

# Optimized Fuzzy Control by Particle Swarm Optimization Technique for Control of CSTR

Saeed Vaneshani and Hooshang Jazayeri-Rad

**Abstract**—Fuzzy logic control (FLC) systems have been tested in many technical and industrial applications as a useful modeling tool that can handle the uncertainties and nonlinearities of modern control systems. The main drawback of the FLC methodologies in the industrial environment is challenging for selecting the number of optimum tuning parameters.

In this paper, a method has been proposed for finding the optimum membership functions of a fuzzy system using particle swarm optimization (PSO) algorithm. A synthetic algorithm combined from fuzzy logic control and PSO algorithm is used to design a controller for a continuous stirred tank reactor (CSTR) with the aim of achieving the accurate and acceptable desired results. To exhibit the effectiveness of proposed algorithm, it is used to optimize the Gaussian membership functions of the fuzzy model of a nonlinear CSTR system as a case study. It is clearly proved that the optimized membership functions (MFs) provided better performance than a fuzzy model for the same system, when the MFs were heuristically defined.

**Keywords**—continuous stirred tank reactor (CSTR), fuzzy logic control (FLC), membership function(MF), particle swarm optimization (PSO)

## I. INTRODUCTION

CONTINUOUS Stirred Tank Reactor (CSTR) is an important branch of studies in chemical processes. Therefore, there is a various range of researches about CSTR in the area of the chemical and control engineering. Various control approaches have been used to control the CSTR parameters. CSTR involves complex reactions with high nonlinearity, and it is very hard to be controlled by the conventional methods [11]. However, to avoid computational complexity brought in by such nonlinear controllers, FLC can be a simple and suitable alternative [10]. The main reasons for starting to appear the powerful and flexible methods are the limitations of traditional approaches in dealing with constraints [13].

Bio-inspired intelligent computing has been successfully applied to solve the complex problem in recent years [14]. The PSO algorithm and fuzzy logic expressed the high capability to overcome the issues mentioned previously [14]. Success of the fuzzy logic, remarks the robustness of this method in real environment application [8]. But, there is the need of efficient method for tuning the MFs with the aim of

minimizing the output error measures or maximizing performance indexes. The PSO optimization technique is a stochastic search through an n-dimensional problem space aiming the minimization (or maximization) of the objective function of the problem [2]. Specifically, PSO-FLC algorithm can be applied for the Concentration Control of CSTR.

This paper uses from a control strategy based on the combination of fuzzy logic and particle swarm optimization techniques. The purpose is to control the concentration of the CSTR in the presence of the set point changes. MATLAB software is used for designing and simulating those controllers and simulating in. The performance of the proposed controllers has been considered based on the sum of the square error (SSE). The results clearly show that the PSO-FLC control strategy gives an acceptable performance with respect to the functional changes of the process. Furthermore, fuzzy based structure strategy gives more flexibility and precise behavior in control action in comparison to the least square based approach.

## II. FUZZY CONTROLLER ALGORITHM

Fuzzy logic controllers (FLCs) are organized based on skilled knowledge that is in the form of rule-based behavior. In general the FLC rules are explicit in the form: If input 1 is A and input 2 is B then output is C.

where antecedents A and B are declared by MFs [4].

There are two types of expressions for consequent C [7]. In Tagaki-Sugeno-type FLCs, the C is expressed as a linear combination of all inputs. On the other hand, if a Mamdani-type of FLC is used, C is expressed by a set of MFs[3]. The procedure that is used to calculate the overall control action in FLC is determined by different type of defuzzification process. In general, a centre of area (CoA) method is usually used, where the output  $u^*$  is computed as [5,8]:

$$u^* = \frac{\int u m_o(u) du}{\int m_o(u) du} \quad (1)$$

The fundamental FLC loop is shown in Fig. 1. It consists of three major serial steps, namely Fuzzification, Inference engine and Defuzzification [9]. Fuzzifications convert a crisp value (real-value) into a member of fuzzy sets, while defuzzification converts the fuzzy output determined by the inference engine into a crisp value [6].

Saeed Vaneshani. Instrumentation and Automation Department, Petroleum University of Technology, Ahwaz, IRAN. (phone:+98-939-988-6330; e-mail: saeed.vaneshani@yahoo.com).

