# Porous Effect on Heat Transfer of Non Uniform Velocity Inlet Flow Using LBM

A. Hasanpour, M. Farhadi and K.Sedighi, H.R.Ashorynejad

***Abstract***—A numerical study of flow in a horizontally channel partially filled with a porous screen with non-uniform inlet has been performed by lattice Boltzmann method (LBM). The flow in porous layer has been simulated by the Brinkman-Forchheimer model. Numerical solutions have been obtained for variable porosity models and the effects of Darcy number and porosity have been studied in detail. It is found that the flow stabilization is reliant on the Darcy number. Also the results show that the stabilization of flow field and heat transfer is depended to Darcy number. Distribution of stream field becomes more stable by decreasing Darcy number. Results illustrate that the effect of variable porosity is significant just in the region of the solid boundary. In addition, difference between constant and variable porosity models is decreased by decreasing the Darcy number.

***Keywords***—Lattice Boltzmann Method, Porous Media, Variable Porosity, Flow Stabilization

## I. INTRODUCTION

CHANELS filled with porous material have been widely used in a variety of engineering applications like flow stabilizer in burners, troche, etc. Many researchers investigated fluid flow and convective heat transfer in channels fully and partially filled with porous medium [1-8]. Poulikakos and Kazmierczak [1] theoretically considered fully developed convection between two parallel plates and in a wall. Guo et al. [2 numerically studied pulsating flow in a circular tube partly filled with an adhered porous matrix to the pipe partially filled with a porous medium by finite volume method.

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In their work, the porous layer is fitted to the pipe wall. Hamdan et al. [3] studied forced convection by inserting porous substrate in the core of a parallel plate channel numerically. Alkam et al. [4] considered forced convection flow in a channel partially filled with porous layers on top and bottom walls. Their results show that the existence of the porous substrate improves the Nusselt number. Jiang et al. [5] simulated forced convection in sintered porous plate channel, numerically. Jen and Yan [6] performed a numerical simulation by a vorticity–velocity method in a channel partially filled with porous medium.

**Nomenclature**

| | |
|---|---|
| $c$ | discrete lattice velocity |
| $c_s$ | Speed of sound in lattice |
| $Da$ | Darcy number, $K/H^2$ |
| $f_i^{eq}$ | equilibrium distribution function |
| $H$ | channel width |
| $K_f$ | thermal conductivity of fluid |
| $K_s$ | thermal conductivity of solid |
| $L$ | channel length |
| $Nu$ | local Nusselt number |
| $Nu_{avg}$ | average Nusselt number |
| $x, y$ | Cartesian coordinates |

*Greek symbols*

| | |
|---|---|
| | Porosity |
| $K$ | permeability of porous medium |
| $\alpha$ | thermal diffusivity of porous medium |
| $\theta$ | dimensionless temperature, $(\theta = T - T_{in}/T_{in} - T_w)$ |
| $\omega_i$ | weighting factor |

In their investigation, the friction factor and Nusselt number are presented as a function of axial position. Liou [7] proposed a new numerical technique for simulating flow in the porous media in terms of pore scale analysis. The effect of wall mounted porous layer in the channel on the heat transfer was investigated by Satyamurty and Bhargavi [8]. In their study, wall heat transfer is changed for establishing the maximum enhancement in heat transfer.

The Lattice Boltzmann Method (LBM) is a practical method for simulating fluid flow and heat transfer [9-12]. This method has been applied to flow in porous media A most

commonly approach to apply LBM to porous flow is to model the flow in the representative elementary volume (REV) scale [13]. This is accomplished by including an additional term to the standard lattice Boltzmann equation to account for the presence of a porous medium. Dardis and McCloskey [14] used this technique for the simulation of flow in porous media using LBM. Spaid and Phelan [15] proposed a model based on the Brinkman equation for single-component flow in porous media. However the Brinkman model has been widely used to describe flows in porous media, some limitations still exist in this model. Vafai and Kim [16] have pointed that there is no mechanism for the development of the flow field without a convective term. In this study, the linear and nonlinear matrix drag components are considered as well as the inertial and viscous forces by Brinkman- Forchheimer model [13 and 17]. In this model, the inertial force is included and the equilibrium distribution function is modified for considering the porosity of the medium. This model is applicable for a medium with both a constant and a variable porosity. Also the magnetic resonance flow imaging (MRI) techniques and LBM for simulating flow of a Newtonian liquid through a dual porosity structure was used by Mantle et al. [18].

