

# Parallel Double Splicing on Iso-Arrays

V. Masilamani, D.K. Sheena Christy, and D.G. Thomas

**Abstract**—Image synthesis is an important area in image processing. To synthesize images various systems are proposed in the literature. In this paper, we propose a bio-inspired system to synthesize image and to study the generating power of the system, we define the class of languages generated by our system. We call image as array in this paper. We use a primitive called iso-array to synthesize image/array. The operation is double splicing on iso-arrays. The double splicing operation is used in DNA computing and we use this to synthesize image. A comparison of the family of languages generated by the proposed self restricted double splicing systems on iso-arrays with the existing family of local iso-picture languages is made. Certain closure properties such as union, concatenation and rotation are studied for the family of languages generated by the proposed model.

**Keywords**—DNA computing, splicing system, iso-picture languages, iso-array double splicing system, iso-array self splicing.

## I. INTRODUCTION

Bio-inspired computing is a field devoted to tackling complex problems using computational methods modeled after design principles encountered in nature. Many of these Bio-inspired computing techniques have been used successfully to find good solutions to difficult problems in a wide range of areas such as combinatorial optimization, classification and decision making, pattern recognition, machine learning, nonlinear dynamics (modeling), computer security; biometrics, time series prediction, data mining, image processing and many more. Some of the topics covered in bio-inspired computing are evolutionary computing; neural computing, DNA computing and membrane computing.

In DNA computing [9] splicing systems were introduced by Head [2], [3] on biological considerations to model the behaviour of DNA molecules. He also introduced 1-splicing, 2-splicing, iterative splicing and double splicing and showed that double splicing is more powerful than normal splicing on strings. Laterly, Krithivasan et al. [7] extended the concept of splicing to arrays and defined array splicing systems. Parallel splicing on iso-arrays has been introduced by Masilamani et al. [8]. In parallel splicing, we use rules in parallel, to cut an image at a specific site. For more details about array languages and iso-picture languages we refer to [1], [5], [6], [10], [11].

In this paper, we define parallel double splicing on iso-arrays based on 1-splicing and we introduce parallel iso-array

self double splicing, parallel iso-array self restricted double splicing. As in parallel splicing on images [4] we also do splicing by applying the set of rules parallelly. In the second section, we give the basic definitions which are necessary to introduce the new model.

In the third section, we define the double splicing on iso-arrays and compare the family of languages generated by iso-array double splicing systems with the family of languages generated by iso-array self splicing systems. In the fourth section, we introduce a variant of iso-array double splicing called iso-array restricted double splicing. We compare the family of languages generated by iso-array self restricted double splicing systems with existing family of local iso-picture languages and with the family of *H*-iso-array splicing systems. We give some closure properties of family of languages generated by iso-array self restricted double splicing system.

## II. BASIC DEFINITIONS

In this section we recall some of the basic definitions of iso-picture languages and iso-array splicing systems to generate the language *L* discussed in [8], [6].

**Definition 1.** Let  $\Sigma = \left\{ S_1 \begin{array}{c} \triangle \\ \text{A} \\ \text{S}_2 \end{array} S_3, S_3 \begin{array}{c} \triangle \\ \text{B} \\ \text{S}_1 \end{array} S_2, S_1 \begin{array}{c} \triangle \\ \text{C} \\ \text{S}_3 \end{array} S_2, S_2 \begin{array}{c} \triangle \\ \text{D} \\ \text{S}_1 \end{array} S_3 \right\}$ .

The sides *S*<sub>1</sub>, *S*<sub>2</sub> and *S*<sub>3</sub> of each tile in  $\Sigma$  are of length  $\frac{1}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}}$  respectively.

An iso-array is an arrangement of isosceles right angled triangles of tiles from the set  $\Sigma$ .

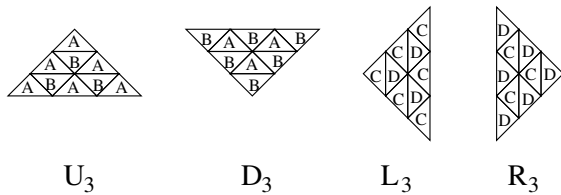


Fig. 1. Iso-arrays of size 3

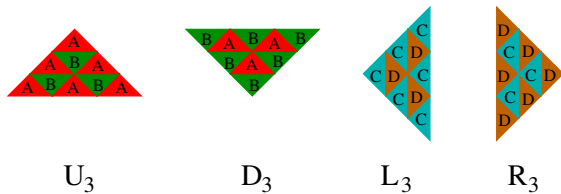


Fig. 2. Corresponding color image

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An *U*-iso-array of size *m* is formed exclusively by *m*  $\begin{array}{c} \triangle \\ \text{A} \end{array}$  tiles on side *S*<sub>2</sub> and it is denoted by *U*<sub>*m*</sub>. It will have *m*<sup>2</sup> tiles

