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Predicting Radiative Heat Transfer in Arbitrary Two and Three-Dimensional Participating Media

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***Abstract*—**The radiative exchange method is introduced as a numerical method for the simulation of radiative heat transfer in an absorbing, emitting and isotropically scattering media. In this method, the integro-differential radiative balance equation is solved by using a new introduced concept for the exchange factor. Even though the radiative source term is calculated in a mesh structure that is coarser than the structure used in computational fluid dynamics, calculating the exchange factor between different coarse elements by using differential integration elements makes the result of the method close to that of integro-differential radiative equation. A set of equations for calculating exchange factors in two and three-dimensional Cartesian coordinate system is presented, and the method is used in the simulation of radiative heat transfer in two-dimensional rectangular case and a three-dimensional simple cube. The result of using this method in simulating different cases is verified by comparing them with those of using other numerical radiative models.

***Keywords*—**Exchange factor, Numerical simulation, Thermal radiation.

I. Introduction

RECENT improvements in computer power have increased the interest of engineers and researchers to simulate their problems with computational methods. A lot of computational tools and methods have been developed in the last decades to analyze fluid dynamics, combustion, and different modes of heat transfer, which can be used in two- and three-dimensional configurations.

Among other practical problems, one of the most important practical problems having a highlighted role in the design and operation of high temperature industrial equipment, analyzing radiative heat transfer in the participating media, has received considerable attention. Radiative heat transfer in the participating media than can absorb, emit and scatter radiation , and is usually surrounded by emitting, absorbing and reflecting walls, is a significant in many industrial applications, such as boilers, furnaces and jet engines. In addition to industrial interest, radiation heat transfer is an effective parameter in the effect of dust, carbon dioxide and other participating gases on the global environment. In most industrial applications, radiative heat transfer is accompanied by turbulent reactive flow and combustion. Thus, an ideal numerical radiative model should be capable of being coupled with other numerical methods in computational fluid dynamics and combustion modeling.

The electromagnetic wave theory and quantum mechanics are two hypotheses for describing the physics of thermal radiation. Even though most of the phenomena in thermal radiation can be explained by the electromagnetic wave theory, there are some exceptions which can not be viewed by the electromagnetic wave theory, such as the spectral distribution of energy emitted from a body, and the radiative properties of gases [1]. Based on these two physical hypotheses, the numerical radiative methods are divided into two main categories ; ray tracing methods and flux methods.

The Monte Carlo method [1-4] and discrete transfer radiation method [5-7] are well established ray tracing methods based on following the energy bundles (photons) by using the concept of random numbers until they are absorbed or exit from the system. The algorithms of these methods are basically different from the algorithms of the methods used in computational fluid dynamics. In addition, they are time consuming and computationally demanding. These problems are the reason why these methods have seldom been used in practical applications.

The flux methods, such as discrete ordinate method [8-14] and finite volume method [15-18], are based on solving the integro-differential equation of radiation in descritized geometry. Chandrasekar [8] presented in 1950 the basics of the discrete ordinate method (DOM), and later Lathrop and Carlson [9] and Truelove, Fiveland and Jamaluddin [10-14] developed this method further. The capability of this method is improved by decreasing the ray effects and false scattering, and using more accurate quadratures. However, in this method, for coupling radiative transfer and other physical phenomena, such as turbulent reactive flow and combustion, the radiative transfer equation should be solved in the same mesh structure as the one used to solve the other balance equations, and therefore radiative interaction between neighbor calls is taken

