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# Positive periodic solutions for a neutral impulsive delay competition system

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*Abstract*—In this paper, a neutral impulsive competition system with distributed delays is studied by using Mawhin's coincidence degree theory and the mean value theorem of differential calculus. Sufficient conditions on the existence of positive periodic solution of the system are obtained.

*Keywords*—neutral impulsive delay system; competitive system; coincidence degree; periodic solution; existence

## I. Introduction

THE qualitative behaviors of neutral delay differential equations in population dynamics have been studied extensively for the past few years [1-5]. Li have established the existence of a positive periodic solution of a neutral Lotka-Volterra model in [6]:

$$\begin{aligned} N_i'(t) &= N_i(t)[r_i(t) - \sum_{j=1}^{n} \alpha_{ij}(t)N_j(t-\tau_{ij}) \\ &\quad - \sum_{j=1}^{n} \beta_{ij}(t)N_j'(t-\sigma_{ij})],\ i=1,2,\dots, \end{aligned} \tag{1}$$

where $r_i, \alpha_{ij}, \beta_{ij} \in C(R,(0,+\infty))$ $(i,j=1,2,\dots,n)$ are functions of period $\omega > 0$, $\tau_{ij}$, $\sigma_{ij} \in [0,+\infty)$ $(i,j = 1,2,\dots,n)$ are positive constants. Recently, Liu [7] considered a neutral delay multispecies ecological competition system, which has the form

$$\begin{aligned} N_i'(t) &= N_i(t)[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)N_j(t) \\ &\quad - \sum_{j=1}^{n} b_{ij}(t)N_j(t-\tau_{ij}(t)) - \sum_{j=1}^{n} c_{ij}N_j'(t-\sigma_{ij})] \\ &\quad i=1,2,\dots,n \end{aligned} \tag{2}$$

where $a_{ij},\ b_{ij} \in C(R,(0,+\infty)),\ r_i,\ \tau_{ij} \in C(R,R)$ are $\omega$-periodic functions, $c_{ij}$, $\sigma_{ij}$ are constants, $c_{ij}$ are nonnegative, $\tau = max_{1\le i,j\le n}\{max_{t\in[0,\omega]} \mid \tau_{ij}(t) \mid, \mid \sigma_{ij} \mid\}$, $\int_0^{\omega} r_i(t)dt > 0$, and Liu obtained a general criteria on the existence of positive solution of system (2).

However, delays of ecological systems include not only discrete delay [8-9] but also continuous distributed delay [10], furthermore, there are some other perturbations in the real world such as fires and floods, that are not suitable to be considered continuously, those are something called impulsive perturbations. In order to describe the system accurately, we need to use impulsive differential equations. Based on system (1) that Li studied in [6], Huo considered equation (1) with impulsive perturbations in [11]. It is known that there have been much research on the theory and applications of impulsive differential equations [12-15]. Motivated by these papers, in this paper, we will study the existence of positive periodic solution of the following neutral impulsive delay multispecies ecological competitive system with distributed delays:

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$$\begin{cases} N_i'(t) = N_i(t)[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)N_j(t) - \sum_{j=1}^{n} b_{ij}(t) \\ \qquad \times N_j(t-\tau_{ij}(t)) - \sum_{j=1}^{n} c_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s)N_j(s)ds \\ \qquad - \sum_{j=1}^{n} d_{ij}(t)N_j'(t-\sigma_{ij}(t))] \\ N_i(t_k^+) - N_i(t_k) = b_{ik}N_i(t_k) i = 1,2,\dots,n, \\ \qquad k=1,2,\dots, \end{cases} \tag{3}$$

where $r_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $c_{ij}(t)$, $d_{ij}(t)$, $\tau_{ij}(t)$, $\sigma_{ij}(t)$ are positive continuous $\omega$-periodic functions. Kernel function $k_{ij}$ : $[0,\infty) \to [0,\infty)$ $(i,j = 1,2,\dots,n)$ are piecewise continuous on $[0,\infty)$ and satisfy $\int_0^{\infty} k_{ij}(s)ds = 1$, $\int_0^{\infty} sk_{ij}(s)ds < +\infty$, $i,j = 1,2,\dots,n$.