in total (including the  $m$   $A$  tiles on  $S_2$ ). Similarly  $D$ -iso-array,  $L$ -iso-array and  $R$ -iso-array are formed exclusively by  $B$ -tile,  $C$ -tile and  $D$ -tile on side  $S_2$  respectively. The following are the iso-arrays of size 3 (or) size (1,3).

Iso-arrays of same-size can be catenated using the following catenation operations. There are four types of catenations of iso-arrays.

- (i) Horizontal catenation ( $\ominus$ ) :  $U \ominus D$
- (ii) Vertical catenation ( $\oplus$ ) :  $L \oplus R$
- (iii) Right catenation ( $\oslash$ ) :  $D \oslash U, R \oslash U, D \oslash L, R \oslash L$
- (iv) Left catenation ( $\oslash$ ) :  $U \oslash D, U \oslash L, L \oslash R, R \oslash D$ .

The iso-array primitives used in Definition 1 uses letters  $A, B, C, D$  where each letter denotes a color.

**Definition 2.** An iso-picture of size  $(n, m)$  over  $\Sigma$  is a picture formed by catenation of  $n$  iso-arrays of size  $m$ .

The number of tiles in any iso-picture of size  $(n, m)$  is  $nm^2$ .

**Definition 3.** Two iso-pictures of sizes  $(n_1, m)$  and  $(n_2, m)$ ,  $n_1, n_2, m \geq 1$  respectively can be catenated using catenation of iso-arrays, if the sides are glueable.

The set of all iso-pictures over  $\Sigma$  is denoted by  $\Sigma_I^{**}$ . An iso-picture language  $L$  over  $\Sigma$  is a subset of  $\Sigma_I^{**}$ .

**Definition 4.** Let  $L$  be an iso-picture language over  $\Sigma = \{ \triangle_A, \nabla_B, \triangleleft_C, \triangleright_D \}$ .  $L$  is said to be local if there exists a finite set  $\Theta$ , which is a subset of the set of

$(1, 2)$  iso-pictures over  $\Sigma \cup \{ \triangle_{\#A}, \nabla_{\#B}, \triangleleft_{\#C}, \triangleright_{\#D} \}$

such that  $L = L(\Theta)$  where  $L(\Theta) = \{ p \in \Sigma_I^{**} / B(\hat{p}) \subseteq \Theta \}$  and  $B(\hat{p})$  is the set of all  $(1, 2)$  sub-pictures of  $\hat{p}$ , where  $\hat{p}$  is a iso-picture obtained by surrounding  $p$  with special symbols

$$\triangle_{\#A}, \nabla_{\#B}, \triangleleft_{\#C}, \triangleright_{\#D} \notin \Sigma.$$

The family of local iso-picture languages will be denoted by  $ILOC$ .

**Definition 5.** Let  $\Sigma$  be an alphabet,  $\#$  and  $\$$  are two special symbols not in  $\Sigma$ . An iso-array over  $\Sigma$  is an isosceles triangular arrangement of tiles  $\triangle_A, \nabla_B, \triangleleft_C$  and  $\triangleright_D$ .

A horizontal splicing rule over  $\Sigma$  is of the form  $\alpha_1 \# \alpha_2 \$ \ominus \alpha_3 \# \alpha_4$  where  $\alpha_1 = U_m$  or  $\lambda$ ,  $\alpha_2 = D_m$  or  $\lambda$ ,  $\alpha_3 = U_m$  or  $\lambda$  and  $\alpha_4 = D_m$  or  $\lambda$ . The set of all horizontal splicing rules is denoted by  $R_{\ominus}$ .

A vertical splicing rule over  $\Sigma$  is of the form  $\alpha_1 \# \alpha_2 \$ \oplus \alpha_3 \# \alpha_4$  where  $\alpha_1 = L_m$  or  $\lambda$ ,  $\alpha_2 = R_m$  or  $\lambda$ ,  $\alpha_3 = L_m$  or  $\lambda$  and  $\alpha_4 = R_m$  or  $\lambda$ . No other possibility can occur. The set of all vertical splicing rules is denoted by  $R_{\oplus}$ .

