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Positive almost periodic solutions for neural multi-delay logarithmic population model

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Abstract—In this paper, by applying Mawhin's continuation theorem of coincidence degree theory, we study the existence of almost periodic solutions for neural multi-delay logarithmic population model and obtain one sufficient condition for the existence of positive almost periodic solution for the above equation. An example is employed to illustrate our result.

Keywords—Almost periodic solution; Multi-delay; Logarithmic population model; Coincidence degree.

I. Introduction

LU and Ge [1] studied the existence of positive periodic solutions for neutral logarithmic population model with multiple delays. Based on an abstract continuous theorem of k-set contractive operator, Luo and Luo [2] investigate the following periodic neutral multi-delay logarithmic population model:

$$\begin{aligned}\dot{x}(t) &= x(t)\Big[r(t)+\sum_{j=1}^{n}a_j(t)\ln x(t-\sigma_j(t)) \\ &\quad -\sum_{i=1}^{m}b_i(t)\frac{\mathrm{d}}{\mathrm{d}t}\ln x(t-\tau_i(t))\Big] \qquad (1)\end{aligned}$$

In recently, there has been considerable interest in the existence of periodic solutions of functional differential equations (see, [1-7]). It is well known that the environments of most natural populations change with time and that such changes induce variation in the growth characteristics of populations. Among many population models, the neutral logarithmic population model has recently attracted the attention of many mathematicians and biologists. Since biological and environmental parameters are naturally subject to fluctuation in time, the effects of a periodically varying environment are considered as important selective forces on systems in a fluctuating environment. Therefore, on the one hand, models should take into account the seasonality of the periodically changing environment. However, on the other hand, in fact, it is more realistic to consider almost periodic system than periodic system.

In 2011, J. Alzabuta, et al. consider the positive almost periodic solutions for a delay logarithmicpopulation model, and obtain some sufficient condition on the existence of positive almost periodic solution by applying Mawhin's continuation theorem of coincidence degree theory.

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By above, this motivates us to investigate the existence of positive almost periodic solution to system (1), to the best of our knowledge, few results are found in literatures.

The main purpose of this paper is to establish sufficient conditions for the existence of positive almost periodic solutions to system (1) by applying Mawhin's continuation theorem of coincidence degree theory.

The organization of this paper is as follows. In Section 2, we make some preparations. In Section 3, by using Mawhin's continuation theorem of coincidence degree theory, we establish sufficient conditions for the existence of positive almost periodic solutions to system (1). An illustrative example is given in Section 4.

II. Preliminaries

Our first observation if that under the invariant transformation $x(t)=e^{z(t)}$, equation (1) reduces to

$$\begin{aligned}\dot{z}(t) &= r(t)-\sum_{j=1}^{n}a_j(t)z(t-\sigma_j(t))-\sum_{i=1}^{m} \\ &\quad \times c_i(t)\dot{z}(t-\tau_i(t)), \qquad (2)\end{aligned}$$

where $c_i(t)=b_i(t)(1-\tau_i(t)), i=1,2,\dots,m$. We consider equation (2) together with the initial condition

$$\begin{aligned}z(t)=\phi(t),\quad t\in[-\Delta,0],\ \ \Delta=\max_{t\in R^+}\{\sigma_j(t),\tau_i(t)\}, \\ i=1,2,\dots,n, j=1,2,\dots,m. \qquad (3)\end{aligned}$$

For equations (2) and (3), we assume the following conditions:

$(F1)$ $r(t),a_j(t),c_i(t)\in C([0,+\infty),[0,\infty))$ are positive functions;

$(F2)$ $\sigma_j(t),\tau_i(t)\in C([0,+\infty),[0,\infty))$ are positive bounded functions;

$(F3)$ $\phi(t)\in C([-\Delta,0],[0,\infty))$.

By a solution of (2) and (3) we mean an absolutely continuous function $z(t)$ defined on $[-\Delta,+\infty)$ satisfying (2) almost everywhere for $t>0$ and (3). As we are interested in solutions of biological significance, we restrict our attention to positive ones.

For the readers' convenience, we first summarize a few concepts from [8].