We will consider the solution of system (3) with initial condition,

$$\begin{aligned} &N_i(s) = \varphi_i(s),\ \ N_i'(s) = \varphi_i'(s),\ \ s \in [-\tau,0],\ \ \varphi_i(0) > 0, \\ &i = 1,2,\dots,n,\ \varphi_i \in C([-\tau,0],\ [0,\infty)) \cap C'([-\tau,0],\ [0,\infty)), \end{aligned} \tag{4}$$

where $\tau = max_{1\le i,j\le n}\{max_{t\in[0,\omega]}(\tau_{ij}(t), \sigma_{ij}(t))\}$.

For system (3), we introduce the following hypotheses:

($H_1$) $0 < t_1 < t_2 < \dots$ are fixed impulsive points with $\lim_{k\to\infty} t_k = \infty$.

($H_2$) $b_{ik}$ are real sequences and $b_{ik} > -1$, $i = 1,2,\dots,n$, $k = 1,2,\dots$

($H_3$) $\prod_{0<t_k<t}(1+b_{ik})$, $i = 1,2,\dots,n$, are $\omega$-periodic functions.

($H_4$) $r_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $c_{ij}(t)$, $d_{ij}(t)$, $\tau_{ij}(t)$, $\sigma_{ij}(t)$ $(i,j = 1,2,\dots,n)$ are positive continuous $\omega$-periodic functions.

($H_5$) Kernel function $k_{ij}$ : $[0,\infty) \to [0,\infty)$ $(i,j = 1,2,\dots,n)$ are piecewise continuous on $[0,\infty)$ and satisfy $\int_0^{\infty} k_{ij}(s)ds = 1$, $\int_0^{\infty} sk_{ij}(s)ds < +\infty$, $i,j = 1,2,\dots,n$.

Here, and in the sequel, we assume that a product equals unity, if the number of factors is equal to zero.

## II. Preliminaries

In this section, we shall introduce some notations and definitions, and state some preliminary results.

**Definition 1.** A function $N_i \in ([-\tau,\infty),[0,\infty))$, $(i = 1,2,\ldots,n)$ is said to be a solution of equation (3) on $[-\tau,\infty)$, if the following are true.
(I) $N_i(t)$ is absolutely continuous on each interval $(0,t_1]$ and $(t_k,t_{k+1}]$, $k=1,2,\ldots$
(II) For any $t_k$, $k=1,2,\ldots$, $N_i(t_k^+)$ and $N_i(t_k^-)$ exist and $N_i(t_k^-)=N_i(t_k)$, $i=1,2,\ldots,n$.
(III) $N_i(t)$ satisfies (3) for almost everywhere (a.e.,) in $[0,\infty)\setminus\{t_k\}$ and satisfies $N_i(t_k^+)-N_i(t_k)=b_{ik}N_i(t_k)$, for every $t=t_k$, $i=1,2,\ldots,n$, $k=1,2,\ldots$.

Under the above hypotheses (H$_1$)-(H$_5$), we consider the neutral nonimpulsive delay multispecies ecological competition system:

$$\begin{aligned} y_i'(t) &= y_i(t)[r_i(t)-\sum_{j=1}^n A_{ij}(t)y_j(t) \\ &\quad -\sum_{j=1}^n B_{ij}(t)y_j(t-\tau_{ij}(t))-\sum_{j=1}^n C_{ij}(t)\times \\ &\quad \int_{-\infty}^t k_{ij}(t-s)y_j(s)ds-\sum_{j=1}^n D_{ij}(t)y_j'(t-\sigma_{ij}(t))] \\ &\quad i=1,2,\ldots,n, \end{aligned} \tag{5}$$

with initial condition

$$\begin{aligned} &y_i(s)=\varphi_i(s),\ \ y_i'(s)=\varphi_i'(s),\ \ s\in[-\tau,0],\ \ \varphi_i(0)>0, \\ &i=1,2,\ldots,n,\ \varphi_i\in C([-\tau,0],[0,\infty))\cap C'([-\tau,0],[0,\infty)), \end{aligned} \tag{6}$$

where

$$\begin{aligned} A_{ij}(t) &= \prod_{0<t_k<t}(1+b_{ik})a_{ij}(t), \\ B_{ij}(t) &= \prod_{0<t_k<t-\tau_{ij}(t)}(1+b_{ik})b_{ij}(t), \\ C_{ij}(t) &= \prod_{0<t_k<s}(1+b_{ik})c_{ij}(t), \\ D_{ij}(t) &= \prod_{0<t_k<t-\sigma_{ij}(t)}(1+b_{ik})d_{ij}(t). \end{aligned} \tag{7}$$

By a solution $y_i(t), i=1,2,\ldots,n$, of (5) and (6), we mean an absolutely continuous function $y_i(t), i=1,2,\ldots,n$, defined on $[-\tau,0]$ that satisfies (5) a.e., for $t\geq 0$ and $y_i(t)=\varphi_i(t), i=1,2,\ldots,n$, on $[-\tau,0]$.