A right splicing rule over  $\Sigma$  is of the form  $\alpha_1 \# \alpha_2 \$ \oslash \alpha_3 \# \alpha_4$  where

- i)  $\alpha_1 = D_m$  or  $\lambda$ ,  $\alpha_2 = U_m$  or  $\lambda$ ,  $\alpha_3 = D_m$  or  $\lambda$  and  $\alpha_4 = U_m$  or  $\lambda$  (or)
- ii)  $\alpha_1 = R_m$  or  $\lambda$ ,  $\alpha_2 = U_m$  or  $\lambda$ ,  $\alpha_3 = R_m$  or  $\lambda$  and  $\alpha_4 = U_m$  or  $\lambda$  (or)
- iii)  $\alpha_1 = D_m$  or  $\lambda$ ,  $\alpha_2 = L_m$  or  $\lambda$ ,  $\alpha_3 = D_m$  or  $\lambda$  and  $\alpha_4 = L_m$  or  $\lambda$  (or)

- iv)  $\alpha_1 = R_m$  or  $\lambda$ ,  $\alpha_2 = L_m$  or  $\lambda$ ,  $\alpha_3 = R_m$  or  $\lambda$  and  $\alpha_4 = L_m$  or  $\lambda$ .

The set of all right splicing rules is denoted by  $R_{\oslash}$ .

A left splicing rule over  $\Sigma$  is of the form  $\alpha_1 \# \alpha_2 \$ \oslash \alpha_3 \# \alpha_4$  where

- i)  $\alpha_1 = U_m$  or  $\lambda$ ,  $\alpha_2 = D_m$  or  $\lambda$ ,  $\alpha_3 = U_m$  or  $\lambda$  and  $\alpha_4 = D_m$  or  $\lambda$  (or)
- ii)  $\alpha_1 = U_m$  or  $\lambda$ ,  $\alpha_2 = L_m$  or  $\lambda$ ,  $\alpha_3 = U_m$  or  $\lambda$  and  $\alpha_4 = L_m$  or  $\lambda$  (or)
- iii)  $\alpha_1 = L_m$  or  $\lambda$ ,  $\alpha_2 = R_m$  or  $\lambda$ ,  $\alpha_3 = L_m$  or  $\lambda$  and  $\alpha_4 = R_m$  or  $\lambda$  (or)
- iv)  $\alpha_1 = R_m$  or  $\lambda$ ,  $\alpha_2 = D_m$  or  $\lambda$ ,  $\alpha_3 = R_m$  or  $\lambda$  and  $\alpha_4 = D_m$  or  $\lambda$ .

The set of all left splicing rules is denoted by  $R_{\oslash}$ .

**Definition 6.** An  $H$ -iso-array splicing scheme is a 5-tuple  $\sigma = (\Sigma, R_{\ominus}, R_{\oplus}, R_{\oslash}, R_{\oslash})$  where  $\Sigma$  is an alphabet,  $R_{\ominus}, R_{\oplus}, R_{\oslash}$  and  $R_{\oslash}$  are the finite set of horizontal, vertical, right and left splicing rules respectively. The iso-array splicing of  $L$  with  $\sigma$  is defined as

$$\sigma(L) = \left\{ p \in \Sigma_I^{**} / \begin{array}{l} (p_1, p_2) \vdash_{\ominus} p \text{ (or)} \\ (p_1, p_2) \vdash_{\oplus} p \text{ (or)} \\ (p_1, p_2) \vdash_{\oslash} p \text{ (or)} \\ (p_1, p_2) \vdash_{\oslash} p \end{array} \right\}$$

for some  $p_1, p_2 \in L$ . The language generated by  $H$ -iso-array splicing system is  $\sigma^*(A)$ .

$\sigma^*(L)$  is defined iteratively as follows:

$$\sigma^0(L) = L, \sigma^{i+1}(L) = \sigma^i(L) \cup \sigma(\sigma^i(L)), \text{ for } i \geq 0, \sigma^*(L) = \bigcup_{i=0}^{\infty} \sigma^i(L).$$

The family of iso-picture languages generated by  $H$ -iso-array splicing systems is denoted by  $FHIA$ .

**Definition 7.** The iso-array self splicing of  $L$  with  $\sigma = (\Sigma, R_{\ominus}, R_{\oplus}, R_{\oslash}, R_{\oslash})$  is defined as  $\sigma(L) = \{ V \in \Sigma^{**} / (u, u) \vdash_{\alpha} v, \text{ for some } u \in \Sigma^{**}, \alpha \in \{ \oslash, \oslash, \oplus, \ominus \} \}$ .

