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# Partially Knowing of Least Support Orthogonal Matching Pursuit (PKLS-OMP) for Recovering Signal

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***Abstract*—**Given a large sparse signal, great wishes are to reconstruct the signal precisely and accurately from lease number of measurements as possible as it could. Although this seems possible by theory, the difficulty is in built an algorithm to perform the accuracy and efficiency of reconstructing. This paper proposes a new proved method to reconstruct sparse signal depend on using new method called Least Support Matching Pursuit (LS-OMP) merge it with the theory of Partial Knowing Support (PSK) given new method called Partially Knowing of Least Support Orthogonal Matching Pursuit (PKLS-OMP).

The new methods depend on the greedy algorithm to compute the support which depends on the number of iterations. So to make it faster, the PKLS-OMP adds the idea of partial knowing support of its algorithm. It shows the efficiency, simplicity, and accuracy to get back the original signal if the sampling matrix satisfies the Restricted Isometry Property (RIP).

Simulation results also show that it outperforms many algorithms especially for compressible signals.

***Keywords*—**Compressed sensing, Lest Support Orthogonal Matching Pursuit, Partial Knowing Support, Restricted isometry property, signal reconstruction.

## I. Introduction

COMPRESSED SENSING (CS) stands for a linear underdetermined problem, where the underlying sampled signal is sparse. The challenge in CS is to reconstruct this sparse signal from few measurements as possible as it could.

The standard CS theorem is based on a sparse signal model and uses an underdetermined system of linear equations [1].

Linear Programming techniques are good for designing computationally CS decoders, but It show kind of complexity for many applications. So, the need for faster decoding algorithms is necessary, even if a procedure raises the measurement number. Several low complexity reconstruction methods are used today as an alternative method for linear programming recovery, which contains a collection of methods and algorithms used for testing [2].

Several algorithms exist for performing the signal reconstruction problem. Some of these include: Convex Optimization: like {Basis Pursuit (BP) and Basis Pursuit De-Noising (BPDN). Iterative Greedy Algorithms like Matching Pursuit (MP) Orthogonal Matching Pursuit (OMP), the Regularized OMP (ROMP), and compressive sampling matching pursuit CoSaMP [3].

The simple idea behind use greedy methods is to find the support for unknown signal sequentially. The support set is containing of indices that are non-zero elements of a sparse vector. To evaluate the support set, iterative greedy search methods use some linear algebraic tools such as the matched filter and least square solution [2].

Greedy algorithms used at each iteration, one or several coordinates of input signal vector x which it elected depend on the maximum correlation value between the columns of$\Phi$ and the measurement vector. The candidates will be added to the currently estimate support set of x. The pursuit algorithm repeats this procedure several times until all the coordinates arrange in the evaluated support set [2], [4].

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## II. Background

### *A. OMP Algorithm*

Notations: let $x$ be a sparse signal, the arbitrary vector $x = \{x_1, x_2 \ldots\ldots, x_N\}^t$, let the support set $T \subset \{1,2 \ldots., N\}$ denote the set of nonzero component indices of x (i.e $up(x) = \{i|x_i \neq 0\}$ ), $A_I \in \mathbb{R}^{Mx|I|}$ consists of the columns of A with indices $i \in I$, $A^*$ denote the transpose of A, and $A^\dagger$ denote the pseudo-inverse $\{(A^*A)^{-1}A^*\}$.

Let us declare the standard CS problem, which achieve a signal $x \in \mathbb{R}^N$ have a K sparse input, via the linear measurements

$$y = \Phi x \tag{1}$$

where$\varphi \in \mathbb{R}^{mxN}$ represents a random measurement (sensing) matrix, and $y \in \mathbb{R}^M$ represent the compressed measurement signal. A K sparse signal vector consists of most K nonzero indices. With the setup of $K < M < N$, the task is to reconstruct x from y as $\hat{x}$ .The aim is to reconstruct sparse signal from a small number of measurements in addition to achieve good reconstruction qualification [4], [5].

Wei Dai and Parichat notes that the compressed measurement signal y is the linear combination of most K atoms (atom means a column of).

One condition for sparse signal recovery is to use the Mutual Incoherence Property (MIP) [6]. The MIP requires the correlations among the column vectors $\Phi$ to be small.