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into account.
Raithby and Chui [15] have introduced the radiative finite volume method (FVM) based on solving the radiative transfer equation (RTE) in descritized geometry. Actually, the radiative balance between the faces of the control volume is done by attenuation and augmentation of radiant energy within a control volume and a control angle. In the last decades, a lot of research has been conducted to improve the radiative FVM with different types of mesh structure and geometries [15-18].
Although the radiative FVM and DOM can be coupled easily with other numerical solutions in fluid flow and combustion modeling and model complex geometry, they basically work on the bases of diffuse radiation, when just the radiation interaction between neighbor cells is taken into account. In most problems for achieving a good level of accuracy, the radiative interaction between all points of the media and surrounding walls should be taken into account for example when the participating media is optically thin because of the low local extinction coefficient or size of the cells.
The zone method [19-22] is one of the oldest numerical methods proposed for predicting radiant heat transfer in participating media. In the zone method, the amount of radiation exchange between each pair of zones is defined by using a special coefficients named "Direct Exchange Area". This coefficient between a source cell "i" and a destination zone "j" is defined as the fraction of the emitting radiative heat power from source zone "i" that is absorbed in "j" (if "j" is a volume zone) or reaching to "j" (if "j" is a surface zone). The amount of net radiative heat transfer for each zone is defined by another coefficient named heat flux area, which should be calculated from the direct exchange area. Finally, the net radiative heat transfer is added to the overall energy balance in each zone. Finally, by solving these equations and inserting the effect of radiative thermal boundary conditions on the wall, the temperature of all the zones are obtained.
Bordbar and Hyppänen [22] have employed the radiative zone method coupled by combustion and flow modeling, to predict the effect of changing the fuel on the operation of a boiler.
Although in the zone method the radiative interaction between all points of the geometry is taken into account, and therefore the radiative zone method has better physical basis, there are still several unsolved problems in this method such as the singularity problem in the calculation of the direct exchange area, and the supporting complex geometries.
In 1993, Maruyama [23] introduced a new definition of view factors and developed the ray emission method for analyzing radiative heat transfer in arbitrary three-dimensional surfaces with specular and diffuse reflection. This idea has grown gradually and was finally used by Maruyama and Aihara [24], to develop a new generalized numerical radiative model for simulating radiative heat transfer in fully participating media surrounded by surfaces with specular and diffuse reflection. They developed a kind of ray tracing method for calculating view factors.
In this article, the theoretical basis of a new method named radiative exchange method is presented. The basic idea, as illustrated in Fig. 1, is to solve the radiative heat transfer in a mesh structure that is coarser than the structure used for solving other balance equations, such as the mass and momentum balance equations.
The calculation of the flow field and other modes of heat and

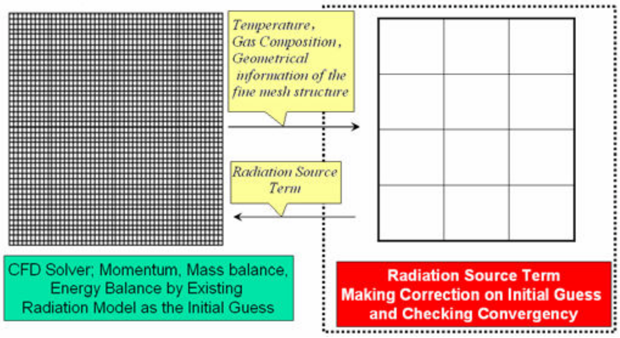

Fig. 1 The basic idea of radiative exchange method

mass transfer (e.g. combustion, conduction and convection) can be done in CFD commercial softwares like FLUENT or CFX, and the calculation of radiative heat transfer is done in a separate box. The information needed for calculating the radiative source term is taken from the CFD solver for the fine mesh structure used in the flow calculation, and is used in the calculation of the properties for the coarser cells, which are used in calculating the radiative source terms. After calculating the radiative source term with the radiative exchange method in the coarse mesh structure, the result is interpolated over the fine structure and the calculated values are added to the overall energy balance equations.
The definition of the exchange factor in the radiative exchange method is different from the definition of the direct exchange area in the radiative zone method. In this new concept of the exchange factor, the radiative interaction between all points of the geometry is taken into account.
In the next sections, after describing the theoretical basis of radiative balance in the radiative exchange method, the new concept of the exchange factor is introduced and the related equations for calculating exchange factors in two and three-dimensional configurations are derived. The result of using this method in the simulation of radiative transfer in a 2D simple rectangular and a 3D simple cube is presented and compared with the result of using other numerical radiative methods, such as the DO and P1 approximation methods.

II. Different Mesh Structure Used in the Radiative Exchange Method

To clarify the meaning of the different terms used to describe the radiative exchange method in this paper, three different mesh structures used in the approach to be coupled with CFD solvers, are introduced in this section before going through the theory of the method.

A. Fine Mesh Structure

This structure is used in the CFD solver for solving other balance equations except the radiative balance equation, such as the mass, momentum, combustion and turbulence equations.

B. Coarse Mesh Structure

As in the zone method, in the radiative exchange method the participating media and its walls are decomposed to some finite volume and surface cells. The radiative balance equation is written for these large size cells and the amount of radiative source terms is calculated in the centers of these cells. The amount of radiative source term on the walls contain the distribution of the net radiative heat flux in the walls, whereas the radiative source terms in the volume cell centers are used to obtain the radiative source term in the fine cell centers by using different interpolation techniques. After finding the radiative source terms in the cell centers, they are added to the overall energy balance equation in the solver to obtain the new temperature field.