The iso-array self splicing system is defined by  $S = (\sigma, A)$  and the family of picture languages generated by the iso-array self splicing systems is denoted by  $ISHA$ .

### III. ISO-ARRAY DOUBLE SPLICING

In this section we give the definitions of iso-array double splicing and iso-array restricted double splicing of a picture language and compare the family of picture languages generated by iso-array restricted double splicing system with iso-array self splicing system.

**Definition 8.**

Let  $\sigma = (\Sigma, R_{\ominus}, R_{\oplus}, R_{\oslash}, R_{\oslash})$ . The iso-array double splicing of a picture language  $L$  with  $\sigma$  is defined as  $\sigma(L) = \{ U \in \Sigma^{**} / (X, Y) \vdash_{\alpha} Z \text{ and } (Z, Z) \vdash_{\beta} U \text{ where } X, Y \in L, \text{ for some } Z \in \Sigma^{**}, \alpha, \beta \in D, \text{ where } D \in \{ \oslash, \oslash, \oplus, \ominus \} \}$  and iso-array double splicing system is  $S = (\sigma, A)$  where  $A$  is a finite subset of images over  $\Sigma_I^{**}$ , called axioms, and the language generated by the system is  $\sigma^*(A)$  where  $\sigma$  is the iso-array double splicing scheme.

**Definition 9.** The iso-array restricted double splicing of a picture language  $L$  with  $\sigma$  is defined as  $\sigma(L) = \{U \in \Sigma^{**}/(X, Y) \vdash_{\alpha} Z, \text{ and } (Z, Z) \vdash_{\beta} U \text{ for some } X, Y \in L, Z \in \Sigma^{**}, \alpha, \beta \in D = \{\emptyset, \ominus, \oplus, \otimes\} \text{ and } \alpha \neq \beta\}$ .

The iso-array restricted double splicing system is  $S = (\sigma, A)$  where  $A \subset \Sigma_I^{**}$ , called axioms and the language generated by the iso-array restricted double splicing system is given by  $\sigma^*(A)$ .

The family of picture languages generated by iso-array restricted double splicing systems is denoted by IRDS.

**Theorem 1.** IRDS is not a subclass of ISHA.

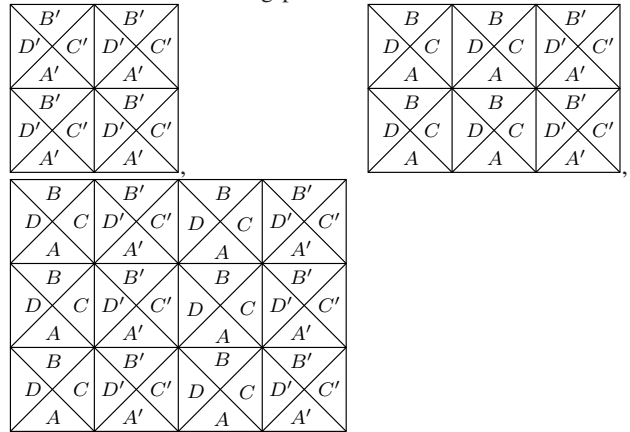
*Proof:* Let

$$\Sigma = \{ \begin{array}{|c|} \hline \triangle \\ \hline \end{array}, \begin{array}{|c|} \hline \nabla \\ \hline \end{array}, \begin{array}{|c|} \hline \triangleleft \\ \hline \end{array}, \begin{array}{|c|} \hline \triangleright \\ \hline \end{array}, \begin{array}{|c|} \hline \triangle \\ \hline \end{array}, \begin{array}{|c|} \hline \nabla \\ \hline \end{array}, \begin{array}{|c|} \hline \triangleleft \\ \hline \end{array}, \begin{array}{|c|} \hline \triangleright \\ \hline \end{array} \}.$$

Let  $L = \{p \in \Sigma^{**}/p = s_m^i t_m^j s_m^k t_m^l \text{ where } m \geq 2, i, j, k, l \geq 0, s_m^i \text{ and } t_m^l \text{ are } m \times i \text{ arrays in which each}$

entry (square) carries either of the tile  $\begin{array}{|c|} \hline B \\ \hline D \quad C \\ \hline A \end{array}$  or  $\begin{array}{|c|} \hline B' \\ \hline D' \quad C' \\ \hline A' \end{array}$  respectively}.

