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Predicting Bridge Pier Scour Depth with SVM

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***Abstract*—**Prediction of maximum local scour is necessary for the safety and economical design of the bridges. A number of equations have been developed over the years to predict local scour depth using laboratory data and a few pier equations have also been proposed using field data. Most of these equations are empirical in nature as indicated by the past publications. In this paper attempts have been made to compute local depth of scour around bridge pier in dimensional and non-dimensional form by using linear regression, simple regression and SVM (Poly & Rbf) techniques along with few conventional empirical equations. The outcome of this study suggests that the SVM (Poly & Rbf) based modeling can be employed as an alternate to linear regression, simple regression and the conventional empirical equations in predicting scour depth of bridge piers. The results of present study on the basis of non-dimensional form of bridge pier scour indicate the improvement in the performance of SVM (Poly & Rbf) in comparison to dimensional form of scour.

***Keywords*—**Modeling, pier scour, regression, prediction, SVM (Poly & Rbf kernels).

I. Introduction

ESTIMATION of accurate bridge-pier scour (Fig. 1) is important for the safe and efficient design of bridges constructed over natural streams and rivers and has been identified as one of the major causes of the bridge failures worldwide. Scour failures occur suddenly without any prior warning and are really very difficult to monitor during the flood events. Since bridge scour is one of the potential causes of bridge failures hence considerable amount of laboratory and field studies have been devoted in the past for examining the effect of variables affecting the pier scour and suggested physical process based models. There are generally three types of scours that affect the performance and safety of the bridges, namely, local scour, contraction scour, and degradation scour, which has been explained in [1]. Factors affecting the bridge scour include channel and bridge geometry, floodplain characteristics, flow hydraulics, bed materials, channel protection measures, channel stability, riprap placement, debris, etc. The mechanism of flow around a pier structure such as horse shoe vortex is so complicated that it is difficult to establish a general empirical model to predict the scour depth.

The details of existing empirical equations for cohesionless sediments for estimating equilibrium local scour around bridge piers based on lab and field data have been studied in detail by [2]. Thus a reliable estimation of maximum local scour depth is of paramount importance in safe, economic, and technically sound bridge pier design.

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II. Bridge Pier Scour Prediction

It has been observed over the past few decades by the bridge engineers and researchers that there are a wide variety of non-physical variables, such as combined effects of turbulent boundary layer, time-dependent flow pattern, and sediment transport mechanism along with physical parameters of flow and sediment characteristics as well as pier geometry, which make the prediction of bridge pier scour a complex problem. Scour prediction practice can be generally divided into four categories: (i) predict bridge scour using theoretical approach (ii) experimental approach empirical equations (iii) Numerical approach and (iv) predict bridge scour using soft computing methods [3], [4]. To describe the scour depth prediction, experimental studies have been presented by combining dimensional analysis with the experimental test of model in the past [5].They developed the following equation for the scour which modeled only upstream half of the pier scour hole, and observed that scour hole erosion depends on diameter and depth of flow of water.

$$\frac{d_s}{D}=\left(\frac{Y}{D}\right)^{0.62}\left(\frac{U}{(gY)^{0.5}}\right)^{0.2}\left(\frac{D}{D_{50}}\right)^{0.08} \quad (1)$$

Experimental relationships may be inadequate because large number of parameters is affecting the scour. Reference [6] studied the effect of drift accumulations on scour in sand around the foot of a model pier of diameter 0.06meter. They proposed an equation for the estimation of pier scour depth as given below.

$$d_s = 1.35D^{0.7}Y^{0.3} \quad (2)$$

References [7]-[10] developed the following equations for estimation of pier scour around bridge piers

$$d_s = 0.00022\left(\frac{UD}{\nu}\right)^{0.619} \quad (3)$$

$$\frac{d_s}{D}=f\left(\frac{U}{Uc}\right)2\tanh\left(\frac{y}{D}\right)^{0.08} \quad (4)$$

$$\frac{d_s}{D}=2.42\left(\frac{2U}{Uc}-1\right)^{0.62}\left(\frac{U_c{}^2}{gD}\right)^{1/3} \quad (5)$$

$$\frac{d_s}{D}=K_3\left(\frac{Y}{D}\right)^{-0.65}\left(\frac{U}{(gY)^{0.5}}\right)^{0.43} \quad (6)$$

