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Optimum Replacement Policies for Kuwait Passenger Transport Company Busses: Case Study

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Abstract**—**Due to the excess of a vehicle operation through its life, some elements may face failure and deteriorate with time. This leads us to carry out maintenance, repair, tune up or full overhaul. After a certain period, the vehicle elements deteriorations increase with time which causes a very high increase of doing the maintenance operations and their costs. However, the logic decision at this point is to replace the current vehicle by a new one with minimum failure and maximum income.

The importance of studying vehicle replacement problems come from the increase of stopping days due to many deteriorations in the vehicle parts. These deteriorations increase year after year causing an increase of operating costs and decrease the vehicle income. Vehicle replacement aims to determine the optimum time to keep, maintain, overhaul, renew and replace vehicles. This leads to an improvement in vehicle income, total operating costs, maintenance cost, fuel and oil costs, ton-kilometers, vehicle and engine performance, vehicle noise, vibration, and pollution.

The aim of this paper is to find the optimum replacement policies of Kuwait Passenger Transport Company (KPTCP) fleet of busses. The objective of these policies is to maximize the busses pure profits. The dynamic programming (D.P.) technique is used to generate the busses optimal replacement policies.

Keywords**—**Replacement Problem, Automotive Replacement, Dynamic Programming, Equipment Replacement, K.P.T.C.

I. Introduction

THE replacement problem is one of the important problems for most organizations. It needs to be solved to find the optimal time to replace the current vehicle by a new one. The survey of the literatures related to maintenance and replacement processes is presented and discussed from the point of view of the most famous methods used for solving such problems. These methods are: the enumeration, the shortest path, regeneration of point approach, integer programming, and dynamic programming (D.P.) techniques. The survey is based on some of the most recent published papers and books available.

Modeling of the replacement problem is presented by many authors. Nakagawa et al [1] introduced the age replacement problem. Roll et al [2] concerned with the derivation of an optimal doctrine regarding continuous attendance to, and preventive replacement of, equipment subject to both gradual deterioration and catastrophic failure. Bartholonew [3] studied the replacement of an N-items equipments. He aimed at deducing the cheapest procedure when a replacement problem of equipment of N-items is considered. Christer [4] considered a decision problem concerning the replacement of members of a fleet of fork lift trucks during a period of inflation and economic uncertainty. Okumoto et al [5] aimed at finding the optimum preventive maintenance policies which minimizes the expected cost per unit time in the steady state by introducing corrective replacement and preventive maintenance costs.

The definition of the enumeration method is discussed in Taha [6] by considering a problem with N-stages, and at each stage there are a number of alternatives. The problem solution is to choose only one decision at each stage and calculating the value of the objective function of the generated replacement policy. Then generate another policy and calculate its own objective function value. The problem stops after enumerating all the replacement policies of the problem and comparing their values of objective functions to choose the optimal policy with maximum profit (or minimum cost). The enumeration method is applied by D'Aversa et al [7] on two actual replacement problems.

The shortest path method is illustrated in Bronson [8], Dreyfus et al [9], Sigal et al [10]. The idea of this method is to draw the problem as a network and use the famous shortest path technique to find the optimal replacement problem which achieves the minimal cost.

The regeneration point approach method is introduced by Dreyfus et al [9]. This method is based on an idea. That is in a certain year, if the decision maker decides to replace the equipment, the firm will enter the next year with a machine of age one year. Then the process is called to regenerate itself.

The integer programming model is introduced by Khalil [11] to solve the vehicle replacement problem as a Zero-One integer programming problem. In [12] Khalil solved the vehicle replacement problem as a large scale multi-objective fuzzy integer programming problem.

The dynamic programming technique is introduced by Abdelwali [13,14], Nicholson et al [15], Hastings [16], D'Aversa [7] and Waddel [17]. The main idea of the D.P. technique is to break the studied problem into stages. Then study each stage separately to find the optimal decision at each state variable at this stage. Then by using a recursive equation, the optimal replacement policy of the problem can be achieved.

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More advanced researches and many references related to the vehicle replacement problem as well as a comparison between the different methods which can be used to solve the replacement problem are included in Abdelwali [13,14].

