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# Physical Parameter Based Compact Expression for Propagation Constant of SWCNT Interconnects

Kollarama Subramanyam, Nisha Kuruvilla, J. P. Raina

***Abstract*****—**Novel compact expressions for propagation constant (γ) of SWCNT and bundled SWCNTs interconnect, in terms of physical parameters such as length, operating frequency and diameter of CNTs is proposed in this work. These simplified expressions enable physical insight and accurate estimation of signal attenuation level and its phase change at any length for a particular frequency. The proposed expressions are validated against SPICE simulated results of lumped as well as distributed equivalent electrical RLC nets of CNT interconnect. These expressions also help us to evaluate the cut off frequencies of SWCNTs for different interconnect lengths.

***Keywords*****—**Attenuation constant, Bundled SWCNT, CNT interconnects, Propagation Constant.

## I. Introduction

CARBON nanotubes based interconnects are a front-runner among futuristic interconnect material for lower technological nodes [1]. They have the potential to outperform the current materials in future nodes where the resistance of the metals rises rapidly due to inherent small dimensions [2], [3]. Interconnects made of traditional interconnecting materials are considered as one of the grandest hurdles that giga-scale integration faces because of the delay they add to the critical paths, the power they dissipate, the noise and jitter they induce on one another, and their vulnerability to electromigration. Carbon nanotubes can potentially address these challenges if they are optimally utilized.

Several attempts were made by many [4]-[8] to study the performance analysis of SWCNT in interconnects applications. Most of these studies got initiated after the proposal RF equivalent model for SWCNT interconnects by [9]. This is the basic electrical model which have used in most of the performance analysis of SWCNT structures. In this work, the high frequency behavior of CNT interconnects is studied. P. J. Burke has rigorously derived an RF electrical equivalent circuit for the performance analysis of SWCNT interconnects. Since then series of phenomenological electrical models for the performance evaluation have been proposed. As the inherent quantum resistance possessed by an individual CNT interconnect is high, a proposal for using them in bundle got attention. A model for tightly packed SWCNT bundles was proposed by [10]. All these works compared the performance of SWCNTs against copper. A review of comparative studies between SWCNT and MWCNTs with respect to copper has done by [11].

However none of available works provided a closed form expressions propagation constant in terms of physical parameters of the SWCNT. As CNTs are proposed mainly as a high frequency interconnects, it is inevitable to know the propagation characteristics of these interconnect. The propagation constant (γ) is a measure of the change in signal amplitude when it propagates in a given direction. This can be measured in terms of voltage or current in a circuit. For a given system it is defined by the ratio of the amplitude at the source of the wave to the amplitude at some distance x. The propagation constant is a complex quantity ($\gamma = \alpha + j\beta$) where real part α, gives the information about signal attenuation level and β the imaginary part gives information about phase change in signal. This work proposes a compact expression for propagation constant (attenuation constant (α) and phase constant (β)) in terms of physical parameters of the CNT interconnects. The physical parameters used in these expressions includes length of interconnects, diameter of the CNTs used and operating frequency.

Section II gives compact expression for propagation constant (attenuation constant (α) and phase constant (β)). Section III narrates about the verification approach used for the validation of proposed expressions. Section IV consolidates the results obtained. Section V concludes the paper.

## II. Compact Expression for Propagation Constant of SWCNT and Bundled SWCNT Interconnects

Equivalent circuit model of CNT interconnect used for the derivation of compact expression is as shown in Fig. 1. A simplified compact expression for propagation constant (attenuation constant (α) and phase constant (β)) is derived in terms physical parameters such as length, diameter and frequency. The electrical phenomenological model of SWCNT in Fig. 1, proposed by [9] is used for the modeling $Z_0$. The L-section equivalent circuit composed of various resistive, inductive and capacitive effects.

Kollarama Subramanyam was with the Center for Nanotechnology Research VIT University, Vellore, Tamilnadu, India (e-mail: subbu.kolla@gmail.com).

Nisha Kuruvilla is with College of Engineering, Chengannur, Kerala, India and also a research scholar at Center for Nanotechnology Research VIT University, Vellore, Tamilnadu, India (e-mail: nisha@ceconline.edu).

J. P. Raina was with IIT Madras, India. He is now with the Center for Nanotechnology Research VIT University, Vellore, Tamilnadu, India (e-mail: jewan1@gmail.com).