Hooshang Jazayeri-Rad. Instrumentation and Automation Department, Petroleum University of Technology, Ahwaz, IRAN. (phone:+98-916-968-6485; e-mail: jaz\_rad\_h@yahoo.com)

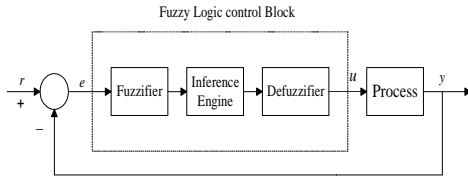


Fig. 1 Block diagram for the FLC algorithm

### III. PARTICLE SWARM OPTIMIZATION

An optimization method that finds the optimal solution using a population of particles [2] is PSO algorithm. Each swarm of PSO is a solution in the solution space. The algorithm can be explained as follows [1]:

- Each individual particle  $i$  has the following properties: A current position in search space,  $x_{id}$ , a current velocity,  $p_{id}$ , and a personal best position in search space,  $p_{id}$ .

- The personal best position,  $p_{id}$ , corresponds to the position in search space where particle  $i$  presents the smallest error as determined by the objective function  $f$ , assuming a minimization task.

- The global best position marked by represents the position yielding the lowest error amongst all the  $p_{gd}$ .

During the iteration every particle in the swarm is updated using the following two equations:

$$V_{id}(t+1) = w \cdot V_{id}(t) + c_1 \cdot r_1 \cdot (p_{id} - X_{id}(t)) + c_2 \cdot r_2 \cdot (p_{gd} - X_{id}(t)) \quad (2)$$

$$X_{id}(t+1) = X_{id}(t) + V_{id}(t+1) \quad (3)$$

where  $V_{id}(t+1)$  and  $V_{id}(t)$  are the updated and current particles velocities, respectively,  $X_{id}(t+1)$  and  $X_{id}(t)$  are the updated and current particles positions, respectively,  $c_1$  and  $c_2$  are two positive constants and  $r_1$  and  $r_2$  are normalized unit random numbers within the range  $[0,1]$ , and  $w$  is the inertia weight.



Fig. 2 Flow chart depicting the General PSO Algorithm

### IV. OPTIMAL FLC DESIGN

The approach of using a PSO for MF tuning in FLC is shown in Fig. 3. In the proposed PSO process, each particle is shaped to represent the MF parameters of the FLC's inputs and outputs. As the aim of the PSO is to minimize the control error of the FLC, the objective function of PSO is defined as:

$$f(x(k)) = \sum_{t=0}^{t_f} \varepsilon^2 \quad (4)$$

Where  $t_f$  is the total running time of the FLC,  $\varepsilon$  is the Control error.

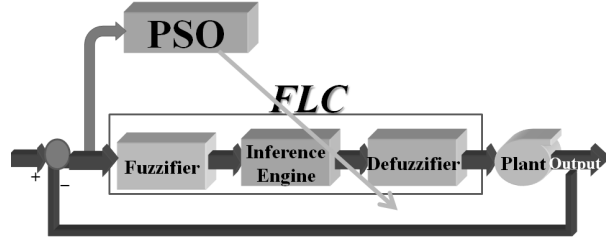


Fig. 3 The PSO-FLC method

The model consists of multi-input single-output (MISO) system with  $n$  number of inputs. The number of fuzzy sets for the inputs are  $m_1, m_2, \dots, m_n$ .

There are some assumptions in the model formulation. These assumptions must be defined and available in advance as a basic integration of this hybrid algorithm. The assumptions are listed as below:

- (i) Gaussian membership functions were used for input and output variables.

- (ii) Complete rule-base was considered. A rule-base is considered complete when all possible combinations of input membership functions of all the input variables participate in fuzzy rule-base formation.

The integration between optimization algorithm and fuzzy logic problem is as follow:

- (i) The parameters are the mean value and standard deviation of each fuzzy membership function.

- (ii) These parameters act as particles and looking for the global best fitness.

- (iii) It starts with an initial set of parameters.

- (iv) After the parameters had been adjusted using optimization method, this parameter will be used to check the performance of the fuzzy logic.

- (v) This process is repeated until the goal is achieved.

The optimization method as shown in Fig. 4 starts with the initial set of parameters and employs the fitness function to obtain new values for the parameters of the membership function. These new values will be used in the case study considered in this paper.