The following lemmas will be used in the proof of our results.

**Lemma 1.** Assume that (H$_1$)-(H$_5$) hold. Then,
I) if $y_i(t)$, $i=1,2,\ldots,n$, is a solution of (5) on $[-\tau,\infty)$, then, $N_i(t)=\prod_{0<t_k<t}(1+b_{ik})y_i(t)$, $i=1,2,\ldots,n$, is a solution of (3);
II)if $N_i(t), i=1,2,\ldots,n$, is a solution of (3) on $[-\tau,\infty)$, then $y_i(t)=\prod_{0<t_k<t}(1+b_{ik})^{-1}N_i(t)$, $i=1,2,\ldots,n$, is a solution of (5) on $[-\tau,\infty)$.

Proof: the proof is similar to lemma 1 in [11], so we omit here.

**Lemma 2.**[16] Let $X$ and $Z$ be two Banach spaces and $L$ be a Fredholm mapping of index zero. Assume that $N:\overline{\Omega}\to Z$ is $L$-compact on $\overline{\Omega}$ with $\Omega$ open bounded in $X$. Furthermore, assume:
(a) for each $\lambda\in(0,1)$, $x\in\partial\Omega\cap DomL$, $Lx\neq\lambda Nx$;
(b) for each $x\in\partial\Omega\cap KerL$, $QNx\neq 0$ and $deg\{QNx,\Omega\cap KerL,0\}\neq 0$.
Then $Lx=Nx$ has at least one solution in Dom $L\cap\overline{\Omega}$.

For convenience, we shall introduce the notation, $\overline{u}=\frac{1}{\omega}\int_0^\omega u(t)dt$, $(u)_0=\max_{t\in[0,\omega]}\mid u(t)\mid$, where $u$ is a $\omega$-periodic continuous function.

## III. Main results

Now we are in a position to state and prove our main results.

**Theorem.** Assume that (H$_1$)-(H$_5$) hold, suppose further that
(i) $\tau_{ij}(t)=\sigma_{ij}(t)$,
(ii) $E_{ij}(t)=\frac{D_{ij}(t)}{1-\sigma_{ij}'(t)},\sigma_{ij}(t),D_{ij}(t)\in C^1(R,R^+)$, $\sigma_{ij}'(t)<1, B_{ij}(t)-E_{ij}'(t)\geq 0,\ i,j=1,2,\ldots,n$,
(iii) $\sum_{j=1}^n\mid D_{ij}\mid_0 e^{H_j}<1,\ i=1,2,\ldots,n$;
(iv) $\sum_{j=1,j\neq i}^n(\overline{A}_{ij}+\overline{B}_{ij}+\overline{C}_{ij}-\overline{E'}_{ij})e^{H_j}<\overline{r}_i,\ i=1,2,\ldots,n$;
(v) the system of algebraic equations $\sum_{j=1}^n(\overline{A}_{ij}+\overline{B}_{ij}+\overline{C}_{ij})e^{u_j}=\overline{r}_i,\ i=1,2,\ldots,n$, has a unique solution $(u_1^*,\ldots,u_n^*)^T\in R^n$, where $H_i=F_{ii}+\sum_{j=1}^n(E_{ij})_0G_{ij}+2\overline{r}_i\omega$, $i=1,2,\ldots,n$, $F_{ij}=\max_{t\in[0,\omega]}[\ln\frac{\overline{r}_i}{A_{ij}(t)}], G_{ij}=\max_{t\in[0,\omega]}[\frac{\overline{r}_i}{B_{ij}(t)-E_{ij}'(t)}]$, $i=1,2,\ldots,n$, where $A_{ij}(t),B_{ij}(t)$, $C_{ij}(t)$, and $D_{ij}(t)$ are defined by (7). Then system (3) has at least one positive $\omega$-periodic solution.