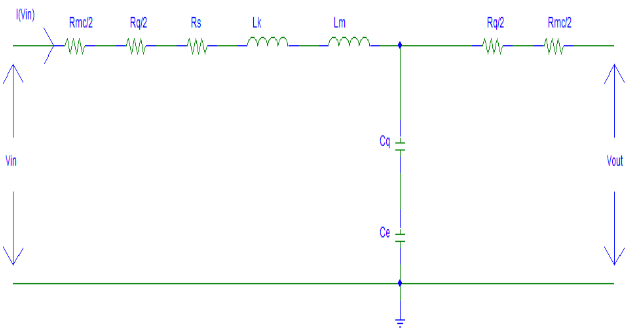


Fig. 1 Equivalent circuit model of SWCNT interconnect

The resistance of CNT consists of contact resistance, quantum resistance and scattering resistance. Quantum resistance of CNT, which arises due to the flow of electrons in the conducting channel, is given by

$$R_Q = \frac{h}{2e^2 N}\,\Omega \tag{1}$$

where = 12.9kΩ, N represents the number of conducting channels in the CNT. The ballistic nature of the CNT depends on whether the length of the nanotube is less or greater than Mean Free Path (λ). Scattering of electrons occurs for length more than the MFP of CNT. Note that scattering also occurs for lengths less than MFP. This scattering resistance is given by [12];

$$R_S = \frac{h}{2e^2 N}\frac{l}{\lambda}\,\Omega \tag{2}$$

where l and λ are length and MFP of CNT respectively. The work done by [13] revealed that MFP depends on diameter given in (3).

$$\lambda = D \cdot v_F/(\alpha T) \tag{3}$$

where D is the diameter, $v_F$ is the Fermi velocity, α is the coefficient of scattering rate and T is the temperature. The work done in [14] indicates that for an SWCNT of diameter 1nm, MFP is 1μm. Hence throughout this work λ is approximated as 1000D. With the advancements in fabrication technology, perfect metal CNT contacts can be possible [15]. Hence in this work contact resistance is ignored.

Inductance of the nanotube consists of kinetic inductance (LK) and magnetic inductance (LM). The kinetic inductance per unit length of the nanotube is given by

$$L_k = \frac{h}{2e^2 v_F} \approx 16\,nH/\mu m \tag{4}$$

The magnetic inductance of SWCNT is given in (5) can be ignored from the equivalent inductance calculations since it is a weak function of [9], [11].

$$L_M = \frac{\mu}{2\pi}\cosh^{-1}\left(\frac{2t}{D}\right) \tag{5}$$

The capacitance of the nanotube consists of quantum capacitance (CQ) and electrostatic capacitance (CE) [9]. The quantum capacitance per unit length of the nanotube is given by

$$C_Q = \frac{2e^2}{h v_F} \tag{6}$$

while the electrostatic capacitance per unit length of CNT is given by

$$C_E = \frac{2\pi\varepsilon}{\cosh^{-1}\left(\frac{2t}{D}\right)} \tag{7}$$

The propagation constant of typical RLC network shown in Fig. 1 is given by

$$\gamma = \sqrt{-\omega^2 LC + j\omega RC} \tag{8}$$

$$\gamma = \left(x + j\frac{R}{2x}\right)\left(\sqrt{\omega C}\right) \tag{9}$$

where,

$$x = \left(\sqrt{\frac{-\omega L + \sqrt{R^2 + \omega^2 L^2}}{2}}\right)$$

The constants α and β are obtained from propagation constant (γ) expression. The real part of ϒ is the attenuation constant and the imaginary part is phase constant. The circuit parameters considered for the derivation of ϒ are quantum resistance ($R_Q$), kinetic inductance ($L_K$), quantum capacitance ($C_Q$) and electrostatic capacitance ($C_E$) for both SWCNT and bundled SWCNT. Magnetic inductance is much smaller compared to kinetic inductance [16] and is ignored in this paper. Contact resistance ($R_{mc}$) is also not taken in to account in this paper. Scattering resistance ($R_s$) is considered in (4) and (6) as per [16]. The compact expressions for SWCNT interconnects are

$$|\gamma| = (12.6\times10^{-7})\left(\frac{\omega l}{1+1.8ln\left(\frac{2t}{D}\right)}\right)^{1/2}(0.65\times10^{12}+\omega^2 l^2)^{1/4}\ for\ (l<\lambda) \tag{10}$$

$$|\gamma| = (12.6\times10^{-7})\left(\frac{\omega l}{1+1.8ln\left(\frac{2t}{D}\right)}\right)^{1/2}\left(0.65\times10^{6}\left(1000+\frac{l}{D}\right)^2+\omega^2 l^2\right)^{1/4} for\ (l>\lambda) \tag{11}$$