II. Dynamic Programming Technique and Applications

Dynamic programming (D.P.) is one of the important branches in operations research. It deals with the complex problems which exist in multi-stages. The procedure can solve these complex problems by dividing the complex problem into a number of independent sub-problems. Each one of these sub-problems is called a stage and can be handled more efficiently from the computational point of view. After solving every sub-problem, the large problem solution can be reached easily by using the state variables. A state variable is a link between stages which allows one to make optimum decisions for the remaining stages without having to check the effect of future decisions on decisions previously made. The state of the problem could be discrete or continuous. There may be more than one state of the problem, which will cause difficulty of the problem solution. The result of the problem will be an optimal policy which contains several decisions. The number of decisions at the optimal policy must equal the number of stages. The D.P. procedure aims at finding the optimal policy, not at finding several optimal decisions. The D.P. technique relates between succeeding stages by means of recursive equation which adds the cost (or profit) of return function to the next stage. The return function means the total cost (or profit) of every decision at each state variable through stages. D.P. can be classified into deterministic and stochastic. D.P. problems can be solved either forward or backward. D.P. is a general strategy for optimization rather than a specific set of rules. Consequently, the particular equations used must be developed to fit each problem.

The D.P. technique had originated in the late 1940s and early 1950s by Richard Bellman. Bellman is the founder of the principle of optimality of D.P. technique. Bellman published D.P. books in 1957, 1961, and 1962. Then many other books started to appear in the area of D.P. by S.D. Dreyfus, Aris, Nemhauser, Wilde, L.G. Mitten, Denardo, Beightler, and many other authors.

The D.P. technique has a wide range of applications in the mechanical engineering and equipments. It can be applied to prepare a plan to renovate and replace equipments and automotive, ship loading problems, minimize the probability of failure, production planning problems, stock control applications, inventory control, selling of stock problems, reliability problems, shortest route problems, solution of linear programming problems, capital budgeting problems, selection of advertising media, world health council problems, employment smoothening problems, determination of required workers, and many other applications.

III. Automotive Replacement Formulation by Dynamic Programming

The D.P. recursive equation of the replacement problem for 2 decisions: Keep and Replace with the aim of minimizing the total cost can be written as in equation (1) if the organization fleet of vehicles generates some income. Equation (2) represents the D.P. recursive equation with the aim of maximizing the company pure profit. But equation (3) represents the D.P. recursive equation for minimizing the total cost if there is no income

$$F_j(X_j) = \min\begin{cases} M_j(X_j) - I_j(X_j) + F_{j+1}(X_j+1) \text{....} Keep \\ M_j(0) - I_j(0) + R_j(X_j) + F_{j+1}(1) \text{...Re} place \end{cases} \quad (1)$$

$$F_j(X_j) = \max\begin{cases} I_j(X_j) - M_j(X_j) + F_{j+1}(X_j+1) \text{....} Keep \\ I_j(0) - M_j(0) - R_j(X_j) + F_{j+1}(1) \text{...Re} place \end{cases} \quad (2)$$

$$F_j(X_j) = \min\begin{cases} M_j(X_j) + F_{j+1}(X_j+1) \text{....} Keep \\ M_j(0) + R_j(X_j) + F_{j+1}(1) \text{...Re} place \end{cases} \quad (3)$$

Where:

$M_j(x_j)$ Represent total cost at each stage (j) of an old bus.
$M_j(0)$ Represent total cost at stage (j) of a new bus.
$I_j(x_j)$ Represent the old bus income at stage (j).
$I_j(0)$ Represent the new bus income at stage (j).
$R_j(x_j)$ Represent the bus replacement cost at stage (j).
$F_j(x_j)$ Represent the total recursive cost for a bus of age (X) at stage (j).
$F_{j+1}(x_j+1)$ Represent the total recursive cost for a bus of age (X_j+1) at stage (j+1).
$F_{j+1}(1)$ Represent the total recursive cost for a bus of age (1) at stage (j+1).
x_j Represent the bus age at stage j, (The state variable).
D_j Represent the decision at stage j.
j Represent the stage.

IV. Illustrative Example

Assuming a 2 years old equipment with the following data in Tables (1), (all values in dollars). It is required to find the optimal replacement policy for this equipment to minimize the total cost over the next 4 years.