C. Integration Structure

The definition of exchange factors and its equations are described in the next sections. Briefly, to calculate the exchange factor between the different coarse cells in the system, it is necessary to calculate some finite integrals over the volume or area of the coarse cells. It is assumed that the integration elements should be optically thin enough to be considered as differential radiant elements, so that no integration is needed for calculating the exchange factor between the integration elements,. By this assumption, also the amount of radiative outgoing power from the volume integration elements, which is absorbed or scattered within the element itself, can be ignored. This parameter is defined as the self-extinction and is discussed in the next sections.

III. Radiative Balance In the Radiative Exchange Method

A. Integro-Differential Equation of Radiation

The general governing equation of radiative heat transfer is an intrgro-differential equation. As illustrated in Fig. 2, for a simple cubic volume cell containing participating media, the radiation intensity at $\vec{r}$ in the direction $\vec{s}$ can be stated from radiative balance as :

$$\frac{dI(\vec{r},\vec{s})}{ds}+(k_a+k_s)I(\vec{r},\vec{s})=k_a n^2\frac{\sigma T^4}{\pi}+\frac{k_s}{4\pi}\int_0^{4\pi}I(\vec{r},\vec{s}')\Phi(\vec{s}.\vec{s}')d\Omega \quad (1)$$

where k_a and k_s are the spectral absorption and scattering coefficients, respectively. s is the path length in direction $\vec{s}$, $\Phi(\vec{s}.\vec{s}')$ is the phase function from direction $\vec{s}$ to $\vec{s}'$, and Ω is the solid angle.

As shown in Fig.2, the change of radiative intensity in distance ds is the sum of the emission power of the gas molecules and additional scattering minus the amount of loss due to absorption and scattering.

For the calculation of radiative attenuation between two points that is approximated to a single beam, the following form of the balance equation can be written:

$$\frac{dI(\vec{r},\vec{s})}{ds}=-\beta I(\vec{r},\vec{s}) \quad (2)$$

where β represents the extinction coefficient, i.e. sum of scattering and absorption coefficients. This equation is used for calculating attenuation between the integration elements during the calculation of the exchange factor between coarse cells.

B. Radiative Balance in Radiative Exchange Method

As mentioned above, in the radiative exchange method, the participating gas and its walls are decomposed to some finite volume and surface cells, and the radiative balance is derived for each of them. If we assume that there is M surface and N volume coarse cells in the system, then for each coarse cell,

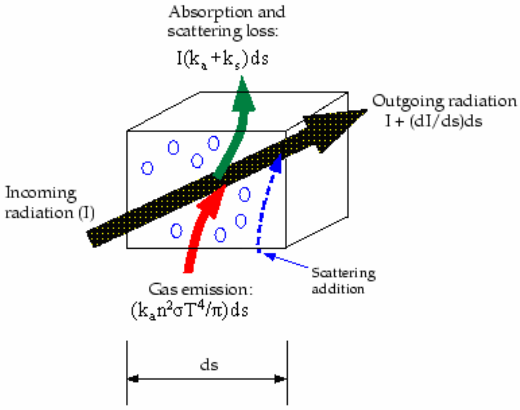

Fig. 2 Radiative balance for simple volume cubic cell containing participating gas

the amount of the radiative source term is equal to the difference between incoming radiative power from other cells in the system to this cell and the outgoing radiative power from this cell to the other cells in the system.

For the individual volume cell, the amount of outgoing radiative power is the sum of the scattering and emission power of the gas molecules. The scattering power is proportional to the incoming radiative power from other cells. Therefore, by defining the exchange factor from cell " j ", which can be a volume or surface cell, to volume cell " i ", Υ_{ji}, is defined as the ratio of the amount of outgoing radiative power from cell " j ", which is absorbed or scattered in volume cell " i " to the amount of outgoing radiative power from cell " j ". Thus, for the differential volume cell when the amount of self-extinction can be ignored, it is driven that

$$q_{out,i}=\int_{V_i}4k_e\sigma T^4 dV+k_{s,i}\left[\sum_{\substack{j=1\\ j\neq i}}^{N}\left(\Upsilon_{ji}\,q_{out,j}\right)+\sum_{j=1}^{M}\left(\Upsilon_{ji}\,q_{out,j}\right)\right] \quad (3)$$