The corresponding equations for SWCNT bundle are

$$|\gamma| = (0.063)\left(\frac{\omega C}{n}\right)^{1/2}(0.65\times10^{12}+\omega^2 l^2)^{1/4}\ for\ (l<\lambda) \tag{12}$$

$$|\gamma| = (0.063)\left(\frac{\omega C}{n}\right)^{1/2}\left(0.65\times10^{6}\left(1000+\frac{l}{D}\right)^2+\omega^2 l^2\right)^{1/4}\ for\ (l>\lambda) \tag{13}$$

$n_{CNT}$ represents number of CNTs in the bundle, t is the thickness of the thick oxide substrate and C is the total capacitance of the bundle. All these three parameters are calculated based on the equations given in reference [10]. The size of the bundle is assumed with an aspect ratio of 1:2, which is the accepted ratio for local interconnects. The width of the bundle is considered equivalent to the pitch of the corresponding technology node.

The expression for the attenuation constant ‘α’ of SWCNT interconnects is given by

$$\alpha = 8.9 \times 10^{-7} \left[\frac{\omega l}{1+1.8\ln\left(\frac{2t}{D}\right)}\right]^{\frac{1}{2}} \left[-\omega l + \sqrt{0.65 \times 10^{12} + \omega^2 l^2}\right]^{\frac{1}{2}} \text{ for } (l < \lambda) \quad (14)$$

$$\alpha = 8.9 \times 10^{-7} \left[\frac{\omega l}{1+1.8\ln\left(\frac{2t}{D}\right)}\right]^{\frac{1}{2}} \left[-\omega l + \sqrt{0.65 \times 10^{6}\left(1000 + \frac{l}{D}\right)^2 + \omega^2 l^2}\right]^{\frac{1}{2}} \text{ For } (l > \lambda) \quad (15)$$

The expression for the attenuation constant ‘α’ of bundled SWCNT interconnects is given by

$$\alpha = 0.044 \left(\frac{\omega C}{n_{CNT}}\right)^{\frac{1}{2}} \left[-\omega l + \sqrt{0.65 \times 10^{12} + \omega^2 l^2}\right]^{\frac{1}{2}} for \ (l < \lambda) \quad (16)$$

$$\alpha = 0.044 \left(\frac{\omega C}{n_{CNT}}\right)^{\frac{1}{2}} \left[-\omega l + \sqrt{0.65 \times 10^{-6}\left(1000 + \frac{l}{D}\right)^2 + \omega^2 l^2}\right]^{\frac{1}{2}} for \ (l > \lambda) \quad (17)$$

The phase constant ‘β’ of SWCNT interconnects is given by

$$\beta = 0.71 \left[\frac{\omega l}{-\omega l + \sqrt{0.65 \times 10^{12} + \omega^2 l^2}}\right]^{\frac{1}{2}} for \ (l < \lambda) \quad (18)$$

$$\beta = 0.71 \left[\frac{\omega l}{-\omega l + \sqrt{0.65 \times 10^{6}\left(1000 + \frac{l}{D}\right)^2 + \omega^2 l^2}}\right]^{\frac{1}{2}} for \ (l > \lambda) \quad (19)$$

The phase constant ‘β’ of bundled SWCNT interconnects is given by

$$\beta = 36 \times 10^{3} \left[\frac{(\omega C / n_{CNT})}{-\omega l + \sqrt{0.65 \times 10^{12} + \omega^2 l^2}}\right]^{\frac{1}{2}} for \ (l < \lambda) \quad (20)$$

$$\beta = 36\left(1000 + \frac{l}{D}\right) \left[\frac{(\omega C / n_{CNT})}{-\omega l + \sqrt{0.65 \times 10^{6}\left(1000 + \frac{l}{D}\right)^2 + \omega^2 l^2}}\right]^{\frac{1}{2}} for \ (l > \lambda) \quad (21)$$

## III. Verification Methodology of Compact Expression for Propagation Constant

For the second part of the work, the propagation constant of the modeled expressions are verified against the published results and γ obtained from the simulation of equivalent circuit model of CNT interconnects using SPICE tool.