These particle dimensions represent fuzzy membership function parameter values. The first column shows the input and output variables. All input and output MFs become different depending on their new position. The particle size for representing the Gaussian membership functions of input and output variables for a model is given by (5) and (6). Particles Dimension for Input Variables:

$$\sum_{i=1}^n (2m_i) \quad (5)$$

where,  $n$  - number of input variables and  $m$  - number of fuzzy sets.

Particles Dimension for Output Variable:

$$\sum_{i=1}^n (2t) \quad (6)$$

where,  $n$  - number of output variables and  $t$  - number of fuzzy sets. The particle dimensions required for encoding the fuzzy model can be obtained in table 1.

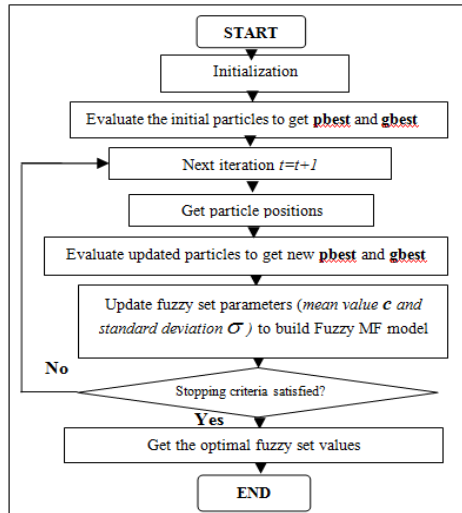


Fig. 4 Flowchart of Particle Swarm Optimization to adjust Fuzzy Membership Function

TABLE I  
PARTICLE DIMENSION FOR REPRESENTING FUZZY MODEL

	c	$\sigma$	c	$\sigma$	...	c	$\sigma$	
Input var #1	X11	X11	X12	X12	...	X1m	X1m	2m1
Input var #2	X21	X21	X22	X22	...	X2m	X2m	2m2
...	...	...	...	...	...	...	...	...
Input var #n	Xn1	Xn1	Xn2	Xn2	...	Xnm	Xnm	2mn
output variabl e	Y1	Y1	Y2	Y2	...	Yt	Yt	2t

These particle dimensions represent fuzzy membership function parameter values. The first column shows the input and output variables. In this column, number represents the input variable. Because MISO system was considered as model only one output variable is used here Because MISO system was considered as model.

First row describe mean value and standard deviation of each membership function. The number of membership function represent until  $m$  variables. In the last column,  $2m$  can be noted which means that 2 positions had been used and unlimited until  $m$  variables.

## V. PROCESS DESCRIPTION AND MODEL

To demonstrate the effectiveness of the proposed PSO-FLC method, a nonlinear system is simulated. In particular, the case considered in this paper is a nonlinear CSTR benchmark model as shown in Fig. 5.

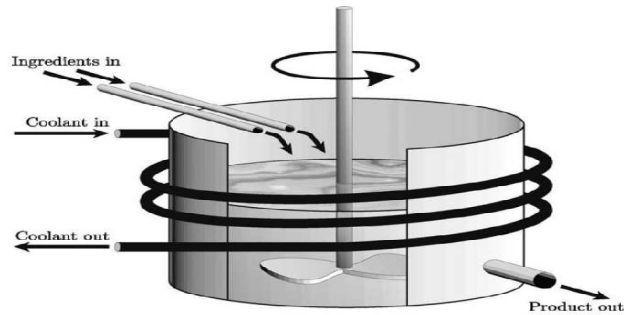


Fig. 5 The schematic of CSTR[10]

A common chemical system in many chemical processing plants, known as a continuous stirred tank reactor (CSTR), is utilized as a suitable test for PSO-Fuzzy control. within the CSTR two chemicals are mixed, and react to produce a product compound with concentration  $C_a(t)$ . The temperature of the mixture is  $T(t)$ . A schematic representation of the system is shown in Fig. 1. The reaction is exothermic, producing heat reduce the reaction rate. By introducing a coolant flow rate  $q_c(t)$ , the temperature can be altered and hence the product concentration controlled. This system can be described by following nonlinear simultaneous equations which effectively combine the laws of chemical reaction and thermodynamic:

$$C_a(t) = q(C_{a0} - C_a(t))/V - k_0 C_a(t) e^{E/RT(t)} \quad (7)$$

$$T(t) = q(T_0 - T(t))/V + k_1 C_a(t) e^{E/RT(t)} + k_2 q C(t).$$

$$.(1 - e^{k_3/q_c(t)})(T_{c0} - T(t)) \quad (8)$$

TABLE II  
THE CSTR PARAMETERS

parameter	value
Process flow rate, $q$ ( $\text{l min}^{-1}$ )	100
Feed Concentration, $C_{a0}$ ( $\text{mol l}^{-1}$ )	1
Feed temperature, $T_0$ (K)	350
Inlet coolant temperature, $T_{c0}$ (K)	350
CSTR volume, $V$ (l)	100
Heat transfer term, $hA$ ( $\text{cal min}^{-1} \text{K}^{-1}$ )	$7 \times 10^5$
Reaction rate constant, $k_0$ ( $\text{min}^{-1}$ )	$7.2 \times 10^{10}$
Activation energy term, $\frac{E}{R}$ (K)	$10^4$
Heat of reaction, $\Delta H$ ( $\text{cal mol}^{-1}$ )	$-2 \times 10^5$
Liquid densities, $\rho, \rho_c$ ( $\text{gl}^{-1}$ )	$10^3$
Specific heats, $C_p, C_{pc}$ ( $\text{cal g}^{-1} \text{K}^{-1}$ )	1

## VI. SIMULATION AND RESULTS

To solve the CSTR model equations numerically an equation based simulator should be used, in this article the MATLAB simulink environment is carried out.

Fig. 6 shows the feedback control system used to construct the control system. In this diagram  $y(k)$  is the output signal of the plant,  $g(k)$  is the set point signal, and  $e(k)$  is the error. It was implemented in MATLAB where controllers were designed independently to follow the input as close as possible.

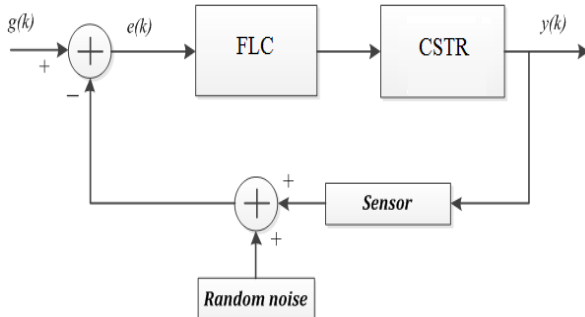


Fig. 6 Block diagram of the fuzzy control systems

As it shown in Fig. 5, uncertainty is added to the system's output as random noise with normal distribution. The reference input is stable and without noise but the feedback at the summing junction is noisy since we introduced deliberately noise for simulating the overall existing uncertainty in the system. In consequence, the controller's inputs  $e(k)$  (error) contain uncertain data.

The FLC outputs are equal to required temperature changes of coolant operator valve to achieve desired concentration. To control the process, two Fuzzy controllers (Conventional FLC, PSO tuned FLC) is used and the results are compared.

Gaussian shapes are considered for the membership functions. For such functions seven input and output are used with the locations and centers that are shown in Fig. 7 and 8. Gaussian shape is selected because it is a continuous function and can be easily coded in a digital computer. The number of fuzzy sets is chosen arbitrary, however increasing them will increase the number of control rules at the little improvement benefit. The relative location of their center will be adjusted automatically using our proposed tuning method as discussed later.

#### A. Conventional FLC

The initial MFs of the FLC for the inputs and outputs are shown in Fig. 6 and 7, seven fuzzy Gaussian MFs are defined: NB, NM, NS, ZE, PS, PM, and PB. The universe of discourse for these MFs are in the range of  $[-1, 1]$  and their initial means are -1, -0.66, -0.33, 0, 0.33, 0.66 and 1 respectively. The initial standard deviation for all MFs is 0.14. Seven fuzzy rules have been considered to construct the fuzzy rule base and are taken follows. Fuzzy controllers have been designed and tested, based on Mamdani inference mechanism.

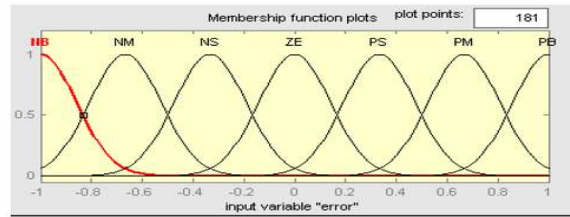


Fig. 7 input MFs for conventional FLC

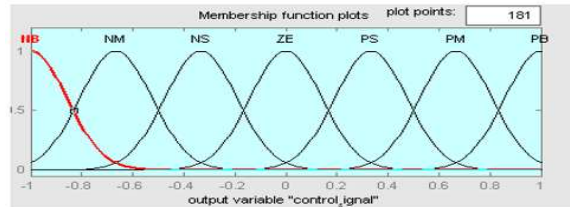


Fig. 8 output MFs for conventional FLC

Fig. 8 represents the schematic of the CSTR simulated model implemented in the MATLAB Simulink environment.