For convenience, we introduce the following notations:

$$\begin{aligned} G_i(t) &= r_i(t)-\sum_{j=1}^n A_{ij}(t)e^{x_j(t)}-\sum_{j=1}^n B_{ij}(t)e^{x_j(t-\tau_{ij}(t))} \\ &\quad -\sum_{j=1}^n C_{ij}(t)\int_{-\infty}^t k_{ij}(t-s)e^{x_j(s)}ds \\ &\quad +\sum_{j=1}^n E_{ij}'(t)e^{x_j(t-\sigma_{ij}(t))}\qquad i=1,2,\ldots,n. \end{aligned}$$

**Proof.** Since solutions of (5) and (6) remains positive for $t\geq 0$, we can make change of variable $y_i(t)=e^{x_i(t)}$, $i=1,2,\ldots,n$. Then the equation (5) is rewritten as

$$\begin{aligned} x_i'(t) &= r_i(t)-\sum_{j=1}^n A_{ij}(t)e^{x_j(t)}-\sum_{j=1}^n B_{ij}(t)e^{x_j(t-\tau_{ij}(t))} \\ &\quad -\sum_{j=1}^n C_{ij}(t)\int_{-\infty}^t k_{ij}(t-s)e^{x_j(s)}ds \\ &\quad -\sum_{j=1}^n D_{ij}(t)x_j'(t-\sigma_{ij}(t))e^{x_j(t-\sigma_{ij}(t))} \\ &\quad i=1,2,\ldots,n. \end{aligned} \tag{8}$$

Let $X=\{x(t)=(x_1(t),\ldots,x_n(t))^T\in C^1(R,R^n):x(t+\omega)=x(t)\}$ and $Z=\{z(t)=(z_1(t),\ldots,z_n(t))^T\in C(R,R^n)\ :\ z(t+\omega)=z(t)\}$ and denote $\mid x(t)\mid=$

$\sum_{j=1}^{n}|x_j(t)|,\ |\ x\ |\ \infty = max_{t\in[0,\omega]}\ |\ x(t)\ |,\ \|\ x\ \|=|\ x\ |_\infty + |\ x'\ |_\infty$. Then $X$ and $Z$ are Banach spaces when they are endowed with norms $\|\cdot\|$ and $|\cdot|_\infty$, respectively. Set

$$Nx = N\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} r_1(t) - \sum_{j=1}^{n} A_{1j}(t)e^{x_j(t)} \\ - \sum_{j=1}^{n} B_{1j}(t)e^{x_j(t-\tau_{1j}(t))} \\ - \sum_{j=1}^{n} C_{1j}(t)\int_{-\infty}^{t} k_{ij}(t-s)e^{x_j(s)}ds \\ - \sum_{j=1}^{n} D_{1j}(t)x_j'(t-\sigma_{1j}(t))e^{x_j(t-\sigma_{1j}(t))} \\ \vdots \\ r_n(t) - \sum_{j=1}^{n} A_{nj}(t)e^{x_j(t)} \\ - \sum_{j=1}^{n} B_{nj}(t)e^{x_j(t-\tau_{nj}(t))} \\ - \sum_{j=1}^{n} C_{nj}(t)\int_{-\infty}^{t} k_{ij}(t-s)e^{x_j(s)}ds \\ - \sum_{j=1}^{n} D_{nj}(t)x_j'(t-\sigma_{nj}(t))e^{x_j(t-\sigma_{nj}(t))} \end{bmatrix},$$

$Lx = x',\ Px = \frac{1}{\omega}\int_0^\omega x(t)dt,\ x\in X,\ Qz = \frac{1}{\omega}\int_0^\omega z(t)dt,\ z\in Z$. Evidently, Ker $L = \{x|x\in X,\ x = k\in R^n\}$ and Im $L = \{z\in Z,\ \int_0^\omega z(t)dt = 0\}$ is closed in $Z$ and dim Ker $L$ =codim Im $L = n$. Hence, $L$ is a Fredholm mapping of index zero. Furthermore, the generalized inverses (to $L$)$K_p$ :Im $L \to$ Ker $P\cap$ Dom $L$ has the form