For instance the following pictures are the members of  $L$



Let  $S = (\sigma, A)$  where  $\sigma = (\Sigma, R_{\oplus}, R_{\ominus}, R_{\otimes}, R_{\emptyset})$  and  $A = \{s_2^i t_2^j s_2^k t_2^l / i, j, k, l = 0 \text{ or } 1\}, R_{\otimes} = R_{\emptyset} = \phi$ . The splicing rules to generate the iso-pictures of  $L$  are given by

$$R_{\oplus} : (1) C' \# \lambda \ \$ C \# D'; \quad (2) C \# \lambda \ \$ \lambda \ # D$$

$$R_{\ominus} : (1) A \# \lambda \ \$ \lambda \ # B \quad (2) A' \# \lambda \ # \lambda \ # B'$$

Clearly these iso-array splicing rules will generate the language  $L$ . Hence  $L \in IRDS$ . We now prove that  $L \notin ISHA$ .

Since the iso-array self splicing involves two copies of the same form, the splicing rule  $C \# \lambda \ \$ \lambda \ # D$  attempts to produce an image  $I : s_m^i t_m^j s_m^{k+i} t_m^l s_m^k t_m^l$  and  $I \notin L$ . Hence the proof. ■

IV. ISO-ARRAY SELF RESTRICTED DOUBLE SPLICING

**Definition 10.** The iso-array self restricted double splicing of  $L$  is defined as  $\sigma(L) = \{p \in \Sigma^{**}/(X, X) \vdash_{\alpha} Y \text{ and } (Y, Y) \vdash_{\beta} p \text{ for some } X \in L, \alpha, \beta \in D = \{\emptyset, \ominus, \oplus, \otimes\} \text{ and } \alpha \neq \beta\}$ .

An iterative iso-array self restricted double splicing  $\sigma^*(L)$

is defined by

$$\sigma^0(L) = L,$$

$$\sigma^{i+1}(L) = \sigma^i(L) \cup \sigma(\sigma^i(L)) \text{ for } i \geq 0,$$

$$\sigma^*(L) = \bigcup_{i=0}^{\infty} \sigma^i(L).$$

The H-iso-array self restricted double splicing system is defined as a pair  $S = (\sigma, A)$  where  $\sigma = (\Sigma, R_{\oplus}, R_{\ominus}, R_{\otimes}, R_{\emptyset})$  and  $A$  is finite subset of  $\Sigma_I^{**}$ , called axioms.

The family of iso-picture languages generated by iso-array self restricted double splicing systems is denoted by FISRD.

**Theorem 2.** FISRD is not closed under union and concatenation.

*Proof:* Let

$$\Sigma = \{ \begin{array}{|c|} \hline \triangle \\ \hline \end{array}, \begin{array}{|c|} \hline \nabla \\ \hline \end{array}, \begin{array}{|c|} \hline \triangleleft \\ \hline \end{array}, \begin{array}{|c|} \hline \triangleright \\ \hline \end{array}, \begin{array}{|c|} \hline \triangle \\ \hline \end{array}, \begin{array}{|c|} \hline \nabla \\ \hline \end{array}, \begin{array}{|c|} \hline \triangleleft \\ \hline \end{array}, \begin{array}{|c|} \hline \triangleright \\ \hline \end{array} / i = 1, 2, 3 \}.$$

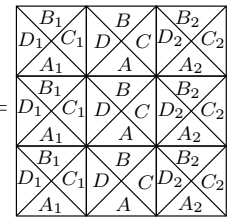
Let  $L_1$  be the set of all isopictures of rectangles over  $\Sigma$  with 3 rows and at least 3 columns, where all the entries (squares)

is in the 1<sup>st</sup> column should have  $\begin{array}{|c|} \hline B_1 \\ \hline D_1 \quad C_1 \\ \hline A_1 \end{array}$ , the last column

should have  $\begin{array}{|c|} \hline B_2 \\ \hline D_2 \quad C_2 \\ \hline A_2 \end{array}$  and the intermediate column should have

$$\begin{array}{|c|} \hline B \\ \hline D \quad C \\ \hline A \end{array}.$$

For example the iso-picture  $I =$



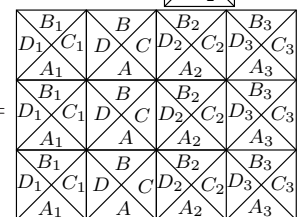
is a member of  $L_1$ . Let  $L_2$  be the set of all iso-pictures of rectangles over  $\Sigma$  with 3 rows and at least 4 columns where

all the squares in the 1<sup>st</sup> column should have  $\begin{array}{|c|} \hline B_1 \\ \hline D_1 \quad C_1 \\ \hline A_1 \end{array}$ , last

column should have  $\begin{array}{|c|} \hline B_3 \\ \hline D_3 \quad C_3 \\ \hline A_3 \end{array}$ , the second column should have

