Performance Analysis of Heterogeneous Cellular Networks with Multiple Connectivity

Sungkyung Kim, Jee-Hyeon Na, Dong-Seung Kwon

***Abstract*—**Future mobile networks following 5th generation will be characterized by one thousand times higher gains in capacity; connections for at least one hundred billion devices; user experience capable of extremely low latency and response times. To be close to the capacity requirements and higher reliability, advanced technologies have been studied, such as multiple connectivity, small cell enhancement, heterogeneous networking, and advanced interference and mobility management. This paper is focused on the multiple connectivity in heterogeneous cellular networks. We investigate the performance of coverage and user throughput in several deployment scenarios. Using the stochastic geometry approach, the SINR distributions and the coverage probabilities are derived in case of dual connection. Also, to compare the user throughput enhancement among the deployment scenarios, we calculate the spectral efficiency and discuss our results.

***Keywords*—**Heterogeneous networks, multiple connectivity, small cell enhancement, stochastic geometry.

I. Introduction

By reason of better link quality, higher spatial reuse, and greater utilization of spectrum resources, hierarchical networks consisting of macro cell and small cell tiers represent an inexpensive alternative for improving the capacity of cellular systems [1]-[3], [12]. Small cells provide improved cellular coverage, capacity and applications for homes and enterprises as well as metropolitan and rural public spaces. With small cells, users can obtain better indoor reception and power savings due to the low transmit powers. In addition, they can offload much data traffic from the macro cell network via backhaul.

The multiple connectivity is defined as operation where a given mobile user consumes radio resources provided by at least two different network points. Recently, for the purpose of greater mobility and throughput, the scheme of multiple connectivity has been proposed in macro and small cell networks [2], [3]. When the small cells are deployed within macro cell coverage, small cells can operate on the same carrier frequency or another carrier frequency. However, one of the major impediments to the success of two-tier heterogeneous cellular networks is the presence of cross-tier interference on the same frequency operation [10]. On the other hand, as deploying on another carrier frequency, the cost of additional radio resources is required.

The performance of wireless communication networks have been evaluated by numerical analysis or simulations [4]-[8], [13]-[15]. Traditionally, cellular networks have been modeled by placing the base stations on a grid, with mobile users either randomly scattered or placed deterministically. However, these regular models tend to overestimate the capacity of cellular networks owing to the perfect geometry of base stations and the neglect of weak interference from outer tier base stations. For this reason, we consider the stochastic geometry approach for performance analysis. This is to model the BS locations are random and drawn from a spatial stochastic process, such as the Poison Point Process (PPP). Stochastic geometry approaches have been advocated in various studies on unplanned networks such as small cells or ad hoc networks; and they are now fairly well-accepted [5]-[7], [11].

In this paper, we investigate the performance of coverage and average throughput in heterogeneous cellular networks constructing especially dual connection. The paper is structured as follows: Section II provides a system model for stochastic geometry analysis and deployment scenarios. Section III derives the coverage probability and the normalized user throughput. Numerical results are presented and discussed in Section IV. And finally, Section V contains conclusions.

II. System Model for Stochastic Analysis

In this paper, we assume that heterogeneous cellular networks consist of macro and small-cell base stations arranged according to a Poisson Point Process (PPP) Φ in the Euclidean plane. The Poisson Point Process Φ is the sum of two independent homogenous Poisson Point Processes Φ_m and Φ_s of intensities λ_m and λ_s respectively. Subscript m and s denote macro cell and small cell respectively. The aggregated random process Φ is also a Poisson Point Process and is given as:

$$\Phi=\Phi_m+\Phi_s \tag{1}$$

We consider that an independent collection of mobile users, located according to some independent stationary point process. For multiple connectivity, a mobile user is associated with the closest macro BS and the closest small-cell BS concurrently. In other words; the users in the Euclidean plane are associated with two BSs, resulting in coverage areas that comprise two-tier Voronoi tessellation on the plane, as shown in Fig. 1. An example for macro and small cells with multiple connections is presented in Fig. 2.

