

# Optimum Design of Pressure Vessel Subjected to Autofrettage Process

Abu Rayhan Md. Ali, Nidul Ch. Ghosh, and Tanvir-E-Alam

**Abstract**—The effect of autofrettage process in strain hardened thick-walled pressure vessels has been investigated theoretically by finite element modeling. Equivalent von Mises stress is used as yield criterion to evaluate the optimum autofrettage pressure and the optimum radius of elastic-plastic junction. It has been observed that the optimum autofrettage pressure increases along with the working pressure. For two different working pressures, the effect of the ratio of outer to inner radius ( $b/a=k$ ) value on the optimum autofrettage pressure is also noticed. The Optimum autofrettage pressure solely depends on  $K$  value rather than on the inner or outer radius. Furthermore, percentage reduction of von Mises stresses is compared for different working pressures and different  $k$  values. Maximum von Mises stress developed at different autofrettage pressure is equated for elastic perfectly plastic and elastic-plastic material with different slope of strain hardening segment. Cylinder material having higher slope of strain hardening segment provides better benedictions in the autofrettage process.

**Keywords**—Autofrettage, elastic plastic junction, pressure vessel, von Mises stress.

## I. INTRODUCTION

HIGH pressure vessels are widely used in food sterilization, hyper-sonic (up to Mach 16) wind tunnels, power generation, water jet cutting, military equipment, fluid transmission and storage applications. Therefore, the prevention of pressure vessel failure to enhance safety and reliability has received considerable attention. To contain a high pressure would typically require a very thick tube wall due to the concentration of tensile hoop stresses at the inner diameter (ID). The magnitude of pressure is also limited by the material yield stress, which must not be exceeded in normal use. On the other hand, worldwide materials scarcity and higher costs have lead researchers attention to the elastic-plastic approach which offers more efficient use of materials. Autofrettage is a well known elastic-plastic technique to

increase the pressure capacity of thick-walled cylinders. In this technique, the cylinder is subjected to internal pressure to cause plastic expansion of some or the entire tube wall. The

pressure is then released and residual compressive hoop stresses are created in the near-bore region while residual tensile hoop stresses are created in the outer-bore region. The resulting residual stress leads to a decrease in the value of maximum von-mises stress in the next loading stage. That means the increase in the pressure capacity of the cylinder in the next loading stage [1] - [2]. A key problem in the analysis of autofrettage process is to determine the optimum autofrettage pressure and corresponding radius of elasto-plastic boundary where the maximum equivalent von mises stress in the cylinder becomes minimal. The analysis of residual stresses and deformation in an autofrettaged thick-walled cylinder has been given by Chen [3] and Franklin and Morrison [4]. Harvey's report [6] gave only a concept about autofrettage but detail result was missing. Brownell and Young [7], and Yu [8] proposed a repeated trial calculation method to determine the optimum radius of elastic plastic junction which was a bit too tedious and inaccurate; moreover this method is based on the first strength theory which is in agreement with brittle materials. But pressure vessels are generally made from ductile materials [9]-[10] which are in excellent agreement with the third or the fourth strength theory [11]-[13]. The graphical method presented by Kong [12] was also a bit too tedious and inaccurate. Based on the third and the fourth strength theory, Zhu and Yang [14] presented an analytic equation for optimum radius of elastic-plastic juncture,  $opt r$  in autofrettage technology. Ghomi & Majzoubi [15] proposed set of equations that used for determining optimum radius of elastic plastic junction. In the present work, Zhu & Yang's equations based on fourth strength theory are employed to predict the optimum autofrettage radius. To compute optimum autofrettage pressure ANSYS software is employed for numerical simulation.

## II. ANALYTICAL APPROACH

Bi-Linear elasto-plastic behavior has been considered in this work.