The propagation constant was then calculated using (15), by measuring $Z_{OC}$ and $Z_{SC}$ of the circuit using simulation.

$$|\gamma| = \sqrt{\frac{Z_{SC}}{Z_{OC}}} \quad (22)$$

Hence the propagation constant can be obtained from the open circuited and short circuited impedance from any of the port of the symmetrical T-network.

## IV. Results

As per the first part of this work we have computed the values of propagation constant of interconnects made of SWCNT and bundled SWCNT for a frequency range of 10Hz to 10THz. These evaluations were done for interconnect length less than mean free path ($l \leq \lambda$) and also for interconnect length greater than mean free path ($l > \lambda$). Fig. 2 gives the comparison of propagation constant. Here γ expressed in terms of R, L and C is compared with γ expressed in terms of l, D and ω and with propagation constant obtained from simulation.

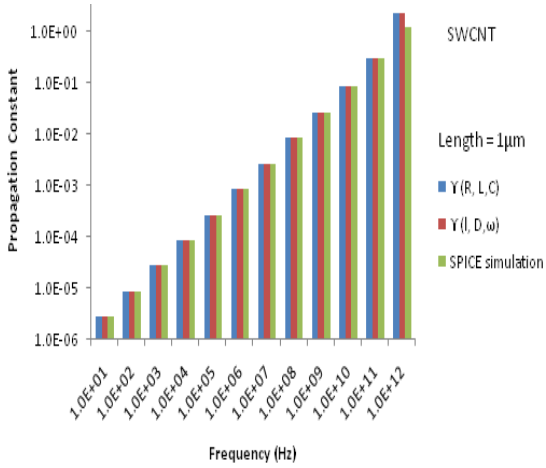


Fig. 2 Comparison of propagation constant, γ expressed in terms of R, L and C is compared with γ expressed in terms of *l*, D, ω and with propagation constant obtained from simulation
Diameter of the nanotube is 1nm

Fig. 3 shows comparison of propagation constant for 1μm and 10μm lengths of nanotube interconnects. Beyond 10GHz, the signal gets attenuated in 10μm length nanotube and for 1μm length it gets attenuated beyond 1THz.

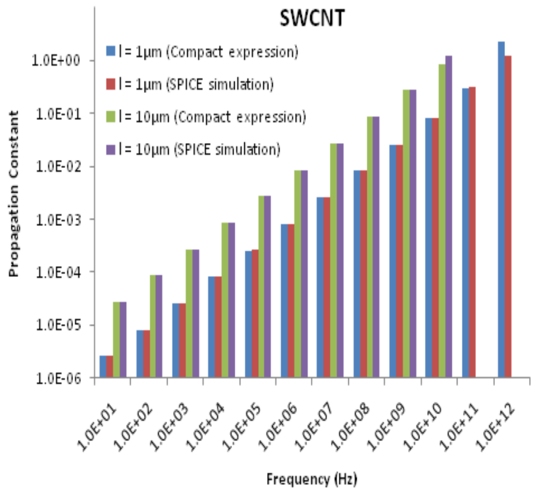


Fig. 3 Compact expression compared to SPICE simulation of SWCNTs for 1μm and 10μm lengths
Diameter of CNT is 1nm

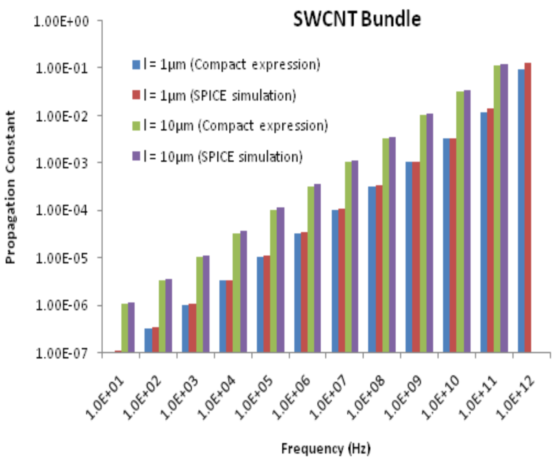


Fig. 4 Compact expression compared to SPICE simulation of SWCNT bundle of 28nm technology node for 1μm and 10μm lengths Diameter of CNT is 1nm

Similar comparison was also done for SWCNT bundle at 28nm technology node as shown in Fig. 4. For SWCNT bundle of 10μm length, the signal gets attenuated beyond 10GHz and for 1μm length it gets attenuated beyond 1THz. In Figs. 3 and 4, propagation constant is in agreement with the SPICE simulated counterpart up to 1GHz for SWCNT bundle and up to 10GHz for SWCNTs. This is because of inductive effects at high frequency. The propagation constant gives us information about the cut off frequencies of interconnect. This cut-off frequency can be compared with the 3 dB frequency obtained from frequency response of SWCNT interconnects [17].