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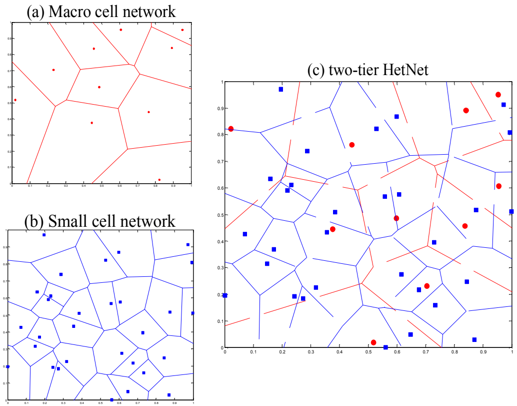

Fig. 1 Voronoi tessellation of two-tier Heterogeneous Networks

When constructing dual connection, we can consider four deployment scenarios in view of inter-tier interference and resource scheduling as:

- Scenario 1: Macro and small cells on the same carrier frequency (intra-frequency) are connected and inter-BSs radio resource aggregation is supported (simultaneous transmission).
- Scenario 2: Macro and small cells on different carrier frequencies (inter-frequency) are connected and inter-BSs radio resource aggregation is supported (simultaneous transmission).
- Scenario 3: Macro and small cells on the same carrier frequency (intra-frequency) are connected and single BS radio resource allocation is supported (alternative transmission).
- Scenario 4: Macro and small cells on different carrier frequencies (inter-frequency) are connected and single BS radio resource allocation is supported (alternative transmission).

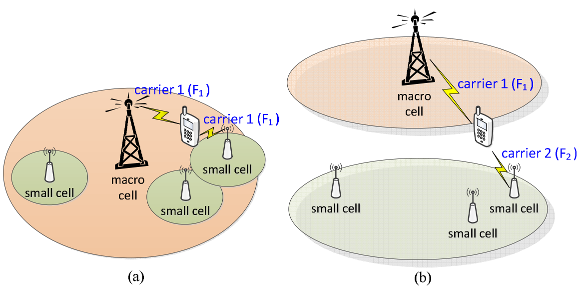

Fig. 2 Deployment scenarios in two-tier macro/small heterogeneous cellular network

III. Performance Analysis

We consider a typical user at the origin associated with the macro and small cells. Denote r as the distance between the user and its serving BS. Since macro and small cells are deployed as each independent PPP, the probability density function (PDF) of r can be derived using the simple fact that the null probability of a 2-D Poisson process. Namely, the cumulative density function (CDF) of r is $F_R(r)=1-$ Pr[No BS closer than r] and the PDF is given as follows:

$$\begin{aligned} f_m(r)&=2\pi\lambda_m r\exp(-2\pi\lambda_m r^2) \\ f_s(r)&=2\pi\lambda_s r\exp(-2\pi\lambda_s r^2) \end{aligned} \tag{2}$$

A user is in coverage when its SINR from its nearest BS is larger than some threshold x and it is dropped from the network for SINR below x. Hence, the coverage probability is defined as $\Pr[SINR>x]$. The SINR of the mobile user at a random distance Y_0 from its associated base station can be expressed as

$$SINR=\frac{P_0 h_0 |Y_0|^{-\alpha}}{\sigma^2+\sum_{Y_i\in\Phi/Y_0} P_i h_i |Y_i|^{-\alpha}}, \tag{3}$$

where P is transmit power of BS and h is the exponentially distributed channel power with unit mean, i.e. interference experiences Rayleigh fading and shadowing is neglected. Subscript 0 and i denote a serving BS and an i_{th} interfering BS respectively. And, $|Y_i|$ is the distance from BS $Y_i\in\Phi/Y_0$ (except the serving BS) to the origin. Also, the interference power is considered as an exponential distribution in various work including [5], [7]. From this assumption, we can get a possibility of significant simplification in a derivation of SINR distribution.

A. Downlink SINR distribution

When frequency resources are isolated in heterogeneous networks, tier-to-tier interference is ignored. In cases of scenario 2 and 4, the SINR distribution per carrier frequency is same as the homogeneous cellular networks. Hence, the coverage probability of macro and small cells is given as $\Pr[SINR_h\geq x]=1/\{1+H(x,\alpha)\}$, where $H(x,\alpha)=x^{2/\alpha}\int_{x^{-2/\alpha}}^{\infty}\frac{1}{1+u^{\alpha/2}}du$ [5].