Table I Data Of The Illustrative Example, Stage 1

Stage	1	
X_j	0	2
$I_j(X_j)$	3000	2200
$M_j(X_j)$	1100	2800
$R_j(X_j)$	---	6200

Table II Data Of The Illustrative Example, Stage 2

Stage	2		
X_j	0	1	3
$I_j(X_j)$	5000	4600	3700
$M_j(X_j)$	1200	2450	6100
$R_j(X_j)$	---	5600	8000

Table III Data Of The Illustrative Example, Stage 3

Stage	3			
X_j	0	1	2	4

$I_j(X_j)$	7000	4800	4600	2700
$M_j(X_j)$	2300	2500	4000	6000
$R_j(X_j)$	---	5700	7500	8200

TABLE IV DATA OF THE ILLUSTRATIVE EXAMPLE, STAGE 4

Stage	4				
X_j	0	1	2	3	5
$I_j(X_j)$	6800	5000	4700	4000	2500
$M_j(X_j)$	2400	2600	4100	5300	6600
$R_j(X_j)$	---	7900	6600	7200	8300

The decision will be taken at the beginning of each year. The problem will be solved by backward dynamic programming by using the recursive equation (1). The problem state variable will be as shown in Table V:

TABLE V STATE VARIABLES FOR 2 YEARS OLD EQUIPMENT

j	1	2	3	4
X_j	X1 = 2	X2 = 1, 3	X3 = 1, 2, 4	X4 = 1, 2, 3, 5

TABLES VI SOLUTION OF STAGES (4)

X_4	$F_4(x_4)$ - Keep	$F_4(x_4)$ - Replace	$F_4(x_4)$	D_4
5	4100	3900	3900	Replace
3	1300	2800	1300	Keep
2	-600	2200	-600	Keep
1	-2400	3500	-2400	Keep

TABLES VII SOLUTION OF STAGES (3)

X_3	$F_3(x_3)$ - Keep	$F_3(x_3)$ - Replace	$F_3(x_3)$	D_3
4	7200	1100	1100	Replace
2	700	400	400	Replace
1	-2900	-1400	-2900	Keep

TABLES VIII SOLUTION OF STAGES (2)

X_2	$F_2(x_2)$ - Keep	$F_2(x_2)$ - Replace	$F_2(x_2)$	D_2
3	3500	1300	1300	Replace
1	-1750	-1100	-1750	Keep

TABLES IX SOLUTION OF STAGES (1)

X_1	$F_1(x_1)$ - Keep	$F_1(x_1)$ - Replace	$F_1(x_1)$	D_1
2	1900	2550	1900	Keep

From Tables (3.4) to (3.1), the optimal replacement policy will be as shown in Table (3.5).

TABLES X THE OPTIMAL REPLACEMENT POLICY AND ITS TOTAL COST

Stage	1	2	3	4	Total Cost
Decision	Keep	Replace	Keep	Keep	$1,900

This means that the company should keep the equipment at the first year, then replace it by a new one, then keep the new equipment till the rest of the planned period. The total optimal cost for this optimal policy {K, R, K, K} equals $1900.

V. CASE STUDY OF KUWAIT PASSENGER TRANSPORT COMPANY (KPTC): COLLECTED DATA

Kuwait Passenger Transport Company (KPTC) is the leading passengers transport company in Kuwait. This company involves more than 500 different busses. Each bus serves about 70,000 passengers every year, while the average kilometers done by each bus is 120,000 kilometers yearly. The company involves 3 types of busses: Mercedes, Volvo, and Daewoo.

Our research study is carried out on 15 busses, 5 busses of each type. The studied planned period is 15 years which starts from the year 2005 to 2019. The actual data are collected for the years 2005 to 2009. Then M.S. Excel is used to predict the future values for the rest of the planned period. Tables XI in the appendix represent our case study collected and predicted data for just one bus (Mercedes). The collected data include: All buses income, all busses operating costs, and busses new and used prices for the planned period years.

VI. SOLUTION OF THE PROBLEM

M.S. Excel solver and Lingo are used to find the optimal replacement policies for the studied busses. The D.P. formulation in Equation (2) is programmed using both Excel solver and Lingo codes.

Table VII in the appendix illustrates the optimal replacement policies for all busses (15 busses). Table 15 illustrates the optimal and the actual replacement policies profits of each bus. There are huge differences between the optimal and current replacement policies. Optimal replacement policies can allow the company to earn about 60% pure profit more than the current profits. Figures (1) to (3) illustrate comparison between optimal policies pure profit and the current policies pure profit.