$K_p(z) = \int_0^t z(s)ds - \frac{1}{\omega}\int_0^\omega\int_0^t z(s)dsdt.$

Thus $QNx = (\frac{1}{\omega}\int_0^\omega G_1(t)dt, \ldots, \frac{1}{\omega}\int_0^\omega G_n(t)dt)^T,\ x\in X$ and

$$\begin{aligned} &K_p(I-Q)Nx \\ &= \begin{bmatrix} \int_0^t G_1(s)ds - \sum_{j=1}^{n}[E_{1j}(t)e^{x_j(t-\sigma_{1j}(t))} \\ -E_{1j}(0)e^{x_j(-\sigma_{1j}(t))}] \\ \vdots \\ \int_0^t G_n(s)ds - \sum_{j=1}^{n}[E_{nj}(t)e^{x_j(t-\sigma_{nj}(t))} \\ -E_{nj}(0)e^{x_j(-\sigma_{nj}(t))}] \end{bmatrix} \\ &- \begin{bmatrix} \frac{1}{\omega}\int_0^\omega\int_0^t G_1(s)dsdt - \frac{1}{\omega}\int_0^\omega[\sum_{j=1}^{n}(E_{1j}(t) \\ \times e^{x_j(t-\sigma_{1j}(t))} - E_{1j}(0)e^{x_j(-\sigma_{1j}(t))})]dt \\ \vdots \\ \frac{1}{\omega}\int_0^\omega\int_0^t G_n(s)dsdt - \frac{1}{\omega}\int_0^\omega[\sum_{j=1}^{n}(E_{nj}(t) \\ \times e^{x_j(t-\sigma_{nj}(t))} - E_{nj}(0)e^{x_j(-\sigma_{nj}(t))})]dt \end{bmatrix} \\ &- \begin{bmatrix} (\frac{t}{\omega}-\frac{1}{2})\int_0^\omega G_1(s)ds \\ \vdots \\ (\frac{t}{\omega}-\frac{1}{2})\int_0^\omega G_n(s)ds \end{bmatrix}. \end{aligned}$$

Clearly, $QN$ and $K_p(I-Q)N$ are continuous. Using Arzela-Ascoli theorem, it is not difficult to show that $QN(\overline{\Omega}),\ K_p(I-Q)N(\overline{\Omega})$ are relatively compact for any open bounded set $\Omega\subset X$. Therefore, $N$ is $L$-compact on $\overline{\Omega}$ for any open bounded set $\Omega\subset X$. Corresponding to equation $Lx = \lambda Nx,\ \lambda\in(0,1)$, we have

$$\begin{aligned} x_i'(t) = \lambda[r_i(t) - \sum_{j=1}^{n} A_{ij}(t)e^{x_j(t)} - \sum_{j=1}^{n} B_{ij}(t)e^{x_j(t-\tau_{ij}(t))} \\ - \sum_{j=1}^{n} C_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s)e^{x_j(s)}ds - \sum_{j=1}^{n} D_{ij}(t) \\ \times x_j'(t-\sigma_{ij}(t))e^{x_j(t-\sigma_{ij}(t))}], \\ i = 1,2,\ldots,n. \end{aligned} \tag{9}$$

Suppose that $x(t)\in X$ is a solution of (9) for certain $\lambda\in(0,1)$. Integrating (9) over the interval $[0,\omega]$, we obtain

$$\begin{aligned} &\int_0^\omega [r_i(t) - \sum_{j=1}^{n} A_{ij}(t)e^{x_j(t)} - \sum_{j=1}^{n} B_{ij}(t)e^{x_j(t-\tau_{ij}(t))} \\ &- \sum_{j=1}^{n} C_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s)e^{x_j(s)}ds \\ &+ \sum_{j=1}^{n} E_{ij}'(t)e^{x_j(t-\sigma_{ij}(t))}]dt = 0, \quad i = 1,2,\ldots,n. \end{aligned} \tag{10}$$

Hence,

$$\begin{aligned} &\sum_{j=1}^{n}\int_0^\omega [A_{ij}(t)e^{x_j(t)} + B_{ij}(t)e^{x_j(t-\tau_{ij}(t))} \\ &+ C_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s)e^{x_j(s)}ds \\ &- E_{ij}'(t)e^{x_j(t-\sigma_{ij}(t))}]dt = \overline{r}_i\omega,\ i = 1,2,\ldots,n. \end{aligned} \tag{11}$$