where ds = depth of scour, D = diameter of pier, Y = depth of flow, g = acceleration due to gravity, D_{50} = mean particle size, υ = kinematic viscosity of fluid, K_3 = shape factor in the above equations. However, there is a lack of reliable formulae for predicting the scour depth to cover all possible ranges from the aforementioned equations. Recognizing these difficulties and the importance of improving prediction capabilities, a number of researches have been involved in exploring and refining methods for improving traditional physical based analysis in the past. Soft computing techniques such as Artificial Neural Networks, SVM, GP and many more are being widely applied to various potential areas of hydraulics and hydrology to overcome the problem of exclusive and the nonlinear relationships. The SVM is being applied in a variety of scientific areas-especially in applications involving diagnosis and forecasting in hydrology and hydraulics engineering. However, some recent examples of SVM in bridge pier scour in predicting the scour depth at bridge piers are pier scour using field data [11], time dependent estimation of pier scour [12], random forest–based regression approach applied to predict the local scour around bridge piers using field data set [13], kernel methods for pier scour [14], bridge pier scour for safe design of bridges [15] and hybrid intelligent techniques for scour depth around bridge piers [16] etc. The results obtained from these studies showed that the SVM provided better results as compared to semi-empirical equations and neural network approach as SVM approach requires fewer user defined parameters and does not face problem of local minima. In this article, a forecasting model is proposed by using the polynomial and radial basis function of Support vector machines to predict the scour depth around the bridge piers.

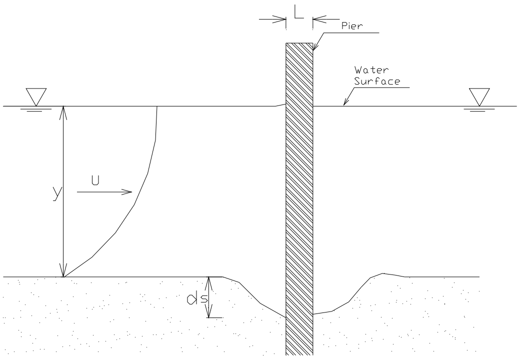

Fig. 1 Definition sketch of bridge pier scour

III. Support Vector Machines (SVMs)

Support vector machines (SVMs) are classification and regression methods and uses concept of supervised learning, which have been derived from statistical learning theory [17], [18]. The SVMs classification methods are based on the principle of optimal separation of classes. If the classes are separable - this method selects, from among the infinite number of linear classifiers, the one that minimize the generalization error, or at least an upper bound on this error, derived from structural risk minimization. Thus, the selected hyper plane will be one that leaves the maximum margin between the two classes, where margin is defined as the sum of the distances of the hyper plane from the closest point of the two classes [18]. The modelling techniques like support vector machines have the capability to reproduce the unknown relationship present between a set of input variables and the output of the system. Support vector machines performance was found to be better due to its use of the structural risk minimization principle in formulating cost functions and of quadratic programming during model optimisation. These advantages lead to a unique optimal and global solution as compared to conventional neural network models. The support vector machines can be applied to regression problems and can be formulated as below:

Investigator [18] proposed ε -support vector regression (SVR) by introducing an alternative ε -insensitive loss function. The purpose of the SVR is to find a function having at most ε deviation from the actual target vectors (y_i) for all given training data and have to be as flat as possible [19]. This can be put in other words as the error on any training data has to less than ε . For a given training data with k number of samples, represented by $(\mathrm{x}_1, y_1), \ldots\ldots\ldots, (\mathrm{x}_k, y_k)$ and a linear function

$$f(\mathrm{x}) = \langle \mathrm{w}, \mathrm{x} \rangle + d \tag{7}$$

where $w \in \mathrm{R}^N$, and d $\in$ R. $\langle \mathrm{w}, \mathrm{x} \rangle$ represents the dot product in space R^N and N is the dimension of input space. A smaller value of w indicates the flatness of (7), which can be achieved by minimizing the Euclidean norm as defined by $\|\mathrm{w}\|^2$ [19]. Thus, an optimization problem for this can be written as:

minimize $\frac{1}{2}\|\mathrm{w}\|^2$ subject to

$$\begin{cases} y_i - \langle \mathbf{w}, \mathbf{x}_i \rangle - d \leq \varepsilon \\ \langle \mathbf{w}, \mathbf{x}_i \rangle + d - y_i \leq \varepsilon \end{cases} \tag{8}$$