The objective of the present study is investigation of flow field and heat transfer in a channel partially filled with a porous screen with non-uniform inlet by Lattice Boltzmann method (LBM). The basis of mathematical formulation for porous media is the Brinkman-Forchheimer model [17]. The results are obtained and the effects of Darcy number and porosity are considered.

## II. MATHEMATICAL MODEL

Flow and heat transfer in a horizontal plane channel with a porous screen have been simulated by LBM. The channel height and length is H and L=8H respectively and non-uniform velocity and temperature is considered at the inlet (Figure 1). At the inlet, the velocity ratio of $U_1/U_2 = 15$, dimensionless inlet temperature $\theta_1=1$, $\theta_2=0$ and wall dimensionless temperature $\theta_w=0$. The porous sample is assumed to be saturated with a fluid that is in local thermal equilibrium (LTE) with the solid matrix [19].The porous screen is located at 0.5H from the entrance of the channel with 0.2H width.
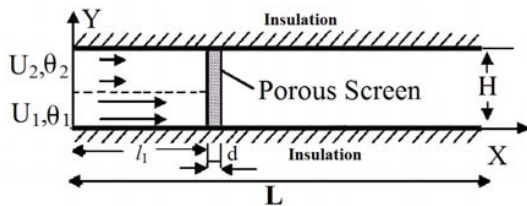


Fig. 1 Geometry of the present study

### A. LBM in MHD Porous Media

The Lattice Boltzmann model for incompressible fluid flow in porous media has been proposed by several investigators [17, 20, and 21]. In LBM, the fluid is modeled by a single-particle distribution function. The distribution functions for porous media are governed by lattice Boltzmann equation as follow [17]:

$$f_i(\vec{x}+\vec{e}_i\delta_t,t+\delta_t)=f_i(\vec{x},t)-\frac{f_i(\vec{x},t)-f_i^{eq}(\vec{x},t)}{\tau_v}+\delta_t F_i \quad (1)$$

$$g_i(\vec{x}+\vec{e}_i\delta_t,t+\delta_t)=g_i(\vec{x},t)-\frac{g_i(\vec{x},t)-g_i^{eq}(\vec{x},t)}{\tau_c} \quad (2)$$

Discrete velocities For $D_2Q_9$ model is:

$$\vec{e}_i=\begin{cases}(0,0) & for \quad i=0 \\ \left(\cos[(i-1)\frac{\pi}{4},\sin[(i-1)\frac{\pi}{4}\right) & for \quad i=0...4 \\ \sqrt{2}\left(\cos[(i-1)\frac{\pi}{4},\sin[(i-1)\frac{\pi}{4}\right) & for \quad i=5...8\end{cases} \quad (3)$$

Here, $\delta_t$ is the lattice time step. The equilibrium functions for the density distribution function ($f_i^{eq}$) for the $D_2Q_9$ model in presence of porous media is:

$$f_i^{eq}=\left\{\omega_i\rho\left[1+\frac{\vec{e}_i.\vec{u}}{c_S^2}+\frac{(\vec{e}_i.\vec{u})^2}{2\varepsilon c_S^4}-\frac{|\vec{u}|^2}{2\varepsilon c_S^2}\right]\right\} \quad (4)$$

Where is the porosity of the porous medium and $\omega_i$ is weighting factor. $c_s$ is speed of the sound and defined by $c_s=\frac{c}{\sqrt{3}}$ [17]. The weighting factors are:

$$\omega_i=\left\{\frac{4}{9}\,for\,i=0,\,\frac{1}{9}\,for\,i=1...4,\,\frac{1}{36}\,for\,i=5...8\right\} \quad (5)$$

The equilibrium distribution functions for thermal energy distribution have been presented by Mohamad [21]:

$$g_i^{eq}=\omega_i T\left(1+\frac{1}{c_s^2}\vec{e}_i.\vec{u}\right) \quad (6)$$

In addition, the Brinkman-Forchheimer equation is: [20]

$$\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\nabla)(\frac{\vec{u}}{\varepsilon})=-\frac{1}{\rho_f}\nabla(\varepsilon P)+\upsilon\nabla^2 u+\vec{F} \quad (7)$$