Obviously, $x(t+\omega) = x(t)$, so $e^{x_j(t-u)}$ is bounded, hence $\int_0^{+\infty} k_{ij}(u)e^{x_j(t-u)}du$ converge, therefore

$$\int_{-\infty}^{t} k_{ij}(t-s)e^{x_j(s)}ds = \int_0^{+\infty} k_{ij}(u)e^{x_j(t-u)}du = I(t) > 0$$

from (11) we obtain

$$\begin{aligned} &\sum_{j=1}^{n}\int_0^\omega [A_{ij}(t)e^{x_j(t)} + B_{ij}(t)e^{x_j(t-\tau_{ij}(t))} + C_{ij}(t)I(t) \\ &- E_{ij}'(t)e^{x_j(t-\sigma_{ij}(t))}]dt = \overline{r}_i\omega, \\ &i = 1,2,\ldots,n. \end{aligned} \tag{12}$$

it follows from (9) and (11) that

$$\begin{aligned}
&\int_0^\omega \mid \frac{d}{dt}[x_i(t)+\lambda\sum_{j=1}^n E_{ij}(t)e^{x_j(t-\sigma_{ij}(t))}] \mid dt\\
=\ &\lambda\int_0^\omega \mid r_i(t)-\sum_{j=1}^n A_{ij}(t)e^{x_j(t)}-\sum_{j=1}^n B_{ij}(t)\\
&\times e^{x_j(t-\sigma_{ij}(t))}-C_{ij}(t)\int_{-\infty}^t k_{ij}(t-s)e^{x_j(s)}ds\\
&+\sum_{j=1}^n E_{ij}^{'}(t)e^{x_j(t-\sigma_{ij}(t))} \mid dt,\\
<\ &\int_0^\omega \mid r_i(t)\mid dt+\sum_{j=1}^n\int_0^\omega [A_{ij}(t)e^{x_j(t)}+B_{ij}(t)\\
&\times e^{x_j(t-\tau_{ij}(t))}+C_{ij}(t)\int_{-\infty}^t k_{ij}(t-s)e^{x_j(s)}ds\\
&-E_{ij}^{'}(t)e^{x_j(t-\sigma_{ij}(t))}]dt\\
=\ &2\overline{r}_i\omega,\ \ i=1,2,\dots,n,
\end{aligned}$$

that is,

$$\begin{aligned}
&\int_0^\omega \mid \frac{d}{dt}[x_i(t)+\lambda\sum_{j=1}^n E_{ij}(t)e^{x_j(t-\sigma_{ij}(t))}] \mid dt<2\overline{r}_i\omega,\\
&i=1,2,\dots,n,
\end{aligned}\tag{13}$$

from assumption (i) and Eq.(12), we have

$$\begin{aligned}
&\sum_{j=1}^n\int_0^\omega [A_{ij}(t)e^{x_j(t)}+(B_{ij}(t)-E_{ij}^{'}(t))e^{x_j(t-\sigma_{ij}(t))}\\
&+C_{ij}(t)I(t)]=\overline{r}_i\omega, i=1,2,\dots,n.
\end{aligned}$$

According to the mean value theorem of differential calculus we can see that there exist points $\xi_i\in[0,\omega]$ $(i=1,2,\dots,n)$, such that

$$\begin{aligned}
&\sum_{j=1}^n A_{ij}(\xi_i)e^{x_j(\xi_i)}+\sum_{j=1}^n(B_{ij}(\xi_i)-E_{ij}^{'}(\xi_i))e^{x_j(t-\sigma_{ij}(\xi_i))}\\
&+\sum_{j=1}^n C_{ij}(\xi_i)I(\xi_i)=\overline{r}_i, i=1,2,\dots,n.
\end{aligned}$$

combined with assumption (ii), which implies

$$x_j(\xi_i)<\ln\frac{\overline{r}_i}{A_{ij}(\xi_i)}\leq\ F_{ij},\qquad i,j=1,2,\dots,n,\tag{14}$$

and

$$\begin{aligned}
&e^{x_j(\xi_i-\sigma_{ij}(\xi_i))}<\frac{\overline{r}_i}{B_{ij}(\xi_i)-D_{ij}^{'}(\xi_i)}\leq G_{ij},\\
&i,j=1,2,\dots,n,
\end{aligned}\tag{15}$$

one can know from Eqs.(13), (14) and (15) that

$$\begin{aligned}
x_i(t)\ \ +\ \ &\lambda\sum_{j=1}^n E_{ij}(t)e^{x_j(t-\sigma_{ij}(t))}\\
<\ \ &x_i(\xi_i)+\lambda\sum_{j=1}^n E_{ij}(\xi_i)e^{x_j(\xi_i-\sigma_{ij}(\xi_i))}\\
&+\int_0^\omega \mid \frac{d}{dt}[x_i(t)+\lambda\sum_{j=1}^n E_{ij}(t)e^{x_j(t-\sigma_{ij}(t))}] \mid dt\\
<\ \ &F_{ii}+\sum_{j=1}^n(E_{ij})_0G_{ij}+2\overline{r}_i\omega:=H_i,\\
&i=1,2,\dots,n,
\end{aligned}\tag{16}$$