Now, we derive the performance of heterogeneous networks operating on the same carrier frequency, i.e., the deployment scenario 1 and 3. In accordance with (3), the SINRs of macro and small cell networks are given as

$$SINR_m=\frac{P_m h_0|Y_0|^{-\alpha}}{\sigma^2+\sum_{Y_i\in\Phi_m/Y_0}P_m h_i|Y_i|^{-\alpha}+\sum_{Y_i\in\Phi_s}P_s h_i|Y_i|^{-\alpha}}, \text{ and}$$

$$SINR_s=\frac{P_s h_0|Y_0|^{-\alpha}}{\sigma^2+\sum_{Y_i\in\Phi_s/Y_0}P_s h_i|Y_i|^{-\alpha}+\sum_{Y_i\in\Phi_m}P_m h_i|Y_i|^{-\alpha}} \text{ respectively.}$$

In the macro cell networks, the probability of coverage of a typical randomly located mobile user is

$$\begin{aligned}
&\Pr[SINR_m \ge x]\\
&= E_{R_m}[\Pr\{SINR_m \ge x | Y_0 = r\}]\\
&= E_{R_m}[\Pr\{\frac{P_m h_0 r^{-\alpha}}{\sigma^2 + \sum_{Y_i\in\Phi_m/Y_0} P_m h_i |Y_i|^{-\alpha} + \sum_{Y_i\in\Phi_s} P_s h_i |Y_i|^{-\alpha}} > x|r\}]\\
&= E_{R_m}[\Pr\{h_0 > x r^{\alpha}(\frac{\sigma^2}{P_m} + \sum_{Y_i\in\Phi_m/Y_0} h_i|Y_i|^{-\alpha} + \sum_{Y_i\in\Phi_s}\frac{P_s}{P_m} h_i |Y_i|^{-\alpha})|r\}]\\
&= E_{R_m}[\Pr\{h_0 > x r^{\alpha}(\frac{\sigma^2}{P_m} + I_{m,Y_0} + \frac{P_s}{P_m} I_s|r)\}],
\end{aligned} \tag{4}$$

where $I_{m,Y_0} = \sum_{Y_i\in\Phi_m|Y_0} h_i|Y_i|^{-\alpha}$ and $I_s = \sum_{Y_i\in\Phi_s} h_i|Y_i|^{-\alpha}$, which I_m and I_s are mutually independent. Hence, the conditional probability within the expectation $E_{R_m}[\bullet]$ is derived as:

$$\begin{aligned}
&\Pr\{h_0 > x r^{\alpha}(\frac{\sigma^2}{P_m} + I_{m,Y_0} + \frac{P_s}{P_m} I_s|r)\}\\
&= E_{I_m, I_s|r}[\Pr\{h > x r^{\alpha}(\frac{\sigma^2}{P_m} + I_{m,Y_0} + \frac{P_s}{P_m} I_s)|I_{m,Y_0}, I_s, r\}]\\
&= \exp(-x r^{\alpha}\frac{\sigma^2}{P_m}) E_{I_m|r}\{\exp(-x r^{\alpha} I_{m,Y_0})\} E_{I_s|r}\{\exp(-x r^{\alpha}\frac{P_s}{P_m} I_s)\}\\
&= \exp(-x r^{\alpha}\frac{\sigma^2}{P_m})\exp\{-\lambda_m \pi r^2 H(x,\alpha)\}\exp\{-\lambda_s \pi r^2 C^{2/\alpha} K(x,\alpha)\},
\end{aligned} \tag{5}$$

where $K(x,\alpha) = x^{2/\alpha}\int_0^\infty \frac{1}{1+u^{\alpha/2}} du$ and $C = P_s/P_m$. Appendix provides more detailed derivations. Using the substitution $r^2 = z$, from (5) we obtain

$$\begin{aligned}
&E_{R_m}[\Pr\{h_0 > x r^{\alpha}(\frac{\sigma^2}{P_m} + I_m + \frac{P_s}{P_m} I_s|r)\}]\\
&= \pi\lambda_m \int_0^\infty \exp\{\frac{\sigma^2}{P_m} x z^{\alpha/2} - \pi\lambda_m z\{1 + H(x,\alpha) + (\lambda_s/\lambda_m) C^{2/\alpha} K(x,\alpha)\} dz
\end{aligned}.$$