The coherence parameter $\mu$ of sensing matrix is defined as,

$$\mu = \max_{i \neq j} \langle \varphi_i, \varphi_j \rangle \tag{2}$$

where $\varphi_i, \varphi_j$ Are two columns of $\Phi$ with unit norm.

For the noiseless case when $\Phi$ is a series of two square orthogonal matrices, that

$$K < \frac{1}{2}\left(\frac{1}{\mu+1}\right) \tag{3}$$

is guarantee the exact recovery of $\hat{x}$ when $\hat{x}$ has at most nonzero entries (such a signal is called k-sparse) [7]. Based on OMP algorithm in [8], [9], LS-OMP also selects one atom in each iteration., but the operation of choosing an index in the current iteration is executed according to its future effect on minimizing the residual norm.

*B. OMP-PKS*

It's derived from classical Orthogonal Matching Pursuit (OMP). In sparse signals some component is more important for others and should be kept as nonzero value. If it compared with OMP, PKS can recovery even when used low measurement rate $(M/N)$ .

While sparse signal can be produced by using wavelet transformer, all the coefficient of LL sub band is selected to be nonzero components without interring it to be tested for correlation [10].

The algorithm for OMP-PKS when using wavelet transform to make the signal sparse, can be found in [10], [11].

*C. Preliminaries*

***Lemma 1*** [7]: (Consequences of RIP) $I \subset \Omega, if\ \delta_{|I|} < 1$ then for any $u \in R^{|I|}$,

$$(1-\delta_{|I|})\|u\|_2 \leq \left\|\Phi_I' \Phi_I u\right\|_2 \leq (1+\delta_{|I|})\|u\|_2 \tag{4}$$
$$\frac{1}{(1+\delta_{|I|})}\|u\|_2 \leq \left\|\left(\Phi_I' \Phi_I\right)^{-1} u\right\|_2 \leq \frac{1}{(1-\delta_{|I|})}\|u\|_2$$

***Lemma 2*** [7]: for disjoint sets $I_1, I_2 \subset \Omega, if\ \delta_{|I_1|+|I_2|} < 1$ then,

$$\left\|\Phi_{I_1}' \Phi v\right\| = \left\|\left(\Phi_{I_1}' \Phi_{I_2} v_{I_2}\right)\right\| \leq \delta_{|I_1|+|I_2|}\|v\| \tag{5}$$

***Lemma 3*** [12] (Consequence of restricted orthogonality constant): For two disjoint sets $I_1, I_2 \subset \Omega$ , let $\delta_{|I_1|,|I_2|}$ be the $|I_1|,|I_2|$-restricted orthogonality constant of $\Phi$.If $|I_1| + |I_2| \leq n$, $\delta_{|I_1|,|I_2|'}$, is the smallest number that satisfies

$$\left\|\Phi_{I_1}' \Phi_{I_2} x_{I_2}\right\| \leq \delta_{|I_1|,|I_2|}\|x\|. \tag{6}$$

***Lemma 4*** [12]: If $\Phi$ satisfies the RIP of both orders $K1$ and $K2$, then $\delta_{K_1} \leq \delta_{K_2}$ for any $K_1 \leq K_2$. This property is referred as the monotonicity of the isometry constant.

***Lemma 5*** [12]: for two disjoint sets $I_1, I_2 \subset \Omega$ with $|I_1| + |I_2| \leq n$, $\theta_{|I_1|,|I_2|} \leq \delta_{|I_1|,|I_2|}$

***Definition 1*** [4]: Let $y \in R^m$ and $\Phi_I \in R^{m\times|I|}$, let $\Phi_I^*\Phi_I$ be invertible matrix , the projection of y onto span $(\Phi_I)$ Can be defined as

$$y_p = \text{proj}(y, \Phi_I) = \Phi_I {\Phi_I}^{\dagger} \tag{7}$$
$$\Phi_I^{\dagger} = (\Phi_I^*\Phi_I)^{-1}\Phi_I^*$$

where$\Phi_I^{\dagger}$ is the represent the Pseudo inverse of matrix $\Phi_I$ and * denote the transpose of $\Phi_I$. Residue vector of the projection can be found as:

$$y_r = \text{resid}(y, \Phi_I) = y - y_p. \tag{8}$$

***Lemma 6*** [1]: Residue Orthogonality: if a vector $y \in R^m$ and $\Phi_I \in R^{m\times K}$ represent sampling matrix which has full column rank, if $y_r = \text{resid}(y, \Phi_I)$, then