where k_e, k_s, V, N, M represent the emission coefficient and scattering coefficient, the volume of the cell, the number of volume cells in the system, and the number of surface cells in the system, respectively. For the non-differential volume cell, the effect of self extinction should be considered in the calculation of outgoing radiative power from the cell, and therefore (3) is modified to the following equation:

$$q_{out,i}=(1-\Upsilon_{ii})\left\{\int_{V_i}4k_e\sigma T^4 dV+k_{s,i}\left[\sum_{\substack{j=1\\ j\neq i}}^{N}\left(\Upsilon_{ji}\,q_{out,j}\right)+\sum_{j=1}^{M}\left(\Upsilon_{ji}\,q_{out,j}\right)\right]\right\} \quad (4)$$

where Υ_{ii} is the self-extinction within volume cell "i".

For the coarse surface cell, the amount of outgoing power is the sum of the emission and reflection power, and the reflection power is proportional to the incoming power from other cells. Therefore, by defining the exchange factor from cell "j" that can be the surface or volume to surface cell "i", Υ_{ji}, is defined as the ratio of the amount of outgoing radiative power from cell "j" that reaches to surface cell "i" to the amount of outgoing radiative power from cell "j". Thus, the following equation can be written for the outgoing radiative power from each surface coarse cell "i";

$$q_{out,i} = \int_{A_i} k_e \sigma T^4 dA + k_{r,i} \sum_{j=1}^{N+M} \left(\Upsilon_{ji} q_{out,j} \right) \quad (5)$$

where A, k_r represent the area and reflection coefficient of the surface cell.

By writing (4) for all the volume cells and (5) for the surface cells, a set of algebraic equations is obtained. By using the temperature field of the previous iteration and the exchange factors, a system of equations for calculating the outgoing radiative heat power from each cell is achieved. After solving this system of equations and calculating the amount of outgoing radiative power from each cell, by using the definition of exchange factors, the incoming radiative power and radiative source term for each cell are obtained. The calculated radiative source term is in use for calculating a new temperature field in the CFD solver and doing the next iteration. This kind of hybrid system, as shown in Fig. 1, is continued until the limit of convergence for radiative source terms is achieved.

IV. Exchange Factor

In the previous section,the definition of exchange factors was introduced. for different states when a volume cell is considered as the destination of radiation and when a surface cell is considered as the destination of radiation. In this section, the equations of exchange factors are derived for different states , i.e. volume to volume, surface to volume, volume to surface and surface to surface, and for two- and three-dimensional configurations.

To satisfy the radiative energy conservation, all the outgoing radiative power from each cell in the system should be absorbed or scattered in the volume cells or reach the surface cells in the system. Therefore, for each cell "i" as the source of radiation, it can be written that

$$\sum_{\substack{j=1 \\ j\neq i}}^{M+N} \Upsilon_{ij} = 1 \quad (6)$$

This equation presents a good criterion for checking the accuracy of the calculation of the exchange factor. However, to decrease the effect of error in the calculation of exchange factors in the overall accuracy of the approach, the calculated exchange factors can be scaled before using in (4) and (5).

A. Three-Dimensional Equations for the Exchange Factors

For three-dimensional configuration based on the definition of the exchange factors, the following equations can be presented for exchange factors in different states:

$$\Upsilon_{V_i-V_j}\Big|_{i\neq j} = \frac{1}{\tau_{V_i}} \left[\frac{1}{\int_{V_i} 4\beta_i \sigma T_i^4 dV_i} \int_{V_i}\int_{V_j} \alpha_{ij} \frac{\beta_i \beta_j}{\pi S_{ij}^2} (4\beta_i \sigma T_i^4) dV_i dV_j \right] \quad (7)$$

$$\Upsilon_{V_i-S_j} = \frac{1}{\tau_{V_i}} \left[\frac{1}{\int_{V_i} 4\beta_i \sigma T_i^4 dV_i} \int_{V_i}\int_{A_j} \alpha_{ij} \frac{\beta_i \cos\theta_j}{\pi S_{ij}^2} (4\beta_i \sigma T_i^4) dV_i dA_j \right] \quad (8)$$

$$\Upsilon_{S_i-V_j} = \frac{1}{\int_{A_i} (k_{e,i}+k_{r,i}) \sigma T_i^4 dA_i} \int_{A_i}\int_{V_j} \alpha_{ij} \frac{\beta_j \cos\theta_i}{\pi S_{ij}^2} (k_{e,i}+k_{r,i}) \sigma T_i^4 dA_i dV_j \quad (9)$$

$$\Upsilon_{S_i-S_j} = \frac{1}{\int_{A_i} (k_{e,i}+k_{r,i}) \sigma T_i^4 dA_i} \int_{A_i}\int_{A_j} \alpha_{ij} \frac{\cos\theta_i \cos\theta_j}{\pi S_{ij}^2} (k_{e,i}+k_{r,i}) \sigma T_i^4 dA_i dA_j \quad (10)$$

where i, j, Υ_{ij}, S, and θ represent the source cell of radiation, the destination cell of radiation, the exchange factor, the center to center distance of the two radiative elements, the angle between the normal vector of the surface elements, and the center to center vector ($\vec{S}$), respectively.