Since the BS density is typically quite high in heterogeneous cellular networks, the interference power easily dominates thermal noise. Thermal noise can often therefore be neglected, i.e., $\sigma^2 = 0$, as the coverage probability can be further simplified to

$$\Pr[SINR_m \ge x] = \frac{1}{1 + H(x,\alpha) + (\lambda_s/\lambda_m) C^{2/\alpha} K(x,\alpha)}. \tag{6}$$

Furthermore, in case of the path loss exponent $\alpha = 4$, the expression of (6) is reduced to the following closed form formula:

$$\Pr[SINR_m \ge x] = \frac{1}{1 + [\frac{\pi}{2} - \arctan(\frac{1}{\sqrt{x}}) + \frac{\pi}{2}\frac{\lambda_s}{\lambda_m}\sqrt{\frac{P_s}{P_m}}]\sqrt{x}} \tag{7}$$

Using the same approach as in (4)-(6), the coverage probability of small cell network is given as

$$\Pr[SINR_s \ge x] = \frac{1}{1 + H(x,\alpha) + (\lambda_m/\lambda_s) C^{-2/\alpha} K(x,\alpha)}. \tag{8}$$

B. Coverage and Best throughput Scheduling

To maximize the user throughput, we apply the best C/I scheduling scheme in the deployment scenarios 3 and 4 as:

$$\max_{k\in\text{serving BSs}} SINR_k.$$

Denote $SINR_b$ as the signal-to-interference ratio of the best C/I scheduling, i.e., the best alternative transmission between macro and small cells with dual connection. By order statistics [9], the CDF of $SINR_b$ is given as

$$\begin{aligned}
&\Pr[SINR_b \le x]\\
&= \Pr[SINR_m \le x]\Pr[SINR_s \le x]\\
&= F_{SINR_m}(x) F_{SINR_s}(x)\\
&= \{1 - \Pr(SINR_m \ge x)\}\{1 - \Pr(SINR_s \ge x)\}.
\end{aligned} \tag{9}$$

In case of operating on different carrier frequency, it can be written by $\Pr[SINR_b \le x] = [F_{SINR_h}(x)]^2$.

The coverage of heterogeneous networks constituting the multiple connectivity is defined as a region having a greater SINR value from the macro cell and small cell. Hence, the coverage probability is derived by order statistics such as (9).

B. Average User Throughput

A typical user where adaptive modulation/coding is used so each user can set their rate such that they achieve Shannon bound for their instantaneous SINR. In this paper, to compare the user-throughput enhancement, average spectral efficiency is calculated as a normalized capacity. Therefore, it is written by

$$\eta = E[\log_2(1 + SINR)]. \tag{10}$$

From (4), the average user throughput served by macro cell is given as

$$\begin{aligned}
\eta_m &= E[\log_2(1 + SINR_m)]\\
&= \int_{r>0} 2\pi\lambda_m r e^{-\pi\lambda_m r^2} E\{\log_2(1 + SINR_m)\} dr\\
&= \int_{r>0} 2\pi\lambda_m r e^{-\pi\lambda_m r^2}\int_{t>0}\Pr\{\log_2(1 + \frac{h r^{-\alpha}}{\sigma^2/P_m + I_{m,Y_0} + C I_s}) > t|r\} dt dr\\
&= \int_{r>0} 2\pi\lambda_m r e^{-\pi\lambda_m r^2}\int_{t>0}\Pr\{h > r^{\alpha}(\sigma^2/P_m + I_{m,Y_0} + C I_s)(2^t - 1)\} dt dr\\
&= \int_{r>0} 2\pi\lambda_m r e^{-\pi\lambda_m r^2}\int_{t>0} E_{I_m, I_s|r}[\exp\{-r^{\alpha}(\frac{\sigma^2}{P_m} + I_{m,Y_0} + C I_s)(2^t - 1)\}] dt dr\\
&= \int_{r>0} 2\lambda_m r e^{-\pi\lambda_m r^2}\int_{t>0}\exp[-\frac{\sigma^2}{P_m} r^{\alpha}(2^t - 1) - \lambda_m \pi r^2 H(2^t - 1,\alpha)\\
&\qquad - \lambda_s \pi r^2 C^{2/\alpha} K(2^t - 1,\alpha)] dt dr,
\end{aligned} \tag{11}$$