Here, $p=\frac{c_s^2\rho}{\varepsilon}$, and the viscosity $\upsilon=c^2(\tau_\upsilon-0.5)\frac{\delta_t}{3}$ . Also, the body force is:

$$\vec{F}=-\frac{\varepsilon\upsilon}{K}\vec{u}-\frac{\varepsilon F_\varepsilon}{\sqrt{K}}|u|\vec{u}+\varepsilon\vec{G} \quad (8)$$

$$K=Da H^2,\quad F_\varepsilon=\frac{1.75}{\sqrt{150\varepsilon^3}} \quad (9)$$

Where $\upsilon$ is the viscosity of the fluid, $K$ is the permeability, $G$ is the acceleration due to gravity, Da is the Darcy number, and $H$ is the characteristic length. The total body force ($\vec{F}$) includes the viscous diffusion, the inertia due to the presence of a porous medium and an external force. It is proved that the most suitable choice for the $F_i$ (see Eq.1) is [17 and 20]:

$$F_i=\omega_i\rho(1-\frac{1}{2\tau_\upsilon})\left[\frac{\vec{e}_i.\vec{F}}{c_s^2}+\frac{(\vec{u}.\vec{F}:\vec{e}_i\vec{e}_i)}{\varepsilon c_s^4}-\frac{\vec{u}.\vec{F}}{\varepsilon c_s^2}\right] \quad (10)$$

The fluid velocity $\vec{u}$ defines as [17 and 20]:

$$\rho\vec{u} = \sum_i \vec{e}_i f_i + \frac{\delta_t}{2}\rho\vec{F} \qquad (11)$$

As shown in Eq. (8), $\vec{F}$ contains the velocity $\vec{u}$. Equation (11) is a nonlinear equation with respect to the velocity $\vec{u}$. This nonlinearity can be ignored by introducing a temporal velocity $\vec{v}$ [17]:

$$\vec{u} = \frac{\vec{v}}{c_0 + \sqrt{c_0^2 + c_1|\vec{v}|}} \qquad \vec{v} = \frac{\sum_i \vec{e}_i f_i}{\rho} + \frac{\delta_t}{2}\varepsilon\vec{G} \qquad (12)$$

$$c_0 = \frac{1}{2}(1 + \frac{\delta_t}{2}\varepsilon\frac{\upsilon}{K}) \qquad c_1 = \frac{\delta_t}{2}\varepsilon\frac{1.75}{\sqrt{150\varepsilon^3 K}} \qquad (13)$$

And finally, he fluid density and temperature are [17 and 21]:

$$\rho = \sum_i f_i, \quad T = \sum_i g_i \qquad (14)$$

### III. RESULTS AND DISCUSSIONS

In this study, the effect of porous screen on non-uniform inlet velocity flow and heat transfer in a horizontal plane channel was considered using Lattice Boltzmann Method (Figure1). The effects of Darcy number and Variable porosity model were investigated on the flow field stabilization and convective heat transfer. The present computation focused on the parameters having the following ranges: Da= $10^{-4}$ to $10^{-1}$, ε=0.4, Pr=0.7 and Re=50. To validate the numerical simulation, the flow and forced convection in a channel filled with or without (Poiseuille flow) a porous medium were simulated and compared with the previous studies (Figure 2).
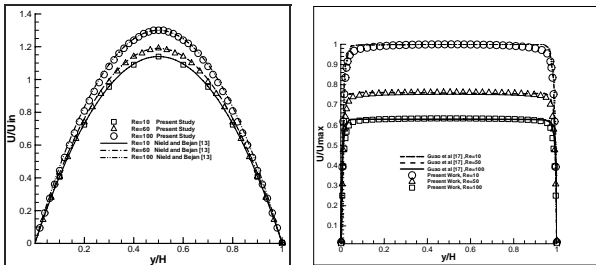


Fig. 2 Velocity profile in (a) Poiseuille flow and (b) channel field by porous medium for present study and previous studies.

Results show a good agreement in comparison with previous studies of Guo and Zhao [17] and Neaild and Bejan [13] (Figure 2).

Figure 3 shows the distributions of non-dimensional velocity in the y direction for different x/H (x/H=0.4, 0.6, 0.8) at Da=$10^{-2}$ and ε=0.4. It is observed that the non uniform inlet flow becomes stabilize after porous layer and a homogeneous velocity distribution is obtained. For instance, before porous screen (x/H=0.4), the velocity gradient between y/H=0.24 and y/H=0.83 is about 60%, but this gradient reduces and reaches near 40% after porous screen (x/H=0.6).
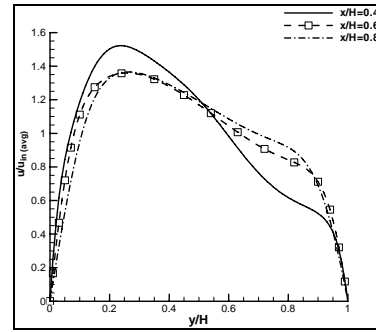