The optimization problem in (8) is based on the assumption that there exists a function that provides an error on all training pairs which is less than ε . In real life problems, there may be a situation like one defined for classification by [20]. So, to allow some more error slack variables ξ, ξ' can be introduced and the optimization problem defined in (7) can be written as:
minimize

$$\frac{1}{2}\|\mathrm{w}\|^2 + C\sum_{i=1}^{k}\left(\xi_i + \xi_i'\right)$$

subject to

$$y_i - \langle \mathrm{w}, \mathrm{x}_i \rangle - d \leq \varepsilon + \xi_i$$
$$\langle \mathrm{w}, \mathrm{x}_i \rangle + d - y_i \leq \varepsilon + \xi_i' \tag{9}$$

and

$\xi_i\ ,\ \xi_i' \geq 0$ for all $i = 1, 2, \ldots\ldots, k$.

The parameter C is determined by the user and it determines the trade-off between the flatness of the function and the amount by which the deviations to the error more than ε can be tolerated. The optimization problem in (9) can be solved by replacing the inequalities with a simpler form determined by transforming the problem to a dual space representation using Lagrange multipliers λ_i , λ_i' , η_i , η_i' i = 1,….,k [21].

The prediction problem can finally be written as:

$$f(\mathrm{x}\ ,\alpha) = \sum_{i=1}^{k} \left(\lambda_i' - \lambda_i\right)\langle \mathrm{x}_i, \mathrm{x} \rangle + d \tag{10}$$

This technique can be extended to allow for non-linear support vector regression by introducing the concept of the kernel function [18]. This is achieved by mapping the data into a higher dimensional feature space, thus performing linear regression in feature space. The regression problem in feature space can be written by replacing $\mathrm{x_i} \cdot \mathrm{x_j}$ with $\Phi(\mathrm{x}_i) \cdot \Phi(\mathrm{x}_j)$ where $K(\mathrm{x_i}, \mathrm{x_j}) \equiv \Phi(\mathrm{x_i}) \cdot \Phi(\mathrm{x_j})$.

The regression function for this can be written as:

$$f(\mathrm{x}, \alpha) = \sum_{i=1}^{k} \left(\lambda_i' - \lambda_i\right) K\left(\mathrm{x}_i\ ,\ \mathrm{x}\right) + d \tag{11}$$

IV. Performance Evaluation Method

The parameters considered to describe the scour depth in a bridge pier in the present study are ds = scour depth, U = mean velocity of flow, Uc = critical flow velocity, D = diameter of the pier, b = Y= flow depth; g = gravitational acceleration, d_{50} = particle mean diameter. The data sets (73 experimental data) mentioned in a study by [22], [23] are used in the present work for model building and validation to assess the potential of the simple regression, linear regression, SVM(Poly and Rbf) modelling techniques, and six empirical equations, in predicting of scour depth for bridge pier. The correlation coefficient and RMSE values are used for the performance evaluation of the models and comparison of the results for prediction scour depth around a pier. Higher value of a correlation coefficient (12) and lower value of RMSE (13) means a better performance of the model. Further, scour depths for the pier were plotted against the computed values obtained with kernel based SVM (Poly and Rbf), simple linear regression, linear regression and empirical equations. To study the scatter of results, a line of perfect agreement (a line at 15^{o}) was also plotted for the data set for the proper analysis.

1. Correlation Coefficient (r)

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}} \tag{12}$$

where x = X - X', y = Y - Y' where X = observed values; X' = mean of X, Y = predicted values, Y' = mean of Y.