α_{ij} is the attenuation of radiation between two cells and can be numerically calculated by using (2).

In (7) and (8), τ_{V_i} is equal to $1-\Upsilon_{V_i-V_i}$ and inserts the effect of self-extinction in the calculation of the exchange factors. The self- extinction for the coarse volume cell is defined as:

$$\Upsilon_{V_i-V_i} = \frac{1}{\int_{V_i} 4\beta_i \sigma T_i^4 dV_i} \int_{V_i}\int_{V_j=V_i} \alpha_{ij} \frac{\beta_i \beta_j}{\pi S_{ij}^2} (4\beta_i \sigma T_i^4) dV_i dV_j \quad (11)$$

B. Two-Dimensional Equations for the Exchange Factors

Einstein [25] has presented a set of equations for calculating the direct exchange area used in the radiative zone method in two-dimensional configuration, when the system has an infinte dimension in the third direction. Using same idea as Einstein, Bordbar and Hyppänen [26] have presented a set of equations for calculating the exchange factor based on above described definition of two-dimensional configurations. These equations are used in two-dimensional sample of this article.

V. Results and Discussion

A. 2D Simple Cubic

As a simple two-dimensional case, the radiative exchange method is used to simulate radiative transfer in a rectangular with the dimensions $6m \times 15m$. A hot stream of carbon dioxide with the temperature of 1600K is comes into the space with a certain velocity profile, which is shown in Fig. 3. The side walls of the space are kept in the temperature of 800K, and the absorption and reflection coefficient of the wall is 0.5.

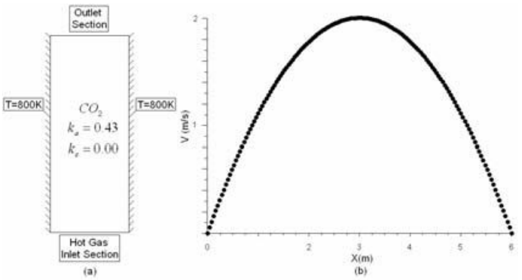

Fig. 3 Description of a two-dimensional sample

Fig. 4 shows the distribution of the exchange factors from a volume cell located in the center of the space as the source of radiation to all other volume cells as the destination of radiation and from a surface cell located in the middle of side wall to all other cells.

As Fig. 4 shows, for the conditions shown in Fig.3, most of the radiative power from the cells is in this case lost within the

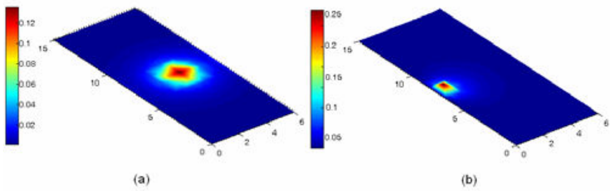

Fig. 4 Distribution of the exchange factors (a) from a volume cell located in the middle of geometry and (b) from a surface cell located in the middle of one of the side walls

closest volume cells.

The Temperature distribution in the gas obtained by the radiative exchange method has been compared with the DO, P1, and DTRM method in Fig. 5. As Fig. 5 shows, the calculated temperature field with the radiative exchange method is in good conformity with those of other numerical radiation methods.

B. 3D Simple Cubic

As an example of using the radiative exchange method in the simulation of three-dimensional participating media, the radiation heat transfer of a simple cubic of carbon dioxide is analyzed. The gas with the temperature of 1600K and a certain velocity profile enters the cube from the bottom, and the side walls are in temperature of 800K. The cube is full of carbon dioxide as shown in Fig. 6.