Let $\mathbf{X}$ and $\mathbf{Z}$ be Banach spaces. Let $L:\mathrm{Dom}\,L\subset\mathbf{X}\to\mathbf{Z}$ be a linear mapping and $N:\mathbf{X}\to\mathbf{Z}$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\mathrm{Im}\,L$ is a closed subspace of $\mathbf{Z}$ and

$$\dim\mathrm{Ker}\,L=\mathrm{codim}\,\mathrm{Im}\,L<\infty.$$

If L is a Fredholm mapping of index zero, then there exist continuous projectors $P : \mathbf{X} \to \mathbf{Z}$ and $Q : \mathbf{Z} \to \mathbf{Z}$ such that Im P=Ker L and Im L=Ker Q=Im $(I-Q)$. It follows that

$$L|_{\text{Dom } L \cap \text{Ker } P} : (I-P)\mathbf{X} \to \text{Im } L$$

is invertible and its inverse is denoted by K_P. If Ω is a bounded open subset of $\mathbf{X}$, the mapping N is called L-compact on $\mathbf{X}$, if $QN(\bar{\Omega})$ is bounded and $K_P(I-Q)N : \bar{\Omega} \to \mathbf{X}$ is compact. Because Im Q is isomorphic to Ker L, there exists an isomorphism $J : \text{Im } Q \to \text{Ker } L$.

In the proof of our existence result, we need the following continuation theorem from Gaines and Mawthin [8].

Lemma 1. *Let L be a Fredholm mapping of index zero and let N be L-compact on $\mathbf{X}$. Suppose*

(1) *for each $\lambda \in (0,1)$, $x \in \partial\Omega$, $Lx \neq \lambda Nx$;*
(2) *for each $x \in \partial\Omega$, $QNx \neq 0$;*
(3) *$deg\{JQN, \Omega \cap Ker\, L, 0\} \neq 0$.*

Then $Lx = Nx$ has at least one solution in $\bar{\Omega} \cap Dom\, L$.

Now we introduce some basic notations. Let $AP(\mathbf{R}) = \{x(t) : x(t)$ is a real valued almost periodic function on $\mathbf{R}\}$. For $f \in AP(\mathbf{R})$ we denote by

$$\wedge(f) = \left\{\tilde{\lambda} \in R : \lim_{T\to\infty} \frac{1}{T}\int_0^T f(s)e^{-i\tilde{\lambda}s}\text{d}s \neq 0\right\}$$

and

$$(\text{mod} f) = \left\{\sum_{j=1}^m n_j\tilde{\lambda}_j : n_j \in \mathbf{Z}, m \in \mathbf{N}, \tilde{\lambda}_j \in \wedge(f), j = 1,2,\dots,m\right\}$$

the set of Fourier exponents and the module of f, respectively. Let $K(f,\varepsilon,S)$ denote the set of ϵ-almost periods for f with respect to $S \subset\in C([-\Delta,0],[0,\infty))$, $l(\epsilon)$ denote the length of the inclusion interval and $m(f) = \frac{1}{T}\int_0^T f(s)\text{d}s$ denote the mean value of f.

Definition 1. *$z(t) \in C(\mathbf{R},\mathbf{R})$ is said to be almost periodic on $\mathbf{R}$ if for any $\varepsilon > 0$ the set $K(z,\varepsilon) = \{\delta : \|z(t+\delta) - z(t)\| < \varepsilon, \forall\ t \in \mathbf{R}\}$ is relatively dense, that is, for any $\varepsilon > 0$ it is possible to find a real number $l(\varepsilon) > 0$ for any interval with length $l(\varepsilon) > 0$, there exists a number $\delta = \delta(\varepsilon)$ in this interval such that $\|z(t+\delta) - z(t)\| < \varepsilon$ for any $t \in \mathbf{R}$.*

Throughout of this paper, we assume that the following condition for equation (2)

(F) $r, a_j, c_i \in AP(\mathbf{R})$ and $m[\sum_{j=1}^n a_j - \sum_{i=1}^m c_i] \neq 0$, where $m[f] = \lim_{T\to\infty}\frac{1}{T}\int_0^\infty f(s)\text{d}s$.

In our case, we shall consider $\mathbf{X} = \mathbf{Z} = V_1 \bigoplus V_2$, where

$V_1 = \{z(t) \in AP(\mathbf{R}) : \text{mod}(x(t)) \subset \text{mod}(F), \forall\mu \in \Lambda(z(t))$ satisfies $|\mu| > \alpha\}$,

$$V_2 = \{x(t) \equiv c \in \mathbf{R}\},$$

where

$$F = F(t,\phi) = r(t) - \sum_{j=1}^n a_j(t)\phi(t-\sigma_j(t)) - \sum_{i=1}^m c_i(t) \times\phi'(t-\tau_i(t)), \quad \phi \in C([-\Delta,0],\mathbf{R}),$$

and α is a given positive constant. Define the norm

$$\|y\| = \sup_{t\in\mathbf{R}}|y(t)| \text{ for all } x \in \mathbf{X}(\text{or } \mathbf{Z}).$$

We start with the following lemmas.