VII. CONCLUSIONS AND RECOMMENDATIONS

1) It is noted that the optimal replacement policies can allow the KPTC company to earn about 60% more than the current pure profit according to the collected data.

2) It is very important to apply the principles of automotive replacement policies for transportation companies' fleet of vehicles. This increases the company profit and gives the company the opportunity for scheduling the maintenance and spare parts.

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3) It is highly recommended to treat the problem by fuzzy logic to overcome the weakness in information, data and the predicted values to achieve more accurate policies.

VIII. APPENDIX

Tables XI The Collected Data.

Stage 15 (2019):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
15	6090	9889	69500
14	7496	8421	69500
13	8530	9098	69500
12	9581	9764	69500
11	10977	10364	69500
10	12397	12449	69500
9	14173	11038	69500
8	15979	11260	69500
7	18152	11358	69500
6	20532	11210	69500
5	23294	11150	69500
4	26796	11042	63400
3	30001	13001	52500
2	34560	9783	42100
1	39713	7134	23800
0	39358	6204	

Stage 14 (2018):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
14	6815	7656	65500
13	8453	9016	65500
12	9496	9676	65500
11	10880	10272	65500
10	12288	12340	65500
9	14050	10942	65500
8	15841	11163	65500
7	17997	11261	65500
6	20358	11115	65500
5	23098	11056	65500
4	26572	10950	59800
3	29754	12894	49500
2	34277	9702	34500
1	39390	7076	22600
0	39358	6154	

Stage 13 (2017):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
13	7685	8197	61500
12	9410	9589	61500
11	10783	10180	61500
10	12180	12231	61500
9	13927	10846	61500
8	15703	11065	61500
7	17842	11164	61500
6	20184	11020	61500
5	22902	10962	61500
4	26349	10858	56200
3	29506	12786	46500
2	33994	9622	32300
1	39067	7018	13000
0	39358	6104	

Stage 12 (2016):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
12	8555	8718	69500
11	10686	10089	69500
10	12071	12122	69500
9	13804	10750	69500
8	15565	10968	69500
7	17687	11067	69500
6	20010	10925	69500
5	22707	10869	69500
4	26126	10766	64600
3	29258	12679	55500
2	33711	9542	42100
1	38744	6960	23800

0	39358	6054	

Stage 11 (2015):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
11	9715	9172	65500
10	11962	12013	65500
9	13680	10654	65500
8	15428	10871	65500
7	17531	10970	65500
6	19836	10830	65500
5	22511	10775	65500
4	25902	10674	61000
3	29010	12571	52500
2	33427	9462	39900
1	38421	6902	22600
0	38686	6004	

Stage 10 (2014):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
10	10875	10921	61500
9	13557	10558	61500
8	15290	10774	61500
7	17376	10872	61500
6	19662	10735	61500
5	22315	10681	61500
4	25679	10582	57300
3	28762	12464	49500
2	33144	9382	37700
1	38098	6844	21400
0	38686	5954	

Stage 9 (2013):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
9	12325	9599	57500
8	15152	10677	57500
7	17221	10775	57500
6	19488	10640	57500
5	22119	10588	57500
4	25456	10490	53600
3	28514	12356	46250
2	32861	9302	35500
1	37775	6786	20200
0	38686	5904	

Stage 8 (2012):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
8	13775	9707	53500
7	17066	10678	53500
6	19314	10545	53500
5	21924	10494	53500
4	25232	10398	49900
3	28266	12249	43000
2	32577	9221	32850
1	37453	6728	19000
0	37004	5854	

Stage 7 (2011):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
7	15515	9708	49500
6	19140	10450	49500
5	21728	10400	49500
4	25009	10306	46200
3	28018	12141	39750
2	32294	9141	30200
1	37130	6670	17100
0	37004	5804	

Stage 6 (2010):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)

6	17400	9500	46500
5	21532	10307	46500
4	24786	10214	43467
3	27770	12034	37500
2	32011	9061	28550
1	36807	6612	16200
0	37004	5754	

Stage 5 (2009):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
5	19575	9370	43500
4	24563	10122	40635
3	27522	11926	35169
2	31728	8981	26900
1	36484	6554	15300
0	35322	5704	