Fig. 3 Axial velocity along the distance from the bottom wall at different locations for Da=$10^{-2}$, =0.4

Before the results of numerical simulations are presented, the bellow non-dimensional parameters are introduced as:

$$\theta = \frac{T - T_w}{T_{in1} - T_w} \qquad (15)$$

The bulk mean temperature and mean velocity for calculating the Nusselt number are:

$$T_m = \frac{1}{HU_m}\int_0^H uTdy \qquad (16)$$

$$U_m = \frac{1}{H}\int_0^H udy \qquad (17)$$

The local Nusselt number is:

$$Nu = \frac{D_H}{\theta_w - \theta_m}\frac{\partial\theta}{\partial y}\bigg|_{wall} \qquad (18)$$

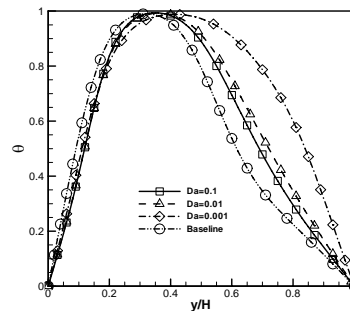In Eq.18, $D_H$ means hydraulic diameter and specified as $D_H$=2H [13].



Fig. 4 Distribution of Temperature over the bottom wall by Darcy variations at x/H=0.7 of for =0.8 and Re=100

The effect of Darcy number on the temperature distribution is pointed up in Figure 4. It is concluded that with declining Darcy number, the temperature profile decreases and becomes stable. The effects of porous screen on temperature contours for various values of Darcy numbers are shown in Figure 5. It can be seen that the maximum temperature gradient is decreased faster using porous screen at lower Darcy number and consequently an improved homogeneous temperature contours are created for smaller channel length.
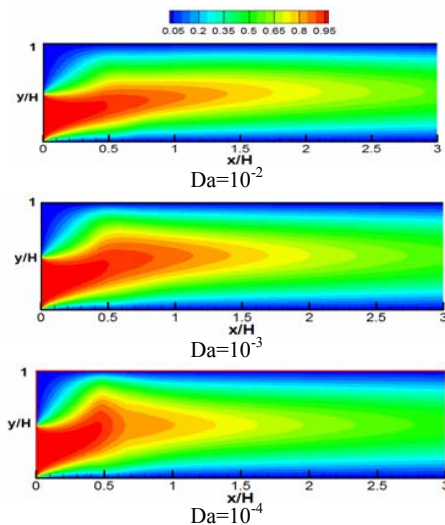
Fig. 5 Distribution of the temperature contours for different
Darcy numbers at =0.8 and Re=100

In Figure 6, the outcome of Darcy variation on local Nusselt number is presented. From this Figure, it is found that lower values of Darcy number lead to higher values of Nusselt number.
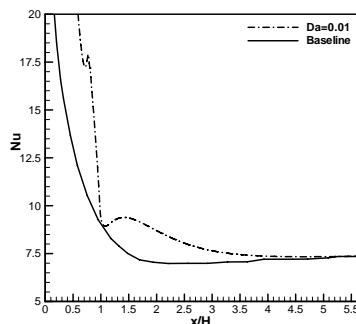


Fig. 6 Local Nusselt number on top (left) and bottom wall
(right) for different Darcy numbers at =0.6 and Re=100

## IV.  CONCLUSION

In this study, the effect of porous screen on heat transfer in a channel with non uniform velocity inlet has been considered using LBM. The final results can be summarizing as follow:

- LBM is an appropriate method to solving fluid flow and heat transfer in porous media.

- Heat transfer instability is damped by decreasing Darcy number and porosity in the porous screen.

-It is found that the heat transfer strongly depends to the Darcy number and porosity and finally, the Nusselt number is increased by declining the Darcy number and porosity.