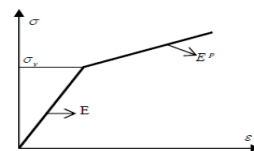


Fig. 1 Bi-linear stress strain curve

The model, shown in fig. 1 is described as follows:

$$\sigma = \sigma_y + E_p \epsilon \quad (1)$$

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In which  $\sigma$  is the effective stress,  $\sigma_y$  is the initial yield stress,  $E^p$  is the slope of the strain hardening segment of the stress strain curve, and  $\epsilon$  is the effective strain.

#### A. Residual Stress Pattern

To observe the residual stress pattern in autofrettage process, a sample cylinder with internal radius  $a = 0.01$  m, and external radius,  $b = 0.02$  m has been considered. Material properties of this cylinder is summarized in table 1.

TABLE I  
MATERIAL PROPERTIES

Mat.	$\sigma_y$ (MPa)	E (GPa)	$E^p$ (GPa)	$\nu$
Al	90	72	1.75	0.33

This cylinder is subjected to an internal pressure (known as autofrettage pressure) so that its wall becomes plastic Up to  $r/\text{inner radius} = 1.56$  and the pressure is then released. Ghomi & Majzoubi [16] proposed set of equations for determining radial and hoop stresses at different location along the cylinder wall in autofrettaged cylinder. By using the equations the resulting residual stress pattern is shown in fig. 2:

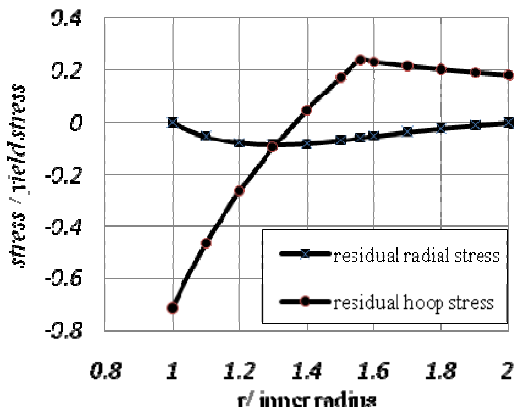


Fig.2 Residual Stress Distribution

From the fig. 2, it is observed that residual compressive hoop stress occurs in near-bore region, while residual tensile hoop stress occurs at outer portion. The resulting residual compressive hoop stress leads to a decrease in the maximum value of the von mises stress in the next loading stage.

#### B. Comparison of Stresses With And Without Autofrettage

By using Lamé's equation for thick-walled cylinder, the stress pattern is obtained for non autofrettaged cylinder. If the same cylinder undergoes autofrettage process then the overall stress pattern will change, which is shown in fig. 3. Here the working pressure is 46 MPa.

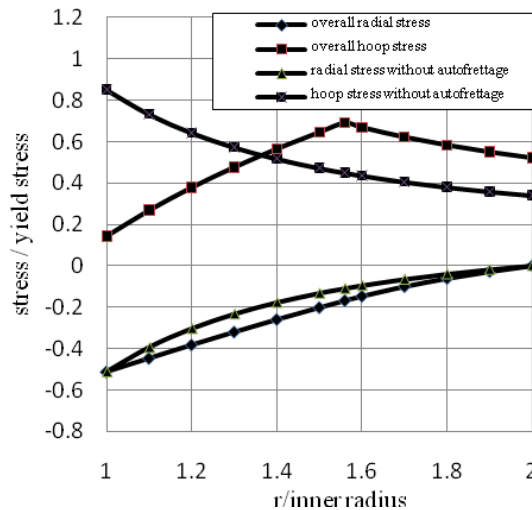


Fig. 3 Comparison of Stresses with autofrettage and without autofrettage.

From fig. 3, following points are observed:

1. Because of residual compressive hoop stress at inner bore, the resultant hoop stress becomes significantly lower in the autofrettaged cylinder than the original hoop stress developed at the same cylinder.
2. Radial stress doesn't vary significantly after autofrettage process.
3. The cylinder which undergoes autofrettage process has maximum stress occurring at the point of elasto-plastic junction rather than inner bore.

#### C. Optimum Elastic Plastic Radius

For different radius of elasto plastic junction developed Von mises stresses are calculated using Ghomi & Majzoubi's [16] proposed set of equations. Then the results are shown in fig. 4.

