08				
CA	0.02	-0.61	-0.32			
FA	0.16	0.61	0.09	-0.95		
fc`7	-0.63	-0.63	0.38	0.50	-0.62	
fc`28	-0.64	-0.57	0.37	0.35	-0.46	0.84

III. METHODOLOGY

Response surface methodology (RSM) consists of a set of statistical methods that can be used to develop, improve, or optimize products [3]. RSM typically is used in situations where several factors influence one or more performance characteristics, or responses. (RSM) used to optimize one or more responses, or to meet a given set of specifications (e.g., a minimum strength specification or an allowable range of slump values). There are three general steps that comprise (RSM): experiment design, modeling, and optimization [4].

Concrete is a mixture of several components. Water, portland cement, fine and coarse aggregates form a basic concrete mixture. Various chemical and mineral admixtures, as well as other materials such as fibers, also may be added. Choosing the design correctly will ensure that the response surface is fit in the most efficient manner. MINITAB provides central composite and Box-Behnken designs [3]. Consider a concrete mixture consisting of q component materials (where q is the number of component materials). Two approaches can be applied to concrete mixture, the mathematically independent variable approach, and optimization the mathematic model.

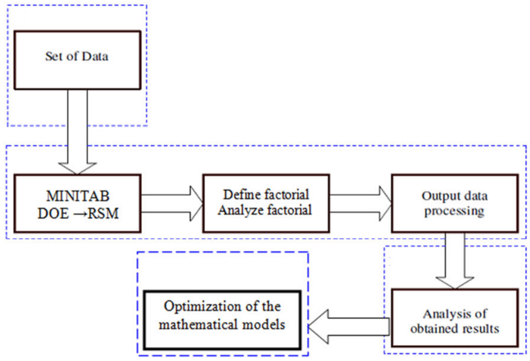

Fig. 1 The research diagram

The empirical models are fit to the data, and polynomial models (linear or quadratic) typically are used. The general case of the full quadratic model for k =3 as an example for independent variables is shown as [3]:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{22}x_1x_2 + b_{33}x_1x_3 + b_{23}x_2x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 \quad (1)$$

In (1), the ten coefficients are represented by the b_k and e is a random error term representing the combined effects of variables not included in the model. The interaction terms (x_ix_j) and the quadratic terms (x_i^2) account for curvature in the response surface. The central composite design (CCD), an augmented factorial design, is commonly used in product optimization [2]. The design consists of 2k factorial points representing all combinations of coded values $x_k = \pm1$, 2*k axial points at a distance $\pm\alpha$ from the origin, and at least 3 center points with coded values of zero for each x_k . The value of α usually is chosen to make the design rotatable, but there are sometimes valid reasons to select other values shown in Fig. 3.

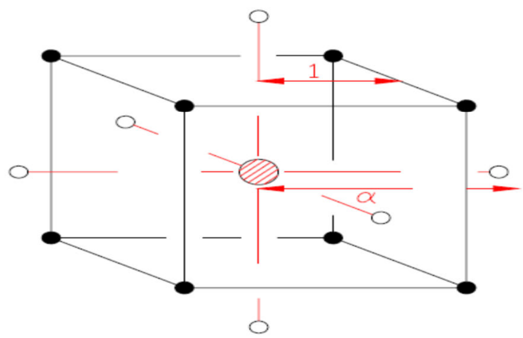

Fig. 2 Schematic of a central composite design for three factors

Optimization may be performed using mathematical (numerical) or graphical (contour plot) approaches. Numerical optimization requires defining an objective function that reflects the levels of each response in terms of minimum (zero) to maximum (one) desirability (D) which is [5]:

$$D = \frac{(w_1 \times d_1 + w_2 \times d_2 + \cdots w_n \times d_n)}{n} \qquad (2)$$

Several types of desirability functions can be defined. Common types of desirability functions are shown in Fig. 4.