2. Root Mean Square Error (RMSE)

$$RMSE = \left[\frac{\sum (X - Y)^2}{n}\right]^{0.5} \tag{13}$$

V. Sensitivity Analysis

The depth of pier scour (ds) was considered as dependent parameter and five others namely U, Uc, D, d_{50}, Y, g were considered as independent parameters. The influence of each independent parameter was studied on the dependent parameter as shown in Table I by considering the functional relation as ds = f (U, Uc, D, d_{50}, Y, g). Table I indicates the results when one of the input parameter is removed. Each time the value of correlation coefficient and rmse values are noted. It is clear from Table I that diameter of the pier (D) is the most influencing parameter affecting the depth of scour as shown by the Model no. 5. There was no effect of considering the value of g on the output of the results as observed during testing. Further, attempts were made to study sensitivity analysis when the maximum depth of scour is non-dimensionalised by dividing it by diameter of the pier (D). The results indicated that the diameter of the pier is the most significant parameter.

TABLE I
Sensitivity Analysis of Independent Parameters of Depth of Scour

Type of model	Dimensional		Non dimensional	
	Correlation coeff	RMSE	Correlation coeff	RMSE
ds = f (U, Uc, D, Y,g, d_{50}) All	0.7769	34.991	0.8142	0.1353
ds = f (Uc, D, Y,g, d_{50}) No U	0.8305	31.032	0.8223	0.1322
ds = f (U, Uc, D, Y,, d_{50}) no g	0.7769	34.991	0.7704	0.1499
ds = f (U, D, Y,g, d_{50}) No Uc	0.6974	40.114	0.8233	0.1322
ds = f (U, Uc, Y,g, d_{50}) No D	**0.5206**	**47.931**	**0.7395**	**0.1600**
ds = f (U, Uc, D,g, d_{50}) No Y	0.5536	46.692	0.8233	0.1322
ds = f (U, Uc, D, g,Y,) No d_{50}	0.7299	38.189	0.8142	0.1353

VI. Material and Methods

In this paper modeling techniques like SVM (poly & rbf), linear regression, and simple regression are being applied in prediction of bridge pier scour. The SVMs (rbf & poly), in addition to the choice of kernel, require setting up of kernel specific parameters. The optimum values of the regularization parameter C and the size of the error-insensitive zone ε need to be determined. To select user-defined parameters i.e. (C, γ and $d^{*)}$ a large number of trials were carried out by using different combination of these parameters on each of the data sets. To reach at a suitable choice of these parameters, the correlation coefficients (CC) and Root Mean Square Error

(RMSE) were compared and a combination of parameters providing smallest value of RMSE and the highest value of correlation coefficient was selected for the final results. Similarly, a number of trials were also carried out to find a suitable value of ε (error-insensitive zone) with a fixed value of C and kernel specific parameters. A number of trials were carried out with different data sets to select a suitable value of regularization parameter C. Variation in the error-insensitive zone ε have no effect on the predicted pier scour, thus a value of 0.0010 was chosen.

Due to the availability of small data sets, a cross validation was used to train and test the performance of the simple regression, linear regression, SVMs (Poly and Rbf) regression techniques using WEKA software [24]. The cross-validation is a method of estimating the accuracy of a classification or regression model. The data was divided into ten equal parts. 90% of data was used in training and remaining 10% was used in testing and in the next trial other 90% were used in training and remaining 10% were used in testing and so on. Each time correlation coefficients and RMSE were computed by the WEKA software [24]. The model giving minimum RMSE and maximum correlation coefficient values was selected finally for the SVM (Poly and Rbf), simple regression, linear regression techniques. However, for SVM (Poly and Rbf) modeling kernel specific parameters were varied and the values for dimensional value of maximum depth of scour and non-dimensional depth of scour are given in Tables II and III respectively. From Tables II and III, the best performing SVM (Poly & Rbf) models have been selected for further study in the present paper.

TABLE II
VALUES OF KERNEL SPECIFIC PARAMETERS OF SVM (POLY & RBF) MODELING FOR DIMENSIONAL PARAMETERS

S. No	Type of parameter	SVM(Poly)			SVM(Rbf)		
		C	γ / d*	Correlation coeff	C	γ / d*	Correlation coeff
1	Ds	1	1	0.8071	1	0.01	0.6059
2	Ds	2	1	0.8177	2	0.01	0.6393
3	Ds	5	1	0.8528	5	0.01	0.7028
4	Ds	10	1	0.8525	10	0.01	0.7401
5	Ds	-	-	-	12	0.01	0.7280