as $\lambda\sum\limits_{j=1}^n E_{ij}(t)e^{x_j(t-\sigma_{ij}(t))}>0$, one can find that

$$x_i(t)<H_i,\qquad i=1,2,\dots,n,\tag{17}$$

by Eqs.(9) and (17), we have

$$\begin{aligned}
&\mid x_i^{'}(t)\mid=\mid\lambda[r_i(t)-\sum_{j=1}^n A_{ij}(t)e^{x_j(t)}-\sum_{j=1}^n B_{ij}(t)e^{x_j(t-\tau_{ij}(t))}\\
&\quad-\sum_{j=1}^n C_{ij}(t)\int_{-\infty}^t k_{ij}(t-s)e^{x_j(s)}ds\\
&\quad-\sum_{j=1}^n D_{ij}(t)x_j^{'}(t-\sigma_{ij}(t))e^{x_j(t-\sigma_{ij}(t))}]\mid\\
&\quad<\mid r_i\mid_0+\sum_{j=1}^n(\mid A_{ij}\mid_0+\mid B_{ij}\mid_0+\mid C_{ij}\mid_0)e^{H_j}\\
&\quad+\sum_{j=1}^n\mid D_{ij}\mid_0\mid x_j^{'}\mid_0 e^{H_j}, i=1,2,\dots,n.
\end{aligned}$$

thus

$$\begin{aligned}
\mid x_i^{'}\mid_0<&\mid r_i\mid_0+\sum_{j=1}^n(\mid A_{ij}\mid_0+\mid B_{ij}\mid_0+\mid C_{ij}\mid_0)e^{H_j}\\
&+\sum_{j=1}^n\mid D_{ij}\mid_0\mid x_j^{'}\mid_0 e^{H_j}i=1,2,\dots,n.
\end{aligned}\tag{18}$$

According to assumption (iii),we have

$$\begin{aligned}
&\mid x_i^{'}\mid_0<\frac{\mid r_i\mid_0+\sum\limits_{j=1}^n(\mid A_{ij}\mid_0+\mid B_{ij}\mid_0+\mid C_{ij}\mid_0)e^{H_j}}{1-\sum\limits_{j=1}^n\mid D_{ij}\mid_0 e^{H_j}}\\
&:=M_i, i=1,2,\dots,n,
\end{aligned}\tag{19}$$

From Eq.(11) and assumption (iv), it is easy to see there exists points $\theta_i\in[0,\omega]$ and constant $C_i$ $(i=1,2,\dots,n)$ such that $\mid x_i(\theta_i)\mid<N_i,\ i=1,2,\dots,n,$
By this and Eq.(19), we obtain
$\mid x_i\mid_0\leq\mid x_i(\theta_i)\mid+\int_0^\omega\mid x_i^{'}\mid dt\leq N_i+M_i\omega,\ i=1,2,\dots,n.$

Clearly, $N_i$ and $M_i$ $(i=1,2,\dots,n)$ are independent of $\lambda$. Denote $B=\sum\limits_{i=1}^n(N_i+(1+\omega)M_i)+h$, where $h>0$ is taken sufficiently large such that the unique solution $x^* = (x_1^*, \ldots, x_n^*)^T$ of the equation

$$\overline{r}_i - \sum_{j=1}^{n}(\overline{A}_{ij} + \overline{B}_{ij} + \overline{C}_{ij})e^{x_j} = 0, \quad i = 1, 2, \ldots, n,$$

satisfies $\| x^* \| < h$. Now we set $\Omega = \{x(t) \in X : \| x \| < B\}$. It is clear that $\Omega$ vertifies the requirement (a) in Lemma 2. When $x = (x_1, \ldots, x_n)^T \in \partial\Omega \cap KerL = \partial\Omega \cap R^n$, $x$ is constant vector in $R^n$ with $| x |= B$, then

$$QN \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \overline{r}_1 - \sum_{j=1}^{n}(\overline{A}_{1j} + \overline{B}_{1j} + \overline{C}_{1j})e^{x_j} \\ \vdots \\ \overline{r}_n - \sum_{j=1}^{n}(\overline{A}_{nj} + \overline{B}_{nj} + \overline{C}_{nj})e^{x_j} \end{bmatrix} \neq \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Furthermore, in view of assumption (v), a straightforward calculation shows that

$$deg\{JQNx, \Omega \cap KerL, 0\} = sgn\{(-1)^n[det(\overline{A}_{ij} + \overline{B}_{ij} + \overline{C}_{ij})]e^{\sum_{j=1}^{n}\mu_j^*}\} \neq 0.$$