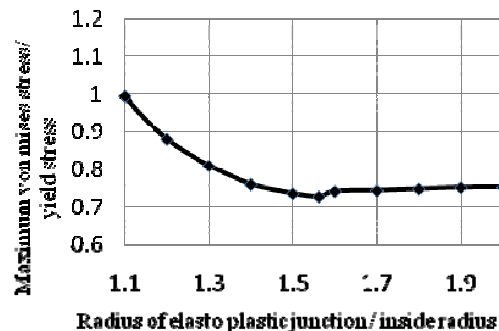


Fig. 4 Maximum Von Mises Stress at Different Radius of Elasto Plastic Junction

As the fig. suggests, Maximum von mises stress starts to decrease as the radius of elastic plastic junction increases. After attaining a certain value of elastic plastic junction, maximum von mises stress started to increase. The point at which maximum von mises stress is minimum is the optimum radius of elasto-plastic junction.

#### D. Zhu & Yang Model for Optimum Elastic Plastic Radius

Zhu & Yang [14] has developed an equation for determining  $opt\ r$  which can be calculated just using a pocket calculator.

$$(a) \text{ based on third strength theory (Tresca-yield)} \\ r_{opt} = a \exp(p_w / \sigma_y) \quad (2)$$

$$(b) \text{ based on fourth strength theory (von Mises)} \\ r_{opt} = a \exp(\sqrt{3}p_w / 2\sigma_y) \quad (3)$$

Ghomi & Majzoobi deduced  $r_{opt}$  by using MATLAB.

For determining optimum radius of elastic plastic junction "Ghomi & Majzoobi's model" and "Zhu & Yang's model" are compared. It has been observed that these values vary between 5-7% only.

#### Sample calculation:

In this case study  $a=0.01\text{m}$ ,  $b=0.02\text{m}$ , working pressure  $p_w=46\text{MPa}$ .

From Zhu & Yang's model

Based on third strength theory,  $r_{opt}=0.01667\text{m}$ .

Based on fourth strength theory,  $r_{opt}=0.0156\text{m}$ .

From Ghomi & Majzoobi's model (fig. 4), it is observed that  $r_{opt}$  is occurring in between 0.015 to 0.016 m. Indeed there is no significant variation between these two models.

Zhu Yang model based on fourth strength theory is considered for calculating  $opt\ r$  that simplifies the calculation

### III. NUMERICAL RESULTS

Commercially available software ANSYS 10.0 has been employed for finite element modeling of the Autofrettaged vessel. The element Quad 4 Node PLANE 42 with the capacity of elastic and plastic material modeling has been used for the modeling [5].

Single cylinder with the dimensions;  $a=0.1\text{m}$ ,  $b=0.2\text{m}$  and an elastic plastic material's model with  $\sigma_y = 800\text{MPa}$ ; Modulus of elasticity  $E = 207\text{GPa}$ ; Slope of the strain hardening segment  $E_p = 4.5\text{GPa}$ ;  $\nu = 0.29$ ; were used for numerical modeling. The two pressure limits  $Py1$  and  $Py2$  can be computed as follows [1 & 17]: Here  $Py1$  is the pressure at which yielding commences at inner surface and  $Py2$  is the pressure at which plasticity has spread throughout the cylinder.

$$Py1 = \sigma_y (1 - 1/k^2)^{1/3} \\ = 347\text{MPa}$$

$$Py2 = \sigma_y \ln(k) \\ = 555\text{MPa}$$

If the autofrettage pressure is lower than 347MPa then there will be no autofrettage effect. If the pressure is higher than 555MPa then there will be converse effect. That means the pressure capacity of the cylinder will decrease instead of increasing.

In this paper, effects of following factors are considered in autofrettage process. The considered factors are 1. Working pressure; 2. Value of  $k$  ( $b/a$ ); 3. Material model (elastic perfectly plastic and elastic plastic with different slope of

strain hardening segment); 4. Autofrettage stages.