TABLE I
COMPARISON OF 3 DB FREQUENCY OBTAINED FROM PROPAGATION CONSTANT AGAINST 3DB FREQUENCY OF CNT INTERCONNECTS UNDER MATCHING IMPEDANCE [17]

| Type of the interconnect | *Length of the interconnect* | *3 dB frequency [this paper]* | *3 dB frequency[17]* |
|---|---|---|---|
| ***SWCNT*** | 1 | 6.52e14 | 3.39e14 |
| | 10 | 6.52e13 | 3.44e13 |
| | 100 | 6.52e12 | 3.4e12 |
| | 1000 | 6.52e11 | 3.37e11 |
| | 10000 | 6.52e10 | 2.29e10 |
| ***SWCNT bundle*** | 1 | 1.06e16 | 3.23e14 |
| | 10 | 9.82e14 | 3.23e13 |
| | 100 | 9.82e13 | 3.23e12 |
| | 1000 | 9.82e12 | 3.22e11 |
| | 10000 | 1.03e12 | 3.66e11 |

## V. CONCLUSION

A closed form expressions for calculation of Propagation constant as a function of physical parameters such as length, diameter and frequency is proposed and validated in this work. Also the cut-off frequencies of different interconnect lengths is observed and compared with published results.

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**Kollarama Subramanyam**: He received his M.Tech (Nanotechnology) degree with specialization in Nanoelectronics and Nanophotonics from VIT University, Vellore in 2012 and B.Tech degree (Electronics and Communications) from VIT University, Vellore in 2010. He is currently working on CNT based interconnects. His areas of interest include Carbon nanotube interconnects, graphene nano ribbon based transistors and heterostructure transistors. He authored / co-authored 2 technical papers in international conferences.

**Nisha Kuruvilla**: She received her M. Tech (Electrical Engineering) degree with specialization in Microelectronics and VLSI from IIT, Kanpur and B.E (Electronics) from Dr. Babasaheb Ambedkar Marathwada University, Aurangabad. She is currently serving College of Engineering, Chengannur as Head of the Department (Electronics) and also peruse towards her doctoral degree under the guidance of Prof. (Dr.) J .P. Raina at Center of Nanotechnology, VIT Vellore. She is a recipient of 'Outstanding IEEE Student Branch Counselor International award' for the academic year 2005-2006.Her areas of interest Include Carbon nanotube Interconnects, interconnect optimization, low power VLSI and optical wireless. She has authored /co-authored around 15 technical papers in international journals/ conferences.

**Prof. (Dr.) J. P. RAINA:** Born 1937 in SGR (KMR), obtained M.Sc.(Physics) degree from M.S. University, Baroda, and subsequently obtained D.Sc. (Engg.) degree from UCL Belgium . Worked in CEERI, Pilani (Raj) in various capacities from 1961 to 1979. Joined I.I.T. Madras in 1979 as Professor and Head, T.V.Eng., Image Signal Processing, Photonics and Fiber Optics Communication, in the Elect. Eng. Dept; Retired from I.I.T. Madras in 1998 and from 1999 till 2004 was Managing Director of A.T.I.I.T. Chennai. Joined V.I.T. University as Dean School of Electrical Sciences in 2005. In 2008 took charge as Director of Center for Nanotechnology Research, were presently continuing. During the past 52 yrs of Teaching and Research experience handled large number of sponsored research projects from DRDO, ARDB, DOE and DST of Govt; of India. Handled number of International assignments, organized many International and National conferences, guided nearly 14 Ph.D. scholars and presently guiding 6 Ph.D. scholars at V.I.T. University. Published nearly 90 papers in International and National journals, Edited number of books and took 7 national patents. Received prestigious National Award from President of India's Gold Shield for Invention Promotion. Present interest in Nanotechnology, Nanophotonics /Plasmonics, Low Power Electronics and Image Signal Processing