$\begin{array}{|c|} \hline B \\ \hline D \quad C \\ \hline A \end{array}$  and the remaining column should have  $\begin{array}{|c|} \hline B_2 \\ \hline D_2 \quad C_2 \\ \hline A_2 \end{array}$ .

$$\text{For example the iso-picture } J =$$



is a member of  $L_2$ . The iso-array self restricted double splicing system that generates  $L_1$  is given by  $\sigma = (\Sigma, R_{\oplus}, R_{\ominus}, R_{\otimes}, R_{\emptyset})$  where  $R_{\otimes} = R_{\emptyset} = R_{\ominus} = \phi$  and  $A = \{I\}$ .

$R_{\oplus} = \{C \# D_2 \ \$ C_1 \ # D\}$   
It can be easily seen that  $\sigma^*(A) = L_1$ . The iso-array self restricted double splicing system that generates the language  $L_2$  is given by

$\sigma = (\Sigma, R_{\oplus}, R_{\ominus}, R_{\otimes}, R_{\odot})$  where  $R_{\otimes} = R_{\odot} = R_{\ominus} = \phi$  and  $A = \{J\}$ .

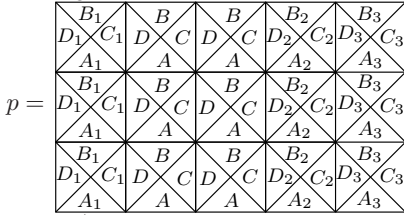
$R_{\oplus} = \{C_2 \# D_3 \$ C \# D_2\}$

It can be easily seen that  $\sigma^*(A) = L_2$ .

Now we are going to prove that  $L_1 \cup L_2 \notin FISRD$ .

Since any splicing rule that attempts to produce members of  $L_1$  and  $L_2$  will also produce an iso-picture  $p$  given below, which is not in  $L_1 \cup L_2$ .

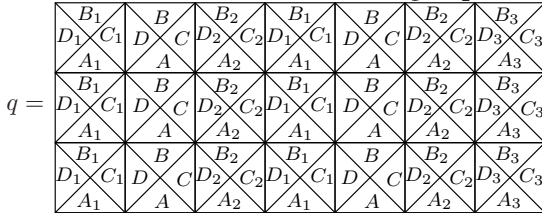
In other words, the splicing rule  $C \# D_2 \$ C_1 \# D$  produces an image



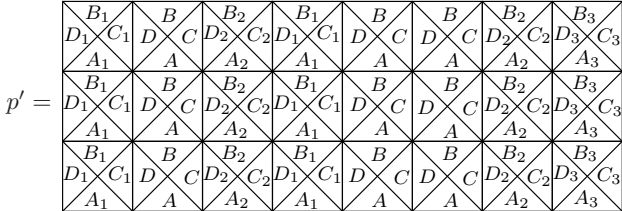
and  $p \notin L_1 \cup L_2$ .

Hence FISRD is not closed under union.

$L_1 \circ L_2 \notin FISRD$  where  $\circ$  represents vertical concatenation. One of the member of  $L_1 \circ L_2$  is



The splicing rule  $C \# D_2 \$ C_1 \# D$  that attempts to generate members of  $L_1$  and  $L_2$  will also produce an image



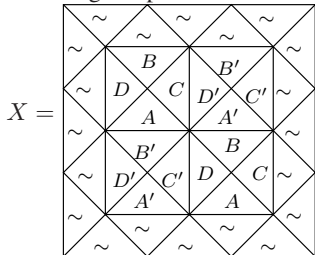
which is not a member of  $L_1 \circ L_2$ . Hence  $L_1 \circ L_2 \notin FISRD$  even though  $L_1$  and  $L_2$  are members of FISRD. ■

**Theorem 3.** FISRD is not a sub class of FHIA.

*Proof:* Let

$$\Sigma = \{ \triangle A, \triangle B, \triangle C, \triangle D, \triangle A', \triangle B', \triangle C', \triangle D' \}$$

Let  $L$  be the set of all  $m \times m$  iso-pictures over  $\Sigma$  having  $\sim$  on the boundary where  $m$  is a power of 2. For instance the following iso-picture  $X$  is the member of language  $L$ .