$$\Phi_I^* y_r = 0$$

Approximation of Projection Residue: consider $\Phi_I \in R^{m\times N}$ , if $I, J \subset \{1 \ldots\ldots N\}$ are two disjoint set (i.e. $I \cap J = \emptyset$) and let $\delta_{|I|+|J|} < 1$ suppose $y \in span\,(\Phi_I)$, $y_p = proj(y, \Phi_J)$, $y_r = resid(y, \Phi_J)$, then

$$\left\|y_p\right\|_2 \leq \frac{\delta_{|I|+|J|}}{1-\delta_{\max\,(|I|,|J|)}}\|y\|_2. \tag{9}$$

## III. LS-OMP

In LS-OMP, the elect of an atom for the current iteration is done by testing its influence on the future iterations. An element is chosen at the beginning of the calculation by finding a set of maximum correlation between $\phi$ and whole signal matrix. This way is faster since it requires less computational complexity.

According to the new stop condition, and compared with OMP, LS-OMP achieves better assessment for underlying support set through iterations without need to test each potential independently.

***Theorem 1***. For any $K$-sparse vector $x$, where $x \in \mathbb{R}^N$ and measurement matrix $\Phi \in \mathbb{R}^{mxN}$, and $y \in \mathbb{R}^M$ represent the measurement vector matrix, the LS-OMP algorithm perfectly recovers $x$from $y = \Phi x$ (depending on Fig. 1),if

$$\left\|y - y_r^{\ell}\right\|_2 \leq \frac{\delta_{2L}}{1-2\delta_{2L}}\left\|y_r^{\ell-1}\right\|_2 \tag{10}$$

Assume $0.4 \leq \delta_{2L} \leq 0.497$

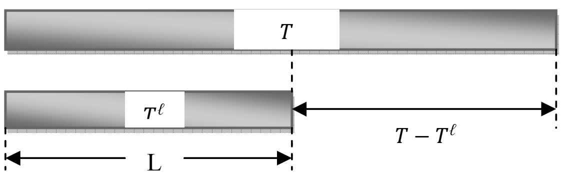



Fig. 1 Illustration of support sets for our theorem 1

## IV. PKLS-OMP

The prior signal information is incorporated in the recovery process. A Discrete Wavelet Transform (DWT) is used to sparsify the signal and all the components in low sub band are selected as nonzero components. The PKLS-OMP algorithm for the data represented in the wavelet domain is shown in Algorithm 1:

***Theorem 2:*** If $x$ is sparse signal and $x \in \mathbb{R}^N$, $y$ is measurement vector $y = \Phi x$, $\Phi$ is sampling matrix satisfies RIP condition, then $x$ can be recovered if

$$\|y_r\|_2 \geq \frac{\delta_{2l}}{(1-\delta_{2l})}\|y_0\|, \tag{11}$$

for $0.005 \le \delta_{2L} \le 0.025$.

Fig. 2 shows the necessary support set need it for driving (11) of theorem 2.

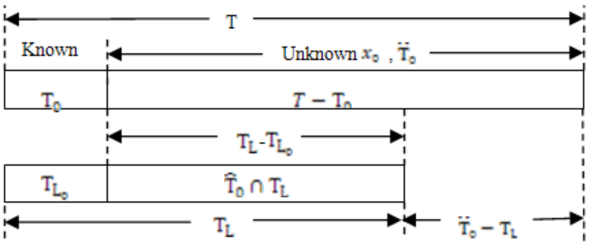



Fig. 2 Illustrations of support set for Theorem 2

**Algorithm 1:** PKLS-OMP Algorithm for Signal Recovery

Input:

- $NxM$ measurement matrix $\Phi$
- $N \times 1$ compressed measurement vector, $y$
- Sparsity level $K$ of the sparse signal
- $L$ Least Support Parameter
- $T_0$Set of indexes of $LL_3$

Output:

An $\hat{X}_{1\times M}$ reconstructed signal, new set of nonzero $\text{Aug\_p}_{(1\times L)}$

Procedure:

1) Initialize the residual, $\text{res}_0 = y$ ,
   support set: $T_0 = [T_{01}, T_{02} \dots T_{0|T_0|}]$
   least Support set: $J_0 = \phi$
   number of Iteration: $\ell = 0$
2) support size: Sup_size=$|T_0|$
3) $\varphi_j = [\varphi_1\varphi_2 \cdots \varphi_{|T_0|}]$
4) find $res_\ell = y - {\varphi_j}^{\dagger} y$
5) index=$T_0$, $I_0 = \varphi_j$
6) increment $\ell = \ell + 1$
7) find the maximum value of auto correlation between $res_\ell$ and $\Phi$,

$$J_\ell = \arg\max_{\ell=1\dots L} \left|{\varphi_\ell}^{*} \text{res}_{\ell-1}\right|$$

8) Augment the index set and matrix of choosing atoms indexed by J, $I_\ell = [I_{\ell-1} \cup \Phi_J]$,
9) find the new augment value $Aug\_p = I_\ell^{\dagger} \times y$ ( $I_\ell^{\dagger}$ denotes the pseudo-inverse operators of set $I_\ell$)
10) find new residual value $res_\ell = y - Aug\_p \times I_\ell$
11) update index, $index(|T_0| + \ell) = J(\ell)$
12) if the termination condition $\|y_r\|_2 \le \frac{\delta_{2l}}{(1-\delta_{2l})} \|y\|_2$, update the position set from [1,L] to [1,$\ell$ ] and go to step (15 ),
13) upgrade the value of $res_{\ell-1} = res_\ell$ and $I_{\ell-1} = I_\ell$,
14) return to step (6) if iteration number $\ell < K$,
15) the reconstructed sparse signal $\hat{X}_{1\times M}$ has nonzero indices at the index listed in $\text{Aug\_p}_{(1\times L)}$, arrange the value of Aug_p in the position listed by J.

$$\hat{X}_{1\times M}(index\,[\,1 : |T_0| + \ell\,]) = \text{Aug\_p}_{(1\times L)}$$

## V. Experimental Results

In this section, numerical experiments that explain the effectiveness of PKLS-OMP will be presents.

Signal characteristic used to experiment as follows: ECG signal with length is set to $n = 1024$, amplitude=200, four level wavelet transformer filter type Symlets8, sparisty level $Kmax = 128$ and Least Support Parameter(L)=60.

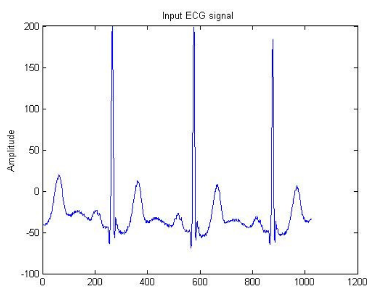
To sample a vector sensing matrices $\Phi$ had been used, that have i.i.d (Independent & Identically Distributed) entries drawn from a standard normal distribution with normalized columns. The R-SNR is used to measure performance of reconstructed original signal.

A study of the effect of using partially knowing support with LS-OMP method based on knowing support of approximation of DWT with size of prior $T_0 = 32$, are presents using Theorem 2 for termination condition. The results show that the new method gives fast and good results to reconstruct the original signal as shown in Fig. 3.

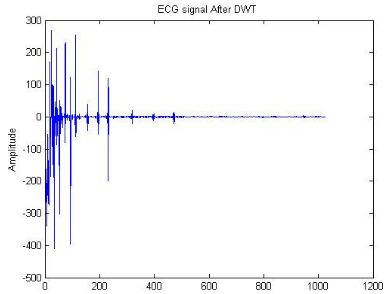
Fig. 4 shows the behavior of the two new methods and OMP-PKS by considering the reconstructed signal to noise ratio for the different measurement rate. As shown in Fig. 4 theorem 1 and 2 give convergent best results for recovering signal as they compared with the OMP-PKS method for the same ECG signal mention above.

Also a comparison is made between two new theorems explain earlier and OMP-PKS method as shown in Fig. 5 to study the effect of these conditions in term of time consumed to recover signal with different measurement rate value, size of known support set $T_0 = 32$.