Lemma 2. *$\mathbf{X}$ and $\mathbf{Z}$ are Banach spaces equipped with the norm $\|\cdot\|$.*

Proof: If $\{z_n\} \subset V_1$ and z_n converges to y_0, then it is easy to show that $y_0 \in AP(\mathbf{R})$ with (mod F). Indeed, for all $\|\tilde{\lambda}\| \leq \alpha$ we have

$$\lim_{T\to\infty}\frac{1}{T}\int_0^\infty z_n(s)e^{-i\tilde{\lambda}s}\text{d}s = 0.$$

Thus

$$\lim_{T\to\infty}\frac{1}{T}\int_0^\infty z_0(s)e^{-i\tilde{\lambda}s}\text{d}s = 0,$$

which implies that $z_0 \in V_1$. One can easily see that V_1 is a Banach space endowed with the norm $||\cdot||$. The same can be concluded for the spaces $\mathbf{X}$ and $\mathbf{Z}$. The proof is complete. ■

Lemma 3. *Let $L : \mathbf{X} \to \mathbf{Z}$ such that $Lz = r(t) - \sum_{j=1}^n a_j(t)z(t-\sigma_j(t)) - \sum_{i=1}^m c_i(t)z'(t-\tau_i(t))$, where $Lz = \frac{\text{d}z}{\text{d}t}$ Then L is a Fredholm mapping of index zero.*

Proof: It is obvious that L is a linear operator and Ker$L = V_2$. It remains to prove that Im$L = V_1$. Suppose that $\phi(t) \in \text{Im}L \subset \mathbf{Z}$. Then, there exists $\phi_1 \in V_1$ and $\phi_2 \in V_2$ such that

$$\phi = \phi_1 + \phi_2.$$

From the definitions of $\phi(t)$ and $\phi_1(t)$, we can easily see that both $\int^t \phi(s)\text{d}s$ and $\int^t \phi_1(s)\text{d}s$ are almost periodic functions, and so we have $\phi_2(t) \equiv 0$ which implies

$$\text{Im } L \subset V_1$$

On the other hand, if $\varphi(t) \in V_1\backslash\{0\}$, then we have $\int_0^t \varphi(s)\text{d}s \in AP(\mathbf{R})$. By the way, if $\lambda \neq 0$, then we obtain,

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T\left[\int_0^t \varphi(s)\text{d}s\right]e^{-i\tilde{\lambda}t}\text{d}t = \frac{1}{i\tilde{\lambda}}\lim_{T\to\infty}\frac{1}{T}\int_0^T \varphi(t)e^{-i\tilde{\lambda}t}\text{d}t.$$

It follows that

$$\bigwedge\left[\int_0^t \varphi(s)\text{d}s - m\left(\int_0^t \varphi(s)\text{d}s\right)\right] = \bigwedge(\varphi(t)).$$

Thus

$$\int_0^t \varphi(s)\text{d}s - m\left(\int_0^t \varphi(s)\text{d}s\right) \in V_1 \subset \mathbf{X}.$$

Note that $\int_0^t \varphi(s)\text{d}s - m(\int_0^t \varphi(s)\text{d}s)$ is the primitive of $\varphi(t)$ in $\mathbf{X}$, therefore we have $\varphi(t) \in \text{Im}L$. Hence, we deduce that

$$V_1 \subset \text{Im}L$$

which completes the proof of our claim. Therefore, $\text{Im}L = V_1$. Furthermore, one can easily show that $\text{Im}L$ is closed in $\mathbf{Z}$ and

$$\text{dimKer}L = 1 = \text{codimIm}L.$$

Lemma 4. *Let* $N : \mathbf{X} \to \mathbf{Z}$*,*

$$\begin{aligned} Nz(t) &= r(t) - \sum_{j=1}^{n} a_j(t) z(t-\sigma_j(t)) \\ &\quad - \sum_{i=1}^{m} c_i(t) z'(t-\tau_i(t)) \quad z \in \mathbf{X} \end{aligned}$$

and

$$P : \mathbf{X} \to \mathbf{X},\ Pz = m(z);\ Q : \mathbf{Z} \to \mathbf{Z},\ Qz = m(z).$$

Then N *is L-compact on* $\bar{\Omega}$(Ω *is a open bounded subset of* $\mathbf{X}$).