Let  $S = (\sigma, A)$  be the iso-array self restricted double splicing system, where  $\sigma = (\Sigma, R_{\oplus}, R_{\ominus}, R_{\otimes}, R_{\odot})$ ,  $R_{\otimes} = R_{\odot} = \phi$ , and  $A = \{X\}$ .

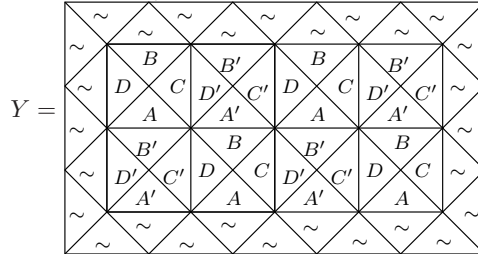
$R_{\oplus}$  consists of

$$(1) C' \# \sim \$ \sim \# D \quad (2) C \# \sim \$ \sim \# D'$$

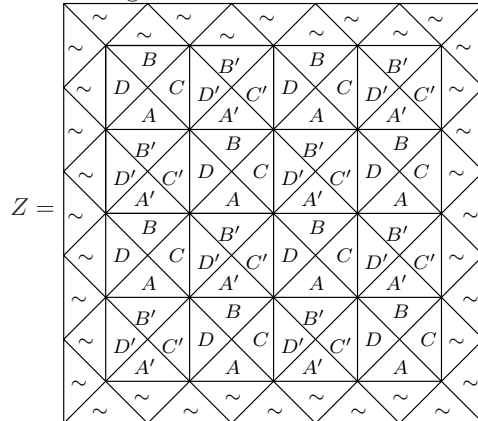
$R_{\ominus}$  consists of

$$(1) A' \# \sim \$ \sim \# B \quad (2) A \# \sim \$ \sim \# B'$$

Let  $(X, X) \vdash_{\oplus} Y$ . We see that



If  $(Y, Y) \vdash_{\ominus} Z$ , then



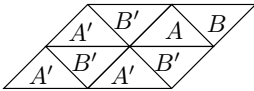
The above system generates the language  $L$ . But this language  $L$  cannot be generated by any  $H$ -iso-array splicing system. ■

**Theorem 4.** The class ILOC of local iso-array languages and the class of FISRD are incomparable but not disjoint.

*Proof:* Let  $K$  be a language consisting of iso-pictures of rhombuses of size  $m \times n$  with  $m, n \geq 2$ , where each tile in the rhombus is either of the form  $\triangle A \triangle B$  (or)

$\triangle A' \triangle B'$  and all the tiles on the  $\angle$  shape should have the tile  $\triangle A' \triangle B'$  and the remaining places in the rhombus

should have the tile  $\triangle A \triangle B$ .

For example the iso-picture  is a member of  $K$ .

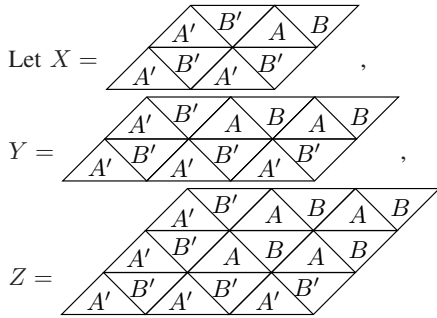
The language  $K$  can be proved to be in ILOC. To prove that  $K \in FISRD$ , consider  $\sigma = (\Sigma, R_{\oplus}, R_{\ominus}, R_{\otimes}, R_{\odot})$  where  $\Sigma = \{ \triangle A', \triangle B', \triangle A, \triangle B \}$  and  $R_{\oplus} = R_{\odot} = \phi$  and  $A = \{X\}$ .

$R_{\otimes}$  consists of the following rules:

$$(1) B \# \lambda \$ B' \# A \quad (2) B' \# \lambda \$ B' \# A'$$

$$R_{\ominus} = \{ A' \# B' \$ \lambda \# B'; A \# B' \$ \lambda \# B \}$$





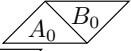
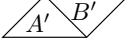
Clearly  $(X, X) \vdash_{\emptyset} Y$  and  $(Y, Y) \vdash_{\Theta} Z$ . In a similar way we can generate all iso-pictures of rhombuses of  $K$ .