Stage 4 (2008):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
4	22330	9202	37796
3	27274	11819	32589
2	31444	8901	25104
1	36161	6496	14400
0	35322	5654	

Stage 3 (2007):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
3	24795	10745	29990
2	31161	8820	22861
1	35838	6438	13273
0	35322	5604	

Stage 2 (2006):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
2	28328	8019	20906
1	35515	6380	11774
0	33640	5554	

Stage 1 (2005):

Age (X)	Income I(X)	Total Op Cost M(X)	Replacement Cost R(X)
1	32287	5800	11214
0	33640	6505	

TABLE XII OPTIMAL POLICY FOR MERCEDES

Stage	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
1	R	R	R	R	R
2	K	K	K	K	K
3	R	R	R	R	R
4	K	K	K	K	K
5	R	R	R	R	R
6	K	K	K	K	K
7	R	R	R	R	R
8	K	K	K	K	K
9	R	R	R	R	R
10	K	K	K	K	K
11	R	R	R	R	R
12	K	K	K	K	K
13	R	R	R	R	R
14	K	K	K	K	K
15	K	K	K	K	K

TABLE XIII OPTIMAL POLICY FOR VOLVO

Stage	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
1	R	R	R	R	R
2	K	K	K	K	K
3	R	R	R	R	R
4	K	K	K	K	K
5	R	R	R	R	R
6	K	K	K	K	K
7	R	R	R	R	R
8	K	K	K	K	K
9	R	R	R	R	R

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10	K	K	K	K	K
11	R	R	R	R	R
12	K	K	K	K	K
13	R	R	R	R	R
14	K	K	K	K	K
15	K	K	K	K	K

TABLE XIV OPTIMAL POLICY FOR DAEWOO

Stage	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
1	R	K	K	R	K
2	K	R	R	R	R
3	R	K	K	R	K
4	R	R	R	R	R
5	K	K	K	R	K
6	R	R	R	R	R
7	R	K	K	K	K
8	K	K	K	K	R
9	R	K	K	K	K
10	K	K	K	K	K
11	R	K	K	K	K
12	K	K	K	K	K
13	R	R	R	R	R
14	R	K	K	K	K
15	K	K	K	K	K

TABLE XV OPTIMAL AND ACTUAL POLICIES PROFITS

Bus Number	Optimal Policy Profit	Actual Policy Profit	% Increased
Mercedes 1	257363	160586	60.26490
Mercedes 2	263228	163108	61.38264
Mercedes 3	197055	135567	45.35617
Mercedes 4	234408	128895	81.85965
Mercedes 5	319884	184442	73.43338
Volvo 1	206821	121960	69.58101
Volvo 2	352976	207432	70.16468
Volvo 3	373110	197533	88.88489
Volvo 4	281529	124053	126.9425
Volvo 5	312298	214066	45.88865
Daewoo 1	199401	84854	134.9930
Daewoo 2	174072	130485	33.40384
Daewoo 3	174763	116653	49.81440
Daewoo 4	199630	153093	30.39786
Daewoo 5	199094	129069	54.25392