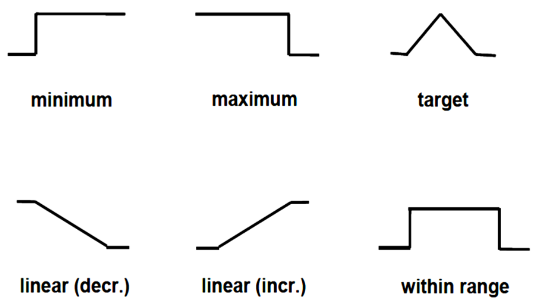

Fig. 3 Examples of desirability functions

When the model was completed, sets of tested data are used to determine the accuracy of regression for comparisons between the predicted compressive strength and true compressive strength. The best performances (observations) are selected according to several criteria. The chosen criteria are defined as follows:

1. Mean Absolute Percentage Error MAPE%

$$MAPE\,\% = \frac{100}{n}\sum_{i=1}^{n}\left|\frac{x_i - y_i}{x_i}\right| \qquad (3)$$

2. Coefficient of Efficiency E

$$E = 1 - \left(\frac{\sum_{i=1}^{n}(x_i - y_i)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right) \qquad (4)$$

IV. Development of Mathematical Model

In a mixture experiment, the response is observed at all mixture design points and the effects of component and interactions between components are investigated simultaneously. However, in concrete mixture design, certain mixture design points are not possible and must be omitted. Therefore, a flattened simplex-centroid mixture experiment design was adopted [4]. All compressive strengths were measured on 150 mm x 150 mm cub. These were fully compacted on a vibrating table, moist cured for 24 h_r, and then cured in water at 20°C until testing at 3, 7, and 28. Therefore, there were 114 training data covered six different levels of strength, about 15, 20, 25, 30,35and 40 MPa. It may be certain that these will form a fairly representative group covering all the ranges of practical use for concrete mixtures and will present the rather complete and independent information required for such an evaluation. The results of the compressive strength tests were subjected to polynomial regression using a computer program (MINITAB) [6]. Two polynomials were tried to represent the measured compressive strength data for five component contents and a 7-day age. The best fit for the compressive strength was obtained with a lowest absolute percentage error (MAPE %) and high coefficient of efficiency (E). The summary of the results were present in Tables II and IV, scatter diagrams showing in Figs. 4 and 5.

Often some of the terms are not significant, the full quadratic model and for each coefficient, calculate the t-statistic for the null hypothesis that the coefficient is equal to zero.

The Model (1) in terms of Mix components:

$$fc`28 = -42261 + 179962\,{}^{W}/_{c} - 446w + 312C - 20FA - 37500({}^{W}/_{c})^2 + w^2 - 311\,{}^{W}/_{c} - 163({}^{W}/_{c})(C) - 4({}^{W}/_{c})CA - 4({}^{W}/_{c})FA \qquad (5)$$

TABLE III
Mixture Experiment Summary Statistics for fc`28 day MPa Strength for Model (1)

Variable	Mean	StDev	Min	Max	
FIT1	34.67	6.76	17.08	43.25	95% CI
fc`28	34.67	7.35	16.40	49.20	33.46 - 36.15
MAPE%=			6.42		
E=		0.85			

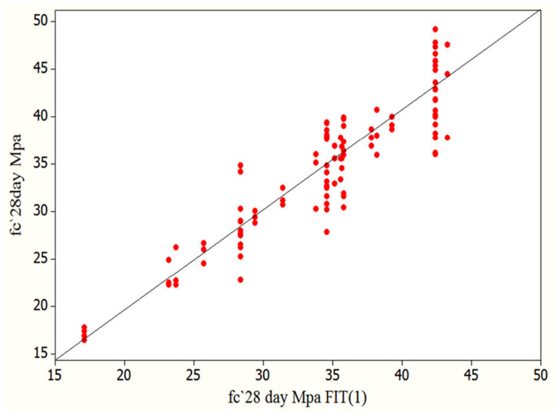

Fig. 4 Measured and predicted compressive strength of model 1

The Model (2) in terms of Mix components and the concrete compressive strength of fc`7 day:

$$fc`28 = -6684 + 58900.7\frac{w}{c} - 180.8w + 99.9c - 11.1CA + 9.4FA + 5.4fc`_{7day} - 12266.1\left(\frac{w}{c}\right)^2 + 0.5w^2 - 0.1c^2 - 99.6\left(\frac{w}{c}\right)w - 54.3\left(\frac{w}{c}\right)c - 1.7\left(\frac{w}{c}\right)CA - 1.1\left(\frac{w}{c}\right)FA - 5.6\left(\frac{w}{c}\right)fc`_{7day} - 0.1w \times c + 0.1w \times CA \qquad (6)$$

TABLE IV
Mixture Experiment Summary Statistics for fc`28 MPa Strength for Model (2)

Variable	Mean	StDev	Max	Min	
FIT 2	34.70	6.96	45.64	15.97	95%CI
Fc`28	34.70	7.30	49.20	16.40	33.46-36.15
MAPE%		4.87			
E		0.91			

It can be seen that, the MAPE % for the results is rather low, and the E for the models are so high as to provide accurate predictions. In other words, the models are good in generalization and close to the observed value within the range of 95% confidence interval. However, there is uncertainty in the fitted functions, because they are estimated from a sample of data. The uncertainty provided is for a 95 percent confidence interval.