Hence the iso-array self restricted double splicing system  $S = (\sigma, A)$ , which is defined above generates the language  $K$ .

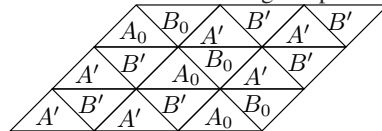
Hence  $FISR D \cap ILOC \neq \phi$ .

Now we exhibit that there exists a language in  $ILOC$  but not in  $FISR D$ .

Let  $\Sigma = \{ \triangle_{A_0}, \triangle_{B_0}, \triangle_{B'}, \triangle_{A'} \}$ . Let  $L = \{ p \in \Sigma^{**} /$

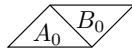
The diagonal positions of  $p$  should have the tile  and other positions should have the tile , and the size of  $p$  is  $m \times m, m \geq 2$ .

For instance the following iso-picture is a member of  $L$ .

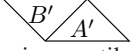



It can be clearly seen that the language  $L$  is proved to be in  $ILOC$ .

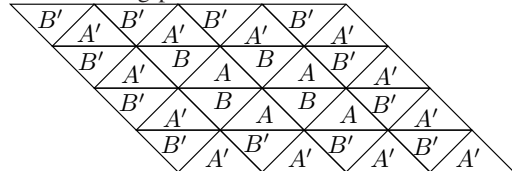
But the language  $L \notin FISR D$ .

If  $L \in FISR D$ , then there exists a  $FISR D$  system  $S = (\sigma, A)$  to generate  $L$ . Any splicing rules that attempts to generate the required language  $L$  will produce an image which is not in  $L$ . i.e., it will produce an image having a tile  in non-diagonal positions also. Hence  $L \notin FISR D$ .

We finally show that there exists a language in  $FISR D$  which is not in  $ILOC$ .

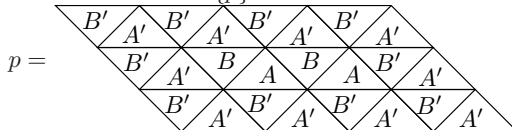
Let  $\Sigma = \{ \triangle_A, \triangle_B, \triangle_{B'}, \triangle_{A'} \}$  and  $L$  be the language of iso-pictures of parallelogram of size  $m \times n$ , where  $m = 2 + 2^i$  for  $i \geq 0$  and  $n = 4$  having a tile  on the boundary and other positions are having a tile .

The following picture



is member of  $L$ . We prove that  $L \in FISR D$ .

The axiom set  $A$  is  $\{p\}$  where

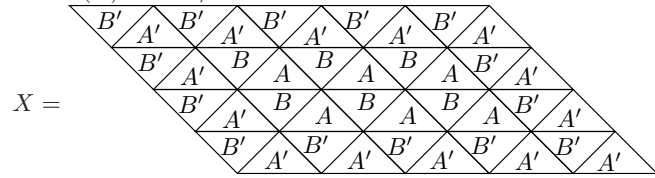


Let  $R_{\ominus} = \{ A' \# B' \$ A' \# B'; A \# B' \$ A' \# B \}$ . and  $R_{\odot} = \{ A' \# B' \$ A' \# B'; A \# B' \$ A \# B' \}$ . Now it can be easily verified that the system  $S = (\sigma, A)$  generates the language  $L$ .

But  $L \notin ILOC$ .

Suppose  $L \in ILOC$ . Then there exists a set  $\Theta$  of  $(1, 2)$  tiles over  $\Sigma \cup \{ \# \}$ , which generates  $L = L(\Theta)$ .

By using the same  $\Theta$  set we can generate the following iso-picture of size  $m \times 5$  which does not belongs to  $L$ . i.e.,  $X \in L(\Theta)$  but  $X \notin L$  where



**Theorem 5.**  $FISR D$  is closed under rotation by  $90^\circ, 180^\circ$  and  $270^\circ$ .

*Proof:* It is known that the class of FHIA is closed under rotations by  $90^\circ, 180^\circ$  and  $270^\circ$  [8]. Similar proof can be given to  $FISR D$  also. ■

V. CONCLUSION

In this paper, we have introduced a new model called double splicing on iso-arrays and its variants to generate iso-pictures. We compared the family of languages generated by self restricted double splicing systems on iso-arrays with the family of local iso-picture languages. Closure properties such as union, concatenation and rotation of iso-array self restricted double splicing are studied.

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