Proof: Obviously, P and Q are continuous projectors such that,

$$(\mathrm{I}-\mathrm{Q})V_2 = \{0\},\ \ (\mathrm{I}-\mathrm{Q})V_1 = V_1.$$

Then, we have

$$\mathrm{Im}(\mathrm{I}-\mathrm{Q}) = V_1 = \mathrm{Im}L.$$

Now, in view of

$$\mathrm{Im}P = \mathrm{Ker}L, \mathrm{Im}L = \mathrm{Ker}Q = \mathrm{Im}(I-Q),$$

we can conclude that the generalized inverse (to L) $K_p : \mathrm{Im}L \to \mathrm{Ker}P \cap \mathrm{Dom}L$ exists and is given by

$$K_p(z) = \int_0^t z(s)\mathrm{d}s - m\big[\int_0^t z(s)\mathrm{d}s\big]$$

Hence,

$$\begin{aligned} QNz &= m[r(t) - \sum_{j=1}^{n} a_j(t) z(t-\sigma_j(t)) \\ &\quad - \sum_{i=1}^{m} c_i(t) z'(t-\tau_i(t))] \end{aligned}$$

and

$$K_p(I-Q)Nz = f[z(t)] - Qf[z(t)]$$

where $f[z]$ is defined by

$$f[z(t)] = \int_0^t \big[Nz(s) - QNz(s)\big]$$

QN and $(I-Q)N$ are obviously continuous. Now we are in position to show that K_p also is continuous.

By our hypothesis, for any $\varepsilon < 1$ and any compact set $s \subset C([-\Delta, 0], R)$, let $l(\varepsilon, S)$ be the inclusion interval of $T(F, \varepsilon, S)$. Suppose $\{z_n(t)\} \subset \mathrm{Im}L = V_1$, and $z_n(t)$ uniformly converge to $z_0(t)$. Because of $\int_0^t z_n(s)\mathrm{d}s \in \mathbf{Z}(n = 0, 1, 2, \ldots)$, there exists $\delta(0 < \delta < \varepsilon)$ such that $T(F, \delta, S) \subset T(\int_0^t z_n(s)\mathrm{d}s, \varepsilon)$. Also, let $l(\delta, S)$ be the inclusion interval of $T(F, \delta, S)$, and

$$l = \max\{l(\delta, S), l(\varepsilon, S)\}.$$

It is easy to see that l is the inclusion interval of both $T(F, \varepsilon, S)$ and $T(F, \delta, S)$. Hence, for $\forall t \notin [0, l]$, there exists $\xi_t \int(F, \delta, S) \subset T(\int_0^t z_n(s)\mathrm{d}s, \varepsilon)$ such that $t + \xi_t \in [0, l]$. Hence, by the definition of almost periodic function we have

$$\begin{aligned} & \| \int_0^t z_n(s)\mathrm{d}s \| \\ = & \sup_{t \in R} | \int_0^t z_n(s)\mathrm{d}s | \\ \le & \sup_{t \in [0,l]} | \int_0^t z_n(s)\mathrm{d}s | + \sup_{t \notin [0,l]} | \Big(\int_0^t z_n(s)\mathrm{d}s \\ & - \int_0^{t+\xi_t} z_n(s)\mathrm{d}s \Big) + \int_0^{t+\xi_t} z_n(s)\mathrm{d}s | \qquad (4) \\ \le & 2 \sup_{t \in [0,l]} | \int_0^t z_n(s)\mathrm{d}s | + \sup_{t \notin [0,l]} | \int_0^t z_n(s)\mathrm{d}s \\ & - \int_0^{t+\xi_t} z_n(s)\mathrm{d}s | \\ \le & 2 \int_0^t |z_n(s)|\mathrm{d}s + \varepsilon, \end{aligned}$$

by applying (4), we can conclude that $\int_0^t z(s)\mathrm{d}s(z \in \mathrm{Im}l)$ is continuous, and consequently K_p and $K_p(I-Q)Nz$ also are continuous.