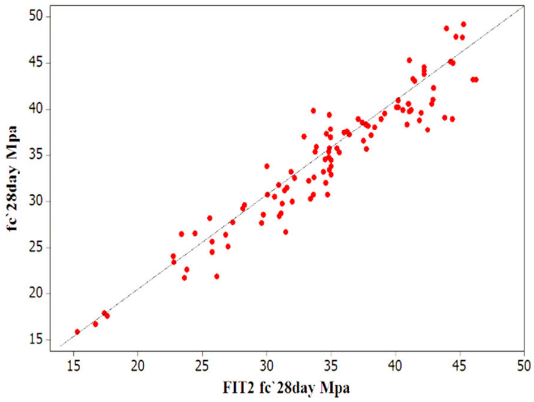

Fig. 5 Measured and predicted compressive strength of model (2)

V. Optimization

Numerical optimization using desirability functions can be used to find the optimum mixture proportions in this situation. A desirability function must be defined for each response (property) [6]. The desirability function takes on values between 0 and 1, and may be defined in several ways. A several targets were optimizing for 28 fc` concrete compressive strength to find out the proper mix component meet the target. In this study, the overall desirability D was defined as the geometric mean of the individual desirability functions d_i over the feasible region of mixtures. Numerical optimization for model (1) gave the following best settings for this concrete mixture as shown in Table V.

TABLE V
Numerical Optimization for Model (1)

The fc` 28 target	W/C	W	C	CA	FA
20	0.46	230	512.8	1401	1119
25	0.69	185	513	480	1117
30	0.70	185	250	1394	480
35	0.70	185	250	1397	480
40	0.69	185	513	480	1114
45	0.69	185	513	480	1113

VI. Summary and Conclusions

In this study shows the following:

1) Data used for this study was taken from experiment results were done at university of Benghazi – civil engineering department. Total 114 sets of data were used to analyze the behavior of the concrete with time. ACI mix design method (ACI 211.1-91) was used for the mix design process. the concrete compressive strength was defined as the strength obtained from standard cubes (150mm) no admixtures or additives were used only the general constituents of concrete [cement (C), coarse-aggregate (CA), fine-aggregate (FA) and water (W)]. Different mix proportions of the ingredients and different w/c ratio were used to study the variations.
2) The statistical error analysis shows the adequacy of the model obtained and also that the terms added to the model have significant effects on the response variable. For both of the models, values of (E) are 0.85 and 0.91 for the first and second models. Therefore, it can be said that the models established for compressive strength are adequate. In both of the models, the MAPE% is in reasonable agreement with the prediction for compressive strength (MAPE% is 6.7% and 4.5%). This indicates that the amounts of results obtained from the models are appropriate.
3) The models are good in generalization and close to the observed value within the range of 95% confidence interval.
4) Based on the data obtained from the RSM models, high correlations between the compressive strength and the component composition of concrete can be developed using the generalization capabilities. Such a model can be efficiently used for simulating the compressive strength behavior.
5) The importance of the influence of mix constituents on the strength of concrete was approved. Two mathematical models for the prediction of concrete compressive strength at the age of 28 was proposed and developed (using RSM) from the knowledge of the mix constituents, and strength at the ages of 7 day.
6) Practical approach has been described for prediction of 28-day compressive strength of concrete and the proposed technique can be used as a reliable tool for assessing the design strength of concrete from quite early test results. This will help in making quick decision for accidental poor concreting at site and reduce delay in the execution time of large civil construction projects.
7) A several targets were optimizing for 28 fc` concrete compressive strength to find out the proper mix component meet the target.

Nomenclature

C	= cement Kg/m^3
CA	= coarse-aggregate Kg/m^3
D	= desirability
DOE	= design of experiment
E	= coefficient of efficiency
FA	= fine-aggregate Kg/m^3
fc`28	= concrete compressive strength (MPa) at 28 day
fc`7	= concrete compressive strength (MPa) at 7day
MAPE%	= Mean Absolute percentage Error
n	= the number of data observed
q	= particle density
RSM	= Response surface method
StDev	= standard deviation
W	= water Kg/m^3
W/C	= water/cement
x_i	= The observed data
y_i	= The predicted data

$\bar{x}$ = the average of data observed
95%CI =95 percent confidence interval MPa

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