#### A. Working Pressure

The cylinders were subjected to autofrettage pressures ranging from 350 MPa to 650 MPa. After removing the autofrettage pressure (AP), the cylinders were subjected to the working pressures of 100, 200 and 300 MPa. From the numerical simulation with ANSYS software, the curve of the von-Mises stress distribution was obtained for each autofrettage and working pressure (WP). From the curve, the values of maximum von-mises stress (MVS) were extracted. These stresses were then plotted against autofrettage pressure for each working pressure. The results are shown in fig. 5.

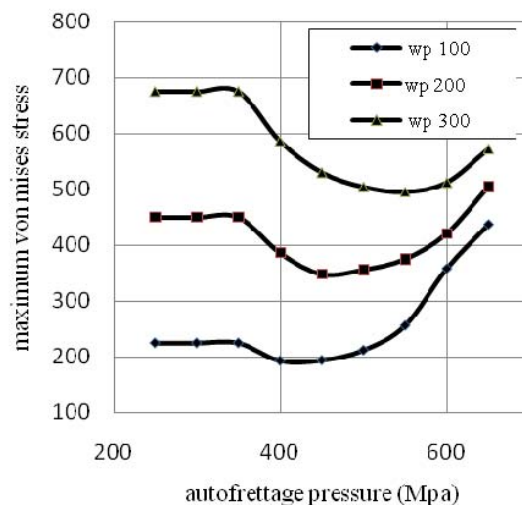


Fig. 5 Variation of MVS versus autofrettage pressure at different working pressure.

It is observed that for each working pressure, the MVS remains constant up to an autofrettage pressure which is nearly equal to  $Py1$ . The curve then begins to decline to a certain point thereafter begins to rise or remains constant. It can be seen that for all working pressures, the rising portion of the curves end at a point which is nearly equal to  $Py2$ .

From the numerical results, it can be concluded that: (i) the MVS depends on the working pressure and for any WP, the best AP lies between  $Py1$  and  $Py2$ ; (ii) for autofrettage pressures lower than  $Py1$  the MVS remains unchanged;

In the preceding case the working pressure is lower than  $Py1$  but if the working pressure itself is higher than  $Py1$  then the effect of autofrettage does not start to occur until exceeding the working pressure. In fig. 6 two working pressure of 400 and 450 MPa are considered.

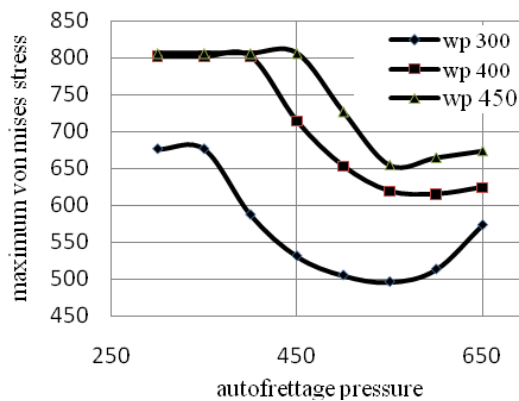


Fig. 6 variation of MVS versus autofrettage pressure at different working pressure

In case of 400 and 450 MPa the developed von mises stress starts to decrease only after exceeding working pressure. If the autofrettage pressure is lower than the working pressure then the flow stress remains unchanged thus no autofrettage effect.

For a constant value of K percent reduction of MVS is calculated for different working pressures.

TABLE II  
EFFECT OF WORKING PRESSURE AT MAXIMUM VON MISES STRESS

WP (MPa)	MVS Without autofrettage (MPa)	MVS With autofrettage (MPa)	%Reduction Of MVS
100	225	193	14.22
200	450	348	22.67
300	676	496	26.62
400	840	615	26.78

From table 2, it is observed that percent reduction of MVS is higher at higher working pressure. This means the autofrettage effect is more beneficial at higher working pressure.