### REFERENCES

[1]  D. Poulikakos, M. Kazmierczak, "Forced convection in a duct partially filled with a porous material", ASME Journal of Heat Transfer,vol. 109, 1987,pp.  653–662.
[2]  Z. Guo, S.Y. Kim, H. J. Sung, "Pulsating flow and heat transfer in a pipe partially filled with a porous medium", International Journal of Heat and Mass Transfer , vol.40, 1997, pp. 4209–4218.
[3]  M. O. Hamdan, M. A. Al-Nimr, M. K. Alkam, "Enhancing forced convection by inserting porous substrate in the core of a parallel-plate channel", International Journal of Numerical Methods Heat & Fluid Flow, vol. 10, 2000, pp. 502–517.
[4]  M. K. Alkam, M. A. Al-Nimr, M.O. Hamdan, "on forced convection in channel partially filled substrate", Heat and Mass Transfer, vol. 38, 2002, pp.337-342.
[5]  P. X. Jiang, M. Li, Y. C. Ma, Z. P. Ren, "Boundary conditions and wall effect for forced convection heat transfer in sintered porous plate channels", International Journal Heat Mass Transfer, vol. 47, 2004, pp. 2073–2083.
[6]  T. C. Jen, T. Z. Yan, "Developing fluid flow and heat transfer in a channel partially filled with porous medium", International Journal Heat Mass Transfer, vol. 48, 2005, pp. 3995–4009.
[7]  M. F. Liou, "A Numerical study of transport phenomena in porous media", PhD thesis, University of Case Western Reserve, Cleveland, 2005.
[8]  V. V. Satyamurty, D. Bhargavi, "Forced convection in thermally developing region of a channel partially filled with a porous material and optimal porous fraction", International Journal Thermal Science, vol. 49, 2010, pp. 319-332.
[9]  C. M. Subhash, M. Bittagopal, K. Tanuj, B. S. R. Krishna, "Solving transient heat conduction problems on uniform and non-uniform lattices using the lattice Boltzmann method", International Communications in Heat and Mass Transfer, vol. 36, 2009, pp. 322-328.
[10]  P. H. Kao, R. J. Yang, "An investigation into curved and moving boundary treatments in the lattice Boltzmann method", Journal of Computational Physics, vol.27, 2008, pp. 5671-5690 .
[11]  M. A. Delavar, M. Farhadi, K. Sedighi,"Effect of the Heater Location on Heat Transfer and Entropy Generation in the Cavity Using the Lattice Boltzmann Method", Heat Transfer Research, vol. 40, 2009, pp. 521-536.
[12]  H. Nemati, M. Farhadi, K. Sedighi, E. Fattahi, A.A.R. Darzi, "Lattice Boltzmann simulation of nanofluid in lid-driven cavity", Accepted for publication)," International Communications in Heat and Mass Transfe., to be published.
[13]  D. A. Nield, A. Bejan, Convection in Porous Media, third ed., New York: Springer-Verlag, 2006, ch. 1-2.
[14]  O. Dardis, J. McCloskey, "Lattice Boltzmann scheme with real numbered solid density for the simulation of flow in porous media", Physics Review E, vol. 57, 1998, pp. 4834-4837.
[15]  M. A. A. Spaid, F. R. Phelan, "Lattice Boltzmann methods for modeling micro scale flow in fibrous porous media", Physics of Fluids, vol. 9, 1997, pp. 2468–2474.
[16]  S. J. Kim, K. Vafai, "Analysis of natural convection about a vertical plate embedded in a porous medium", International Journal Heat Mass Transfer, vol. 32, 1989, pp. 665–677.
[17]  Z. Guo, T. S. Zhao, "Lattice Boltzmann model for incompressible flows through porous media", Physics Review E, vol. 66, 2002, pp. 304-309.
[18]  M. D. Mantlea, B. Bijeljicb, A. J. Sederman, L. F. Gladdena, "MRI velocimetry and lattice Boltzmann simulations of viscous flow of a Newtonian liquid through a dual porosity fibre array", Magnetic Resonance Imaging, vol. 19, 2001, pp.527–529.
[19]  M. Kaviany, "Laminar flow through a porous channel bounded by isothermal parallel plates", International Journal Heat and Mass Transfer, vol. 28, 1985, pp.  851-858.
[20]  T. Seta, E. Takegoshi, K. Kitano, K. Okui, "Thermal Lattice Boltzmann Model for Incompressible Flows through Porous Media", Journal of Thermal Science and Technology, vol. 1, 2006, pp. 90-100.
[21]  A. A. Mohamad, Applied lattice Boltzmann method for transport phenomena, momentum, heat and mass transfer, Calgary: Sure Printing, 2007, ch. 2.
[22]  P. Cheng, A. Chowdhury, C. T. Hsu,"Forced Convection in Packed Tubes and Channels with Variable Porosity and Thermal Dispersion Effects", Convective Heat Mass Transfer in Porous Media, vol.  35, 1991, pp. 625-653.