From inequality (4), we also have $\int_0^t z(s)\mathrm{d}s$ and $K_p(I-Q)Nz$ are uniformly bounded in $\bar{\Omega}$. In addition, we can easy conclude that $QN(\bar{\Omega})$ is bounded and $K_p(I-Q)Nz$ is equicontinuous in $\bar{\Omega}$. By now, by using the Arzela-Ascoli theorem, we can immediately conclude that $K_p(I-\bar{Q})N\bar{\Omega}$ is compact. Thus N is L-compact on $\bar{\Omega}$. The proof of Lemma 2.4 is complete. ■

Lemma 5. [10] *Assume that* $x(t) \in AP(\mathbf{R})$*, then* $x(t)$ *is bounded on* $\mathbf{R}$*.*

For the sake of convenience, we introduce notations as follows:

$$f^L = \max_{t \in \mathbf{R}} f(t), \quad f^l = \min_{t \in \mathbf{R}} f(t),$$

where f is a positive continuous almost periodic function.

III. Existence of positive almost periodic solution

Theorem 1. *Assume that* (F) *hold, then system* (1) *has at least one positive almost periodic solution.*

Proof: In order to use the continuation theorem of coincidence degree theory to establish the existence of a solution of system (1), we set Banach spaces $\mathbf{X}$ and $\mathbf{Z}$ the same as those in Lemma 2.2 and set mappings L, N, P, Q the same as those in Lemmas 2.3 and 2.4, respectively. Then we can obtain that L is a Fredholm mapping of index zero and N is a continuous operator which is L-compact on $\bar{\Omega}$.

Now, we are in the position of searching for an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation

$$Lz = \lambda Nz, \quad \lambda \in (0, 1),$$

we obtain

$$\frac{\mathrm{d}z(t)}{\mathrm{d}t} = \lambda \Big[r(t) - \sum_{j=1}^{n} a_j(t) z(t-\sigma_j(t))$$

$$-\sum_{i=1}^{m} c_i(t)z'(t-\tau_i(t))\Big]. \qquad (5)$$

Assume that $z \in \mathbf{X}$ is a solution of system (5) for some $\lambda \in (0,1)$. From Lemma 2.5, there must exist ξ, $\eta \in \mathbf{R}$ such that

$$z(\xi) = \sup_{t\in\mathbf{R}} z(t) \quad \text{and} \quad x(\eta) = \inf_{t\in\mathbf{R}} z(t).$$

It is clear that $z'(\xi) = 0$ and $z'(\eta) = 0$. From this and system (5), we have

$$m[r(t)] = m\Big[\sum_{j=1}^{n} a_j(t)z(t-\sigma_j(t))\Big]$$

and consequently,

$$m[r(t)] \geq z(\xi)m\Big[\sum_{j=1}^{n} a_j(t)\Big],$$

or equivalently

$$z(\xi) \leq \frac{m[r]}{m\Big[\sum_{j=1}^{n} a_j\Big]}. \qquad (6)$$

Similarly, we can get

$$z(\eta) \geq \frac{m[r]}{m\Big[\sum_{j=1}^{n} a_j\Big]}. \qquad (7)$$

By the inequalities (6) and (7), we can find that there exists $t_0 \in \mathbf{R}$ such that

$$z(t_0) \leq M_1,$$

where

$$M_1 = \left|\frac{m[r]}{m\big[\sum_{j=1}^{n} a_j\big]}\right| + 1.$$

By virtue of (4), we obtain

$$\begin{aligned} \|z(t)\| &\leq |z(t_0)| + \sup_{t\in\mathbf{R}}\left|\int_{t_0}^{t} z'(s)ds\right| \\ &\leq M_1 + 2\sup_{t\in[t_0,t_0+l]}\left|\int_{t_0}^{t} z'(s)\mathrm{d}s\right| + \epsilon \end{aligned}$$

or

$$\|z(t)\| \leq M_1 + 2\int_{t_0}^{t_0+l} |z'(s)|\mathrm{d}s + 1. \qquad (8)$$

Choose a point $\nu - t_0 \in [l, 2l] \cap K(F,\rho,S)$, where $\rho(0 < \rho < \epsilon)$ satisfies $K(F,\rho) \subset K(y,\epsilon)$. Integrating (5) form t_0 to ν, we obtain

$$\begin{aligned} &\lambda\int_{t_0}^{\mu}\Big[\sum_{j=1}^{n} a_j(s)(s-\sigma_j(s))\Big]\mathrm{d}s \qquad (9)\\ =\;& \lambda\int_{t_0}^{\mu}\Big[r(s) + \sum_{i=1}^{m} c_i(s)z'(s-\tau_i(s))\Big]\mathrm{d}s - \int_{t_0}^{\mu} z'(s)\mathrm{d}s \\ \leq\;& \lambda\int_{t_0}^{\mu}|r(s)|\mathrm{d}s + \epsilon. \qquad (10)\end{aligned}$$