#### B. Value of K

For a constant inner radius  $a=0.1\text{m}$ , cylinder with different K values ( $K=2, 2.5, 3.0$ ) are subjected to autofrettage process. Here the working pressure remains constant at  $N_w=0.25$ .

Where  $N_w = \text{Working pressure } P_w / \text{yield stress } \sigma_y$ .

For  $K=2.0$ :  $P_{y1} = 347 \text{ MPa}$ ,  $P_{y2} = 555 \text{ MPa}$ .

For  $K=2.5$ :  $P_{y1} = 388 \text{ MPa}$ ,  $P_{y2} = 733 \text{ MPa}$ .

For  $K=3.0$ :  $P_{y1} = 411 \text{ MPa}$ ,  $P_{y2} = 879 \text{ MPa}$ .

From numerical simulation, the value and the position of MVS were extracted. This MVS is then plotted against the autofrettage pressure for each K value. The results are shown in fig. 7.

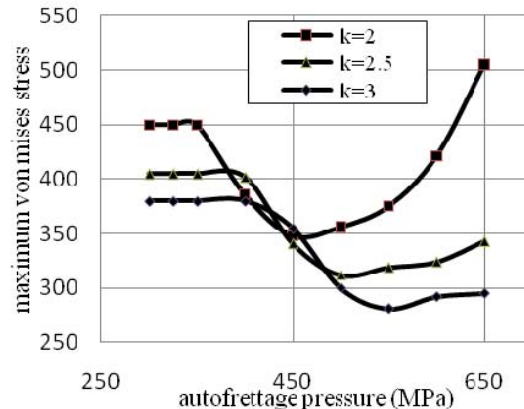


Fig. 7: Effect of K (b/a) On Optimum Autofrettage Pressure (constant inner radius = 0.1m)

As the fig. suggests, for different values of K, the optimum autofrettage pressure obtain a higher value for thicker cylinder.

For a constant working pressure ( $N_w=0.25$ ), percent reduction of MVS is calculated for different K values.

TABLE III:  
EFFECT OF K (B/A) ON MVS

K	MVS Without autofrettage (MPa)	MVS With autofrettage (MPa)	%Reduction Of MVS
2.0	450	348	22.67
2.5	405	312	22.97
3.0	380	281	26.05

From table 3 it is observed that, percent reduction of MVS is higher for higher value of K. That means the autofrettage effect is more beneficial with the increase of the thickness of the cylinder wall.

In the preceding case the inner radius of the cylinder was kept constant. Now by assuming the outer radius constant ( $b = 0.3\text{m}$ ), cylinders with three K values of 2.0, 2.5 and 3.0 were considered. From the numerical simulations, the curve of von-Mises stress distribution was obtained for each K value. The developed von mises stresses are then plotted against autofrettage pressure at fig. 8.

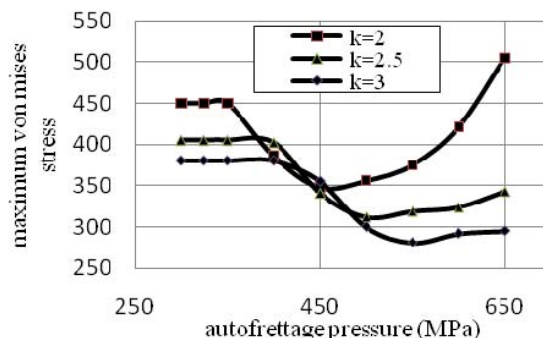


Fig. 8 Effect of K (b/a) On Optimum Autofrettage Pressure (constant outer radius = 0.3m)

From fig. 8, it is noticed that for same K value, developed MVS & the optimum autofrettage pressure is same though the value of inner and outer radius have been changed. Thus, the optimum autofrettage pressure depends on K value only. If the inner and outer radius are changed keeping the K value constant, then there will be no change in the optimum autofrettage pressure.