In the interference limited system, (11) is expressed to a single numerical integration as follows:

$$\begin{aligned}
&\int_{t>0}\int_{r>0} 2\lambda_m r \exp\{-\lambda_m \pi r^2\{1 + H(2^t - 1,\alpha) + \frac{\lambda_s}{\lambda_m} C^{2/\alpha} K(2^t - 1,\alpha)\} dr dt\\
&= \int_{t>0}\left(\frac{1}{1 + H(2^t - 1,\alpha) + (\lambda_s/\lambda_m) C^{2/\alpha} K(2^t - 1,\alpha)}\right) dt.
\end{aligned} \tag{12}$$

Similarly, the average user throughput served by small cell is written by

$$\eta_s = \int_{t>0}\left(\frac{1}{1 + H(2^t - 1,\alpha) + (\lambda_m/\lambda_s) C^{-2/\alpha} K(2^t - 1,\alpha)}\right) dt. \tag{13}$$

In case of simultaneous transmission on the same carrier frequency, namely the deployment scenario 1, the aggregated average user throughput is the sum of (12) and (13). For the alternative transmission applying the best C/I scheduling, the average user throughput is given as $\eta_b = E[\log_2(1+SINR_b)]$, where the CDF of the $SINR_b$ is obtained from (6), (8), and (9).

IV. Numerical Results

In this section, we investigate the performance of coverage and normalized user throughput as varying network density ratio and transmit power ratio of macro cell to small cell. In our numerical results, it is assumed that the noise is negligible, namely interference limited system. And, we use a path loss exponent $\alpha=4$ in Figs. 3-5.

Fig. 3 shows the coverage probability as the deployment scenario 1 or 3 is applied. Red lines in Figs. 3 (a) and (b) present enhanced coverage by multiple connectivity in case of operating on the same carrier frequency. The coverage area has been extended by multiple connectivity. Moreover, as increasing the network density or the transmit power; the coverage of corresponding network is increased. When λ_s is higher than λ_m, the coverage of small cells is superior to that of macro cells as shown in Fig. 3 (a) while the coverage of macro cells is dominant in Fig. 3 (b). From these results, we can get some guidelines how to install the small cells while maintaining the performance requirements of the macro cell. If there are some requirements that the coverage of macro cell is 50% or more at -5dB and $P_s/P_m=0.01$ (Fig. 3 (b)), the ratio of small cell density to macro cell density do not become greater than 5 times.

As varying the transmit power ratio or the network density ratio, the coverage by multiple connectivity on the same carrier frequency is shown as Fig. 4. When the transmit power ratio is smaller than 0.001 or the network density ratio is greater than 100, the coverage probabilities are almost the same. Furthermore, it approach to the coverage of homogeneous networks, i.e., $\Pr[SINR_h \geq x] = 1/\{1+H(x,\alpha)\}$ as shown in Fig. 5.

Fig. 5 compares the coverages of the deployment scenarios, which they operate on the same carrier frequency (scenario 1/3) or the different carrier frequencies (scenario 2/4). In Fig. 5, the black line presents the coverage of homogeneous networks. In this paper, the homogenous cellular network means that only the macro cells are deployed in a certain area and the mobile users are associated with single base station. The performance of the coverage of the homogenous networks provides an upper bound to the scenario 1/3. Also, we observe that the coverage of scenario 2/4 is much larger than that of scenario 1/3. This is because that the dual carrier frequency enables mobile users to use the better signal compared with single carrier frequency. Namely, although the frequency resource is insufficient and costly, the multiple connectivity operating on the different frequency resources provide a significant improvement in the communication region.