TABLE III
VALUES OF KERNEL SPECIFIC PARAMETERS OF SVM (POLY & RBF) MODELING FOR NON-DIMENSIONAL PARAMETERS

S. No	Type of parameter	SVM(Poly)			SVM(Rbf)		
		C	γ / d*	Correlation coeff	C	γ / d*	Correlation coeff
1	ds/D	2	0.01	0.805	1	2	0.8743
2	ds/D	3	0.01	0.8052	1	3	0.9186
3	ds/D	2	0.02	0.8083	2	2	0.8955
4	ds/D	3	0.03	0.7901	2	3	0.9474

VII. PREDICTION OF DEPTH OF PIER SCOUR

The first set of analysis was carried out by using input parameters namely U, Uc, D, d_{50}, Y from the data sets [22] for predicting the scour depth of bridge pier (ds). The results for scour depth for pier in terms of correlation coefficient and RMSE are obtained by polynomial and radial basis function kernels of support Vector Machines i.e.SVM (Poly and Rbf), simple regression; linear regression and six empirical equations proposed by [5]-[10] are shown in Table IV. Measured versus calculated values of the scour depth for pier are shown graphically as scatter plots in Fig. 2. In Fig. 2, ideal fit line along with two 15^{o} lines is also marked.

TABLE IV
VALUES OF PERFORMANCE PARAMETERS OF SIMPLE REGRESSION, LINEAR REGRESSION, SVM (POLY & RBF), SIX EMPIRICAL EQ FOR PREDICTION OF DS

S no	Type of technique/ empirical equation	Dimensional		Non dimensional	
		Cor coeff	RMSE	Cor coeff	RMSE
1	Simple regression	0.3187	53.01	0.7517	0.15
2	Linear regression	0.7769	34.99	0.8142	0.13
3	SVM(Poly)	0.8528	29.01	0.9634	0.06
4	SVM(Rbf)	0.7401	37.36	0.8052	0.16
5	Laursen & Toch Eq	0.6794	13.92	0.9443	1.14
6	Ettema Eq	0.8249	08.46	0.9555	0.77
7	Shen Eq	0.4499	4.262	0.5140	0.32
8	Hauch Eq	0.2769	5.284	-0.354	0.34
9	US DOT Eq	-0.034	14.08	-0.701	0.69
10	Breusers et al. Eq	0.8431	11.31	0.8226	1.04

For maximum depth of scour prediction (ds), a correlation coefficient and RMSE values for simple regression (0.3187, 53.01), linear regression (0.7769, 34.99), SVM Poly (0.8528, 29.01), SVMRbf (0.7401, 37.36), eq by [5] (0.8249,08.46) , eq by [6] (0.6794, by [7] (0.4499,4.262),eq by [8] (0.8431,11.31), eq by [9] (0.2769,5.284), and eq by [10] (-0.0346,14.08) are obtained (Table IV).

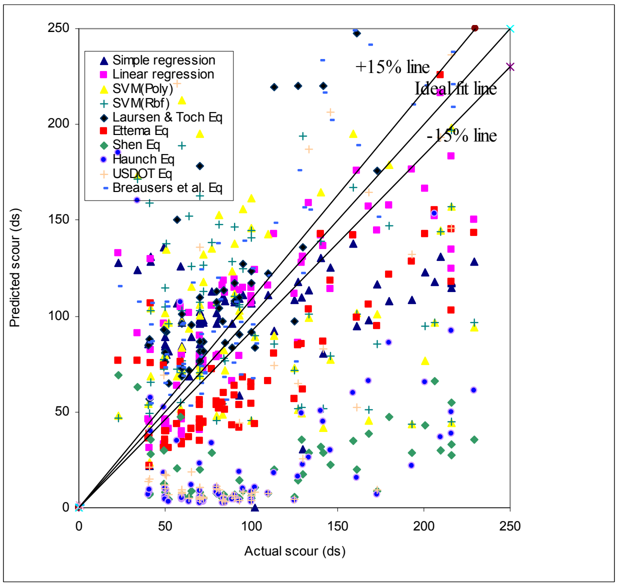

Fig. 2 Comparison for the actual scour depth with predicted scour depth by simple regression, linear regression, SVM (Poly and Rbf) model, and six empirical equations