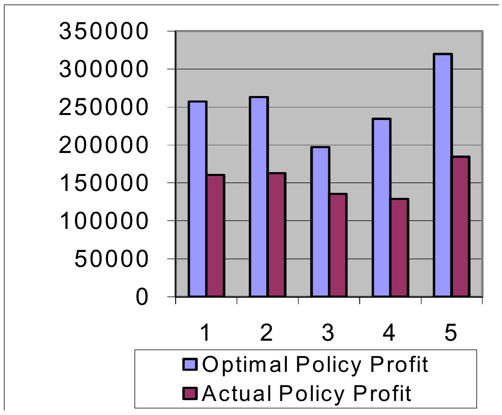

Fig. 1 Pure Profits For Optimal and Actual Policies For Mercedes Busses

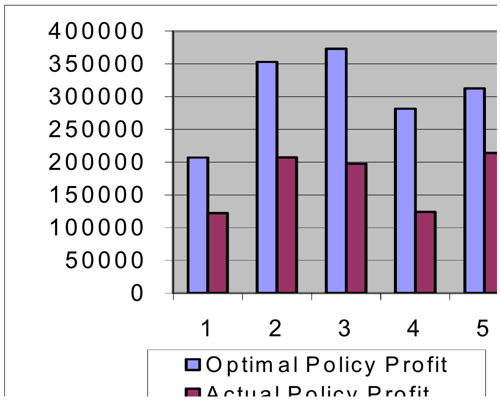

Fig. 2 Pure Profits For Optimal and Actual Policies For Volvo Busses

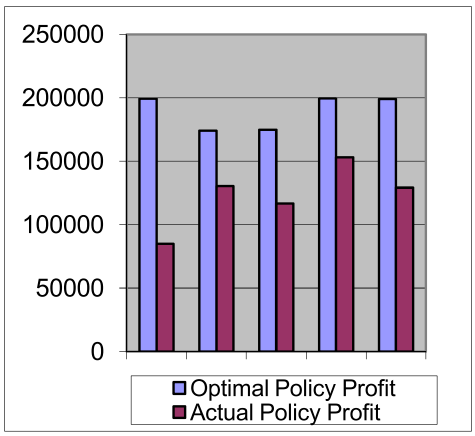

Fig. 3 Pure Profits for Optimal and Actual Policies For Daewoo Busses

REFERENCES

[1] Toshio Nakagawa and Shunji Osaki, "Discrete Time Age Replacement Policies", J. Opl. Res. Q., Vol. 28, no. 4, 1977, PP. 881-885.

[2] Y. Roll and P. Naor, "Preventive Maintenance of Equipment Subject To Continuous Deterioration and Stochastic Failure", J. Opl. Res. Q., Vol. 19, no. 1, 1968, PP. 61-71.

[3] D. J. Bartholomew, "Two Stage Replacement Strategies", J. Opl. Res. Q., Vol. 14, no. 1, 1962, PP. 71-87.

[4] A.H. Christer and W. Goodbody, "Equipment Replacement in An Unsteady Economy", J. Opl. Res. Q., Vol. 31, no. 6, 1987, PP. 497-506.

[5] Kazuhira Okumoto and Shunji Osaki, "Optimum Policies For A Stand By System With Preventive Maintenance", J. Opl. Res. Q., Vol. 28, no. 2, 1977, PP. 415-423.

[6] Hamdy A. Taha, "Operations Research An Introduction, Macmillan Publishing", New York, 7th Edition, 2003.

[7] J.S. D'Aversa and J.F. Shapiro, "Optimal Machine Maintenance and Replacement by Linear Programming and Enumeration", J. Opl. Res. Q., Vol. 29, no. 8, 1978, PP. 759-768.

[8] Richard Bronson, "Theory and Problems of Operations Research", Mc Graw Hill, 1986.

[9] Stuart E. Dreyfus and Averill M. Law, "The Art and Theory of Dynamic Programming", Academic Press, New York, 1977.

[10] C.E. Sigal , A.A. Pritsker and J.J. Solberg, "The Stochastic Shortest Route Problem", J. Opl. Res. Q., Vol. 28, no. 5, 1977, PP. 1122-1129.

[11] M.I. Khalil, "On Integer Programming Problems and Their Applications In Vehicle Rerplacement Model", M.Sc. Thesis, Helwan University, Cairo, Egypt, 1999.

[12] M.I. Khalil, "On Vehicle Replacement Models Using Large Scale Multi-Objective Fuzzy Integer Programming", Ph.D. Thesis, Helwan University, Cairo, Egypt, 2005.

[13] H.A. Abdelwali, "Study of The Parametric Dynamic Programming Applied To Vehicle Replacement Problem", M.Sc. Thesis, Minia University. Egypt, 1992.

[14] H.A. Abdelwali, "On Parametric Multi-objective Dynamic Programming With Applications To Automotive Problems", Ph.D. Thesis, Minia University. Egypt, 1997.

[15] T.A.J. Nicholson and R.D. Pullen, "Dynamic Programming Applied To Ship Fleet Management", J. Opl. Res. Q., Vol. 22, no. 3, 1971, PP. 211-220.

[16] N.A.J. Hastings, "Some Notes On Dynamic Programming and Replacement", J. Opl. Res. Q., Vol. 19, no. 4, 1968, PP. 453-464.

[17] Richard Waddel,, "A Model For Equipment Replacement Decisions and Policies, Interfaces", Vol. 13, no. 4, 1983, PP. 1-7.