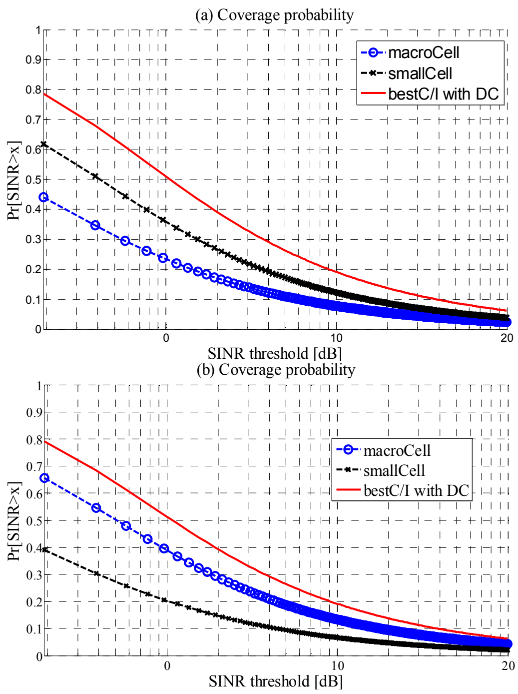

Fig. 3 Probability of coverage in the deployment scenario 1 or 3: (a) $(\lambda_s/\lambda_m=5, P_s/P_m=0.1)$; (b) $(\lambda_s/\lambda_m=5, P_s/P_m=0.01)$

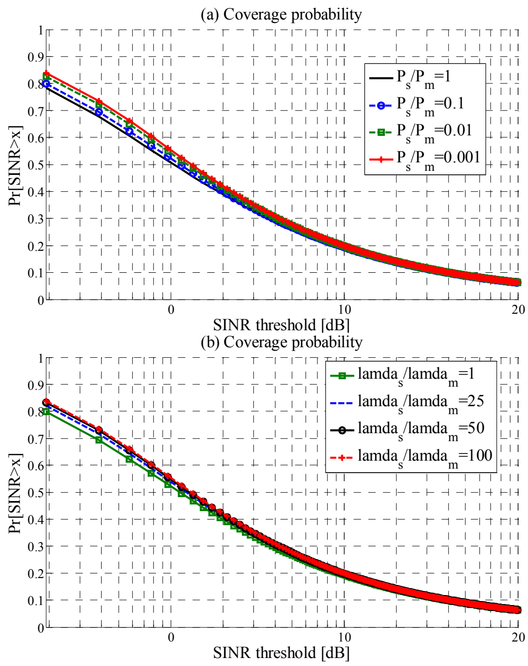

Fig. 4 Probability of coverage varying the transmit power ratio (a) and the network density ratio (b)

Fig. 6 shows the normalized user throughput as the path loss exponent varies between 3 and 5. Using the numerical integration, we can obtain the approximated value of η_m, η_s, and η_b that involve a non-closed-form integral. In this figure, we can see that the user capacity of networks increases as the path loss exponent becomes higher. This is due to the fact that the higher path loss will filter co-channel interference among the cells. Furthermore, the normalized user throughput of the scenario 1 (i.e., simultaneous transmission & same carrier frequency) is the best of the proposed deployment scenarios. At $\alpha=5$, the capacity gain of scenario 1 is about 40 percent compared to scenario 4. Also, there is not much difference between scenario 2 and scenario 3 in view of spectral efficiency.

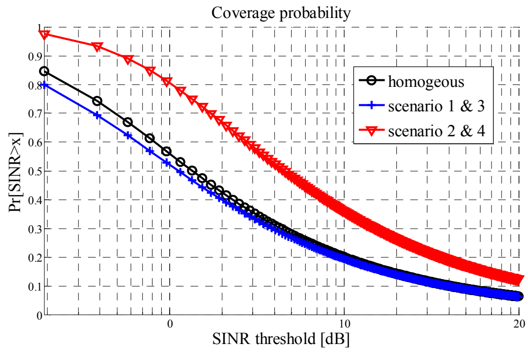

Fig. 5 Probability of coverage for the comparison among the deployment scenarios $(\lambda_s/\lambda_m=10, P_s/P_m=0.1)$

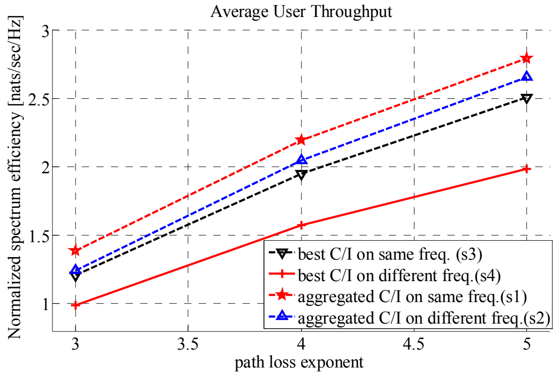

Fig. 6 Average user throughput $(\lambda_s/\lambda_m=10, P_s/P_m=0.1)$

In other words, when constituting multiple connectivity with single carrier frequency, we can get the benefit of user throughput. Note that the coordination for cross-tier interference is required in case of the co-channel deployment of macro cells and small cells.