A perusal of Fig. 2 indicates the SVM (Poly) technique is giving best performance on this data set as compared to linear, simple regression equations and empirical equations by [6], [7], [9], [10] are giving results comparable with Polynomial kernel based SVM. A 15% error line was also plotted and it can be observed from Fig. 2 that most of the points are falling in this range. More scatter has been observed in case of simple regression, and equations proposed by [7], [9], [10]. Further, examination of this figure indicates that this scatter is higher at the larger depth of scour due to turbulent conditions prevailing around the bridge pier during the unsteady flow. Also efforts were made to estimate the maximum scour depth in non-dimensional form by repeating the whole procedure. The maximum depth of scour was made non dimensional by dividing it by diameter of the pier (D), which was found to be most sensitive amongst all the input parameters as shown in Table I. The results obtained for (ds/D) in terms of correlation coefficient and RMSE for nondimensional depth of bridge pier scour for simple regression (0.7517, 0.15), linear regression (0.8142, 0.13), SVM Poly (0.9634, 0.062), SVM Rbf (0.8052, 0.162), eq by [5] (0.9555,0.77), eq by [6] (0.9443, 1.14), eq by [7] (0.5140, 0.32), eq by [8] (0.8226, 1.04), eq by [9] (-0.354,0.34), and eq by [10] (-0.701, 0.69) are obtained (Table IV) and are plotted in Fig. 3.

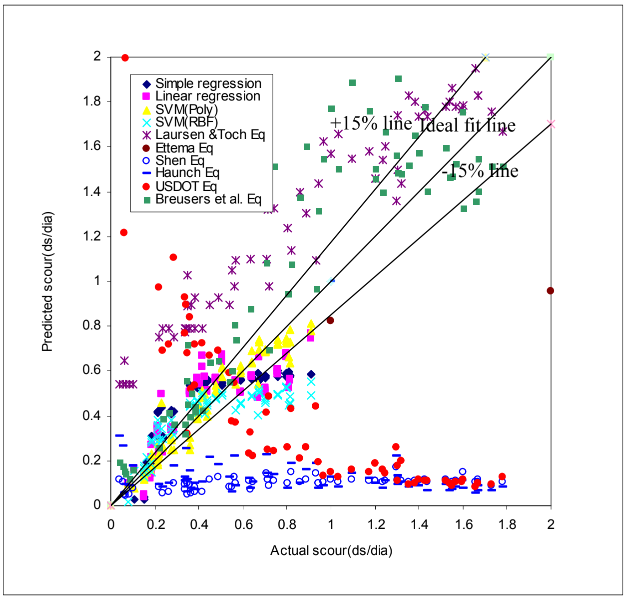

Fig. 3 Comparison for the non-dimensional actual scour depth with predicted scour depth by simple regression, linear regression, SVM (Poly and Rbf) model, and six empirical equations

In non-dimensional form of pier scour depth, SVM (Poly) is also performing the best modelling technique. However, eqs by [5], [6]are also perofrming equally well. It is evident from Fig. 3 that the data points are falling along the best fit line and results are improved significantly in most of the cases except in eq by [9], when nondimensional form of pier scour depth is used. It is inferred from the study that non dimensional representation of maximum depth of scour is superior to only maximum depth of scour in polynomial and RBF kernek based SVM techniques. Further, the results obtained by empirical equations suggested are not very consistent as these equations have been developed under varying pier geometry, flow and material conditions.

VIII. Conclusions

In this study, kernel based SVM (Poly and Rbf) models, simple regression, linear regression, and six important empirical equations are applied to determine the maximum scour depth for pier and have been compared with each other on the basis of correlation coefficient and RMSE. The test results revealed that SVM (Poly & Rbf) model predicted the measured values with nearly same accuracy as those of by Ettema equation [5]. Further analysis was also carried out to estimate non dimensional maximum scour depth of bridge pier following the same procedure. It was found that the results of non-dimensional estimation of maximum scour are much better than general maximum scour depth in SVM (Poly and Rbf), linear & simple regressions and most of the empirical equations. The comparison between the present SVM (Poly and Rbf) model and simple regression, linear regression, and empirical equations expressions found that the SVM (poly and Rbf) model has good ability of forecasting the maximum scour depth for bridge pier for dimensional and non-dimensional depth of scour. Hence kernel based SVM (Poly & Rbf) can be successfully employed as an alternative option to predict local scour depth for bridge pier.

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