The radiative heat transfer within this sample is modeled with the radiative exchange method, and the result for the

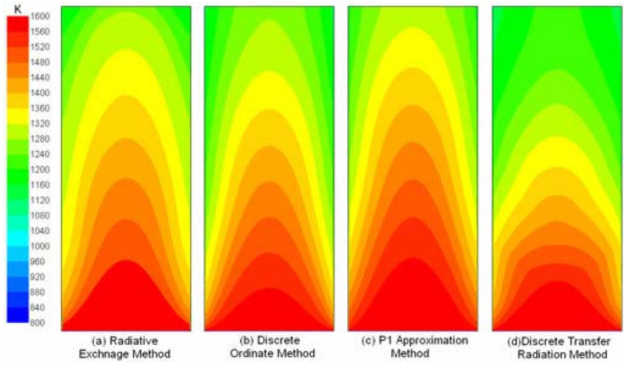

Fig. 5 Comparison between the result of radiative exchange method with the results of other numerical methods for the temperature field of the gas;(a) Radiative Exchange Method,(b) Discrete Ordinate Method,(c)P1 Method,(d) Discrete Transfer Radiation Method

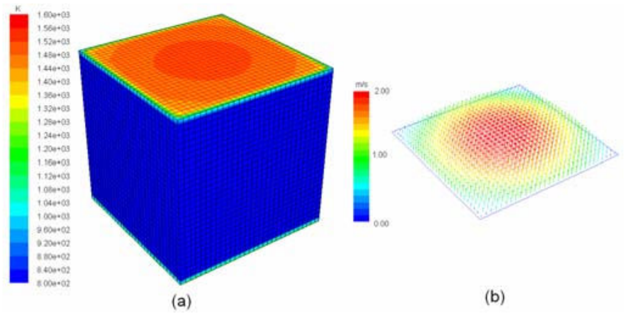

Fig. 6 Description of the 3D sample

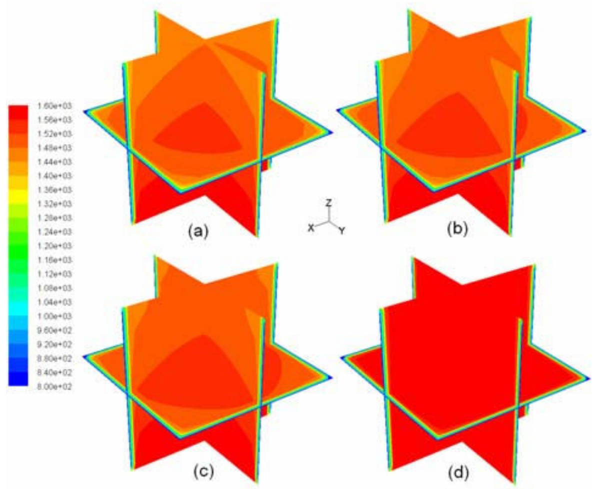

Fig. 7 Comparison between the result of the radiative exchange method with the results of other numerical methods for the temperature field of the gas; (a) Radiative Exchange Method,(b) Discrete Ordinate Method,(c) P1 Method,(d) No radiation model used

temperature profile of the gas is shown in Fig. 7. The coarse cells with the optical thickness of 0.35 were used for modeling this sample, while the minimum optical thickness of integration elements used in the calculation of the exchange factor was 0.02. A good conformity is observed between the result of the radiative exchange method with the results of other numerical radiation methods.

VI. Conclusion

The radiative exchange method was introduced as a generalized numerical method for the simulation of radiative heat transfer in absorbing, emitting, and isotropically scattering media surrounded by emitting, absorbing, and isotropically reflecting walls. The basis of the theory of radiative balance in this method was described. A new concept of exchange factors needed for the theory of this method was introduced and the related equations for three-dimensional case were reviewed. As two simple problems, the radiative exchange method was used to simulate radiative transfer in a two-dimensional rectangular and a three-dimensional cube problem. The result of using this method showed a good conformity with the results of using other numerical methods, such as DO, P1 and DTRM.

Although the number of discrete points for defining the radiative source term within the domain is related to the size of the coarse cells, the accuracy of the whole approach is highly dependent on the accuracy of the exchange factor calculation. The accuracy of the exchange factor calculation depends on the optical thickness of the integration elements, and thus a separate study to drive suitable correlations for the exchange factors will be highly valuable and will have a great effect on the accuracy and the computational time of the approach. This is one of the activities of our group that may be reported in our next publications.

The convergence of the method is very rapid, and even for large geometries the amount of source term is converged into the final value after a few iterations.

From the physical point of view, this method has a good physical basis in considering radiative interaction between all points of the space, not just between neighbor cells.

Using integration elements with very low optical thickness, the result of the method gets close to the solution of the integro-differential equation of radiative balance.

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