V. Conclusions

This paper has presented a stochastic model for heterogeneous macro and small cell networks with multiple connectivity. Using stochastic geometry approach, we derived the SINR distributions of the downlink and calculated the coverage of networks and normalized user capacity according to deployment scenarios. From our results, multiple connectivity enhanced the overall network coverage and capacity. In terms of macroscopic, the stochastic analysis such as our approach can provide the main performances and some guidelines for deployment of complicated and unplanned networks.

Appendix

$$
\begin{aligned}
&E_{I_s|r}[\exp(-xr^\alpha C I_s)]=E_{I_s|r}[\exp(-xr^\alpha C\sum_{Y_i\in\Phi_s}h_iY_i^{-\alpha})]\\
&=E_{I_s|r}[\exp(-sC\sum_{Y_i\in\Phi_s}h_iY_i^{-\alpha})],\ \ \lrcorner(xr^\alpha=s)\\
&=E_{\Phi_s|r}[E_h\{\exp(-sC\sum_{Y_i\in\Phi_s}h_iY_i^{-\alpha})\}]\\
&=E_{\Phi_s|r}[\prod_{Y_i\in\Phi_s}E_h\{\exp(-sCh_iY_i^{-\alpha})\}]=E_{\Phi_s|r}\{\prod_{Y_i\in\Phi_s}\frac{1}{1+sCY_i^{-\alpha}}\}\\
&=\exp\{-\lambda_s\int_{\Re}(1-\frac{1}{1+sCy^{-\alpha}})dy\}=\exp\{-2\pi\lambda_s\int_0^\infty(1-\frac{1}{1+sCt^{-\alpha}})tdt\}\\
&=\exp\{-2\pi\lambda_s\int_0^\infty(\frac{x}{x+C^{-1}(t/r)^\alpha})tdt\}\qquad \lrcorner(s=xr^\alpha)\\
&=\exp\{-\pi\lambda_s r^2x^{2/\alpha}C^{2/\alpha}\int_0^\infty\frac{1}{1+u^{\alpha/2}}du\}\quad \lrcorner(u=\left(\frac{t}{rC^{1/\alpha}x^{1/\alpha}}\right)^2)\\
&=\exp\{-\pi\lambda_s r^2C^{2/\alpha}K(x,\alpha)\}
\end{aligned}
$$

$$
\begin{aligned}
&E_{I_m|r}[\exp(-xr^\alpha I_{m,Y_0})]=E_{I_m|r}[\exp(-xr^\alpha\sum_{Y_i\in\Phi|Y_0}h_iY_i^{-\alpha})]\\
&=E_{\Phi|r}[\prod_{Y_i\in\Phi|Y_0}E_h\{\exp(-sh_iY_i^{-\alpha})\}]=E_{\Phi|r}\{\prod_{Y_i\in\Phi|Y_0}\frac{1}{1+sY_i^{-\alpha}}\}\\
&=\exp\{-\lambda_m\int_{\Re/r}(1-\frac{1}{1+sy^{-\alpha}})dy\}=\exp\{-2\pi\lambda_m\int_r^\infty(1-\frac{1}{1+st^{-\alpha}})tdt\}\\
&=\exp\{-2\pi\lambda_m\int_r^\infty(\frac{x}{x+(t/r)^\alpha})tdt\}\qquad \lrcorner(s=xr^\alpha)\\
&=\exp\{-\pi\lambda_m r^2x^{2/\alpha}\int_{x^{-2/\alpha}}^\infty\frac{1}{1+u^{\alpha/2}}du\}\quad \lrcorner(u=\left(\frac{t}{rx^{1/\alpha}}\right)^2)\\
&=\exp\{-\pi\lambda_m r^2H(x,\alpha)\}
\end{aligned}
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