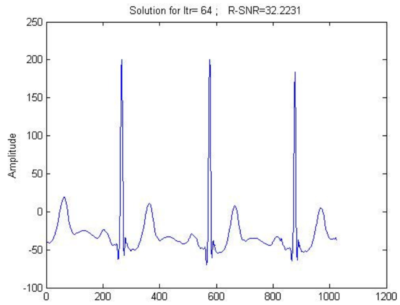
Finally, a comparison made between many methods used for compressive sensing and two suggested theorems to summarize the performance of the new algorithm that used in this paper. In Fig. 6 a comparison made for the performance of some method like: OMP, CoSaMP, OMP-PKS, CoSaMP-PKS, MP--PKS, LS-OMP, and our PKLS-OMP. Size of known support set=32. From the Fig. 6, it can be seen that, the best performance is given by the new PKLS-OMP, LS-OMP and CoSaMP-PKS. CoSaMP gives good results but only when the measurement rate is high (0.38) that's meant its need more measurement to produce good recovering signal.




(a) Input ECG signal




(b) ECG in wavelet domain




(c) Reconstructed signal using PLSK-OMP with theorem 2 for Iteration=64 and R-SNR=32.2231

Fig. 3 (a)-(c) Decomposition of input EGC signal after using PLSK

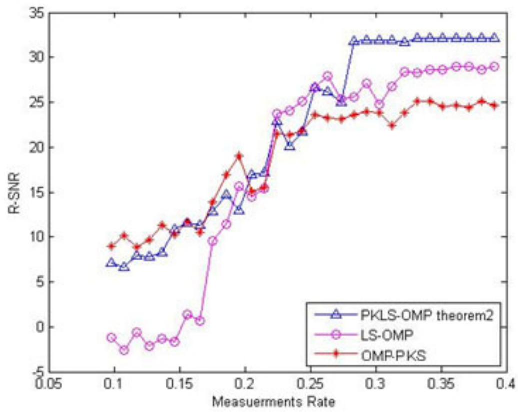



Fig. 4 Comparison between our two proved theorem for PKLS-OMP and PSK-OMP

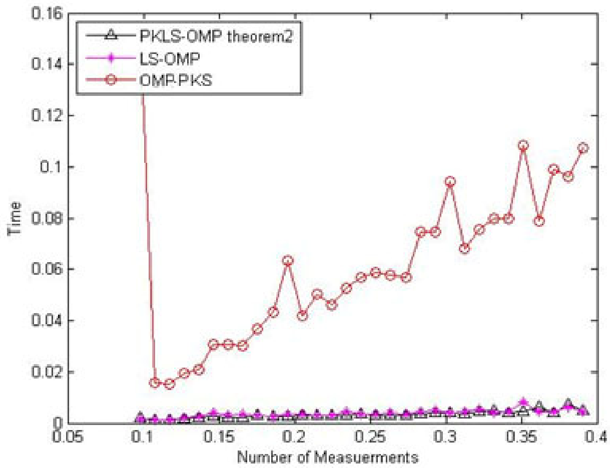



Fig. 5 Time consumes of the two theorems compeer with OMP-PKS method

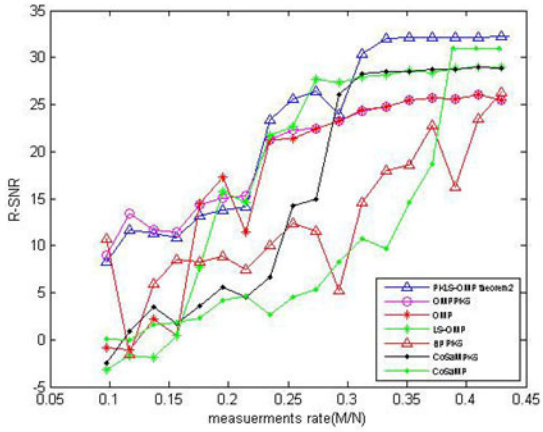



Fig. 6 Comparison between seven methods used for experiments test

## VI. CONCLUSIONS

In this paper, we try to produce two new methods for recovering signal by using compressive sensing greedy method, by improving new method depend on classic OMP procedure, this new method called LS-OMP. Also we produce a new method depend on partial knowing support, called PKLS-OMP. This new method improves some old method like PSK-OMP. We try to prove our new methods mathematically and then used these new methods in real signal like an ECG.

Experiment results show that new theorems improved the interpretation of some iterative algorithms like MP, OMP, and CoSaMP by producing faster calculation to get better approximate recovering original signal.

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