By now we know that $\Omega$ verifies all the requirements of Lemma 2, hence Eq.(8) has at least one $\omega$-periodic solution. By the medium $N_i(t) = e^{x_i(t)}$, Eq.(5) has at least one positive $\omega$-periodic solution. According to Lemma 1, we obtain that the equation (3) has at least one positive $\omega$-periodic solution.The proof of the Theorem is completed.

**Remark 1.** For system (3), let $a_{ij} = 0$, $c_{ij} = 0$, then system (3) will be reduced into the system that Huo discussed in Ref.[11]. Furthermore, for a system with distributed delays and satisfies the conditions (i),(ii),(iii),(iv),(v), then Theorem 2.1[11] fails, but according to the Theorem in this paper, the system has at least one positive periodic solution. So our main results generalized the results of Theorem 2.1 in Ref.[11].

**Remark 2.** If we don't consider impulsive perturbation and distributed delays, obviously, system (2) that Liu studied in Ref. [7] and system (1) that Li discussed in Ref.[6] are special cases of system (3), so our main Theorem generalized the corresponding Theorem in Ref.[7] and Theorem 2.1 in Ref.[6].

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## REFERENCES

[1] Y. Li, Positive periodic solution for neutral delay model, Acta Math. Sinica 39 (1996) 789-795 (in Chinese).
[2] Y. Li, Periodic solution of a periodic neutral delay equation, J. Math. Anal. Appl. 214 (1997) 11-21.
[3] K. Gopalsamy, Stability and Oscillations in Delay Differential Equations of Population Dynamics, Kluwer Academic, Boston, 1992.
[4] Y. Kuang, On neutral delay two species Lotka-Volterra competitive system, J. Austral. Math. Soc. Ser. B 32 (1991) 311-326.
[5] Y. Zhu, Periodic solutions for a higher order nonlinear neutral functional differential equation. Inter. J. Comp. Math. Sci., 5 (2011) 8-12.
[6] Y. Li, On a periodic neutral delay Lotka-Volterra system, Nonlinear Anal. 39 (2000) 767-778.
[7] Z. Liu, Positive periodic solution for a neutral delay competitive system, J. Math. Anal. Appl. 293 (2004) 181-189.
[8] X. Ding, S. Cheng, Positive periodic solution for a nonautonomous logistic model with linear feedback regulation, Appl. Math. J. Chinese Univ. Ser. B, 21 (2006) 302-312.
[9] X. Ding, Periodicity in a delayded semi-ratio-dependent predator-prey system, Appl. Math. J. Chinese Univ. Ser. B, 20 (2005) 151-158.
[10] Y. Li, W. Xing, L. Lu, Existence and global exponential stability of periodic solution of a class of neural networks with impulses, Chaos, Solitions and Fractals (2006) 437-445.
[11] H. Huo, Existence of positive periodic solution for a neutral delay Lotka-Volterra system with impulses, Computers and Mathematics with Applications 48 (2004) 1833-1846.
[12] H. Huo, W. Li, Existence of positive periodic solution of a neutral impulsive delay predator-prey system, Applied Mathematics and Computation 185 (2007) 499-507.
[13] D. Wang, C. Feng, Three positive periodic solutions to a kind of Impulsive Differential Equations with delay, Journal of Jilin University, Science Edition, 3 (2011) 391-396.
[14] Z. Liu, L. Chen, Periodic solution of a two-species competitive system with toxicant and birth pulse, Chaos, Solitons and Fractals 32 (2007) 1703-1712.
[15] Y. Li, Z. Xing, Existence and global exponential stability of periodic solutions of CNNs with impulses, Chaos, Solitons and Fractals 33 (2007) 1686-1693.
[16] R. Gaines, J. Mawhin, Coincidence Degree and Nonlinear Differential Equations, Springer-Verlag, Berlin, 1977.