However, from (5) and (9), we have

$$\begin{aligned} &\int_{t_0}^{\mu}|z'(s)|\mathrm{d}s \\ \leq\;& \lambda\int_{t_0}^{\mu}|r(s)|\mathrm{d}s + \lambda\int_{t_0}^{\mu}\Big[\sum_{j=1}^{n} a_j(s)z(s-\sigma_j(s))\mathrm{d}s \\ &+ \sum_{i=0}^{m} c_i(s)z'(s-\tau_i(s))\Big]\mathrm{d}s \\ \leq\;& 2\int_{t_0}^{\mu}|r(s)|\mathrm{d}s + \epsilon \leq 2\int_{t_0}^{\mu}|r(s)|\mathrm{d}s + 1. \qquad (11)\end{aligned}$$

Substituting back in (8) and for $\mu \geq t_0 + l$, we have

$$\|z(t)\| \leq M_2,$$

where

$$M_2 = M_1 + 4\int_{t_0}^{\mu}|r(s)|\mathrm{d}s + 3.$$

Let $M = M_2 + \left|\frac{m[r]}{\sum_{j=1}^{n} a_j}\right| + 1$. Obviously, is is independent of λ. Take

$$\Omega = \{z \in \mathbf{X} : z \leq M\}.$$

It is clear that Ω satisfies assumption (1) of Lemma 2.1. If $y \in \partial\Omega \cap \mathrm{Ker}L$ then z is a constant with $\|z\| = M$. It follows that

$$\begin{aligned} QNz &= m\Big[r(t) - \sum_{j=1}^{n} a_j(t)z(t-\sigma_j(t)) \\ &\quad - \sum_{i=1}^{n} c_i(t)z'(t-\tau_i(t))\Big] \\ &\neq 0, \end{aligned}$$

which implies that assumption (2) of lemma 2.1 is satisfied. Finally, we show that show (3) in Lemma 2.1 holds. The isomorphism $J : \mathrm{Im}Q \to \mathrm{Ker}L$ is defined by $J(y) = y$ for $y \in \mathbf{R}$. Thus, $JQNz \neq 0$. In order to compute the Brouwer degree, we consider the homotopy

$$H(z,s) = -sy + (1-s)JQNz, \quad 0 \leq s \leq 1.$$

For any $y \in \partial\Omega \cap \mathrm{Ker}L, s \in [0,1]$, we have $H(z, s \neq 0)$. By the homotopic invariance of topological degree, we obtain

$$\deg\{JQN, \Omega_i \cap \mathrm{Ker}\,L, 0\} = \deg\{-z, \Omega \cap \mathrm{Ker}L, 0\} \neq 0.$$

Here, J is taken as the identity mapping. So far we have proved that Ω satisfy all the conditions in Lemma 2.1. Hence system (2) has at least one almost periodic solution in $\bar{\Omega}$, that is equation (1) has at least one positive almost periodic solution. This completes the proof. ■

IV. An example

Now, we give an example to demonstrate our results.

Example 1. *Let* $r(t) = \cos t$, $a_1(t) = e^{\pi}(2+\cos\sqrt{3}t)$,$c_1(t) = e^{\pi}(\sin\sqrt{2}t)$, $\sigma_1(t) = \tau_1(t) = \frac{\pi}{2}$. *Then system* (2.1) *reduce as flowing*

$$\dot{z}(t) \;=\; \sin t - e^{\pi}(2+\cos\sqrt{3}t)z(t-\frac{\pi}{2})$$

$$-e^{\pi}(\sin\sqrt{2}t)\dot{z}(t-\frac{\pi}{2}). \tag{12}$$

By calculation, $m[a_1] = e^{\pi} \neq 0$ and thus condition (F) holds. Therefore, by the consequence of lemma 2.1, system (12) has at leat one positive almost periodic solution $z(t)$.

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