### C. Material model

For a particular working pressure of  $N_w=0.25$  the cylinder is subjected to autofrettage pressure ranging from 250 to 700 MPa. Here, the material of cylinder wall is considered as elastic perfectly plastic ( $E^p=0$ ) to elastic plastic with different slope of strain hardening segment ( $E^p=4.5$ ,  $E^p=30$ ,  $E^p=50$ ). From the numerical simulations, the curve of von-Mises stress distribution was obtained for each autofrettage and different material model. From the curve, the value and the position of the maximum von-mises stress (MVS) were extracted and plotted against autofrettage pressure.

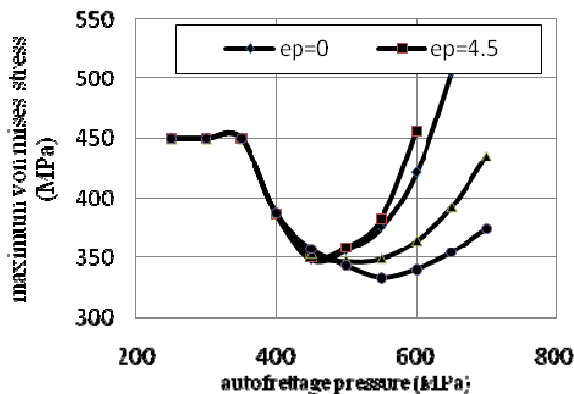


Fig. 9 Effect of Material Model on Optimum Autofrettage Pressure

From fig.9, it is observed that for autofrettage pressure between  $P_{y1}$  (347 MPa) and  $P_{y2}$  (555 MPa) von mises stress varies in nominal manner. The variation becomes significant after exceeding  $P_{y2}$ . The optimum autofrettage pressure is higher for higher value of the slope of strain hardening segment. The resultant von mises stress starts to decrease as the slope of the strain hardening segment increases. So if the cylinder wall material has higher slope of the strain hardening segment then the autofrettage process can give us much more benedictions.

### D. Autofrettage Stages

A cylinder is considered with working pressure of  $N_w=0.375$  where autofrettage pressure is 500 MPa. At first step the autofrettage is done in three loading stages and in the second step autofrettage is done in nine loading stages.

#### 3 stage autofrettage

Stage 1	Stage 2	Stage 3
500 MPa	0 MPa	300 MPa

#### 9 stage autofrettage

Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
350 MPa	0 MPa	400 MPa	0 MPa	450 MPa

Stage 6	Stage 7	Stage 8	Stage 9
0 MPa	500 MPa	0 MPa	300 MPa

From the numerical simulation, it is remarked that in both cases the MVS is 505 MPa and the stress pattern is almost similar. So there is no effect of loading stages on autofrettage process.

### IV. CONCLUSION

From the present investigation the following conclusions can be drawn:

1. In autofrettaged cylinder, maximum stress does not occur at inner bore instead of that, it occurs at the radius of elastic plastic junction. As the autofrettage pressure increases the point of MVS move towards the outer bore.
2. Optimum autofrettage pressure is not a constant value rather it depends on the working pressure. The optimum autofrettage pressure increases along with the increase of working pressure.
3. For same working pressure, increasing the ratio of outer to inner radius (K) leads to an increase in the optimum autofrettage pressure.
4. Optimum autofrettage pressure depends on K value rather than on the internal or external radius.
5. It has also been observed that if the slope of strain hardening segment increases, then the optimum autofrettage pressure also increases.
6. Because of autofrettage, percent reduction of maximum von mises stress increases for higher K value and for higher value of the slope of the strain hardening segment.
7. Number of autofrettage stages has no effect on MVS and hence on pressure capacity.

### APPENDIX

TABLE  
NOMENCLATURE

Symbol	Meaning	Unit
a	Internal radius	(m)
b	External radius	(m)
$\sigma_{\theta r}$	Residual hoop stress	(MPa)
$\sigma_{rr}$	Residual radial stress	(MPa)
$\sigma_{rf}$	Overall radial stress	(MPa)
$\sigma_{\theta f}$	Overall hoop stress	(MPa)
$P_w$	Working pressure	(MPa)
$N_w$	Working pressure $P_w$ / yield stress $\sigma_y$	Null

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