Seven fuzzy rules have been considered to construct the fuzzy rule base. These rules are as follows:

1. If (e is NB) then (valve is PB)
2. If (e is NM) then (valve is PM)
3. If (e is NS) then (valve is PS)
4. If (e is ZE) then (valve is ZE)
5. If (e is PS) then (valve is NS)
6. If (e is PM) then (valve is NM)
7. If (e is PB) then (valve is NB)

For the FLC, the minimum operator is used as the  $\mu$  norm, and centroid method for defuzzification.

To evaluate the merit of each fuzzy controller, Sum of the Square Error (SSE) that is given by equation (9) is used as performance criteria.

$$F = \sum_{j=1}^N [e(j)]^2 \quad (9)$$

where  $e$  the difference between the set point and the actual is output at the  $j$ th sampling, and,  $N$  is the number of sampling instants.

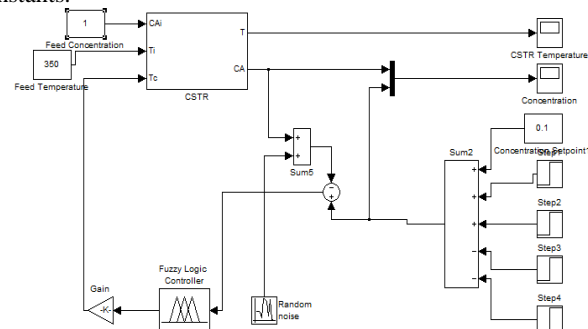


Fig. 9 CSTR simulated model in Simulink with PSO- FLC

### B. PSO tuned FLC

All of The MFs used in this FLC are Gaussian forms. The parameters that define the MFs are the mean value  $c$  and the standard deviation  $\sigma$  of each MF. The membership function is defined as:

$$f_{mf}(x) = e^{-(x-c)^2/(2\sigma^2)} \quad (10)$$

Fig. 10 and 11, show the optimized MFs of FLC respectively. This criterion is used by PSO to evaluate the fitness of each candidate solution. This criterion is used by PSO to evaluate the fitness of each candidate solution. Since there are 7 input MFs and 7 output MFs, there are a total of 28 parameters that need to be tuned. Therefore, in the PSO, each particle is to have 28 dimensions. It is set that there are 50 particles in the swarm and the total searching iterations are set to be 200. The inertia  $w$  factor was set to be 0.5 and weighting factors  $c_1$  and  $c_2$  were set to be 1.2 and 0.8, respectively (see Table III). The objective function that evaluates the fitness of each particle was defined as (9). Therefore, after the proper tuning of the MFs, the FLC will have a minimized control error. Table IV shows the MF parameters before and after the PSO tuning process.

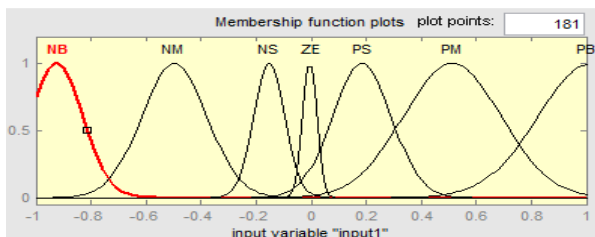


Fig. 10 Optimized input MFs for PSO- FLC

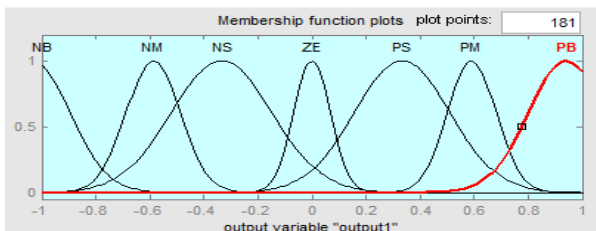


Fig. 11 Optimized output MFs for PSO- FLC

TABLE III  
PSO PARAMETERS FOR Csth PROBLEM

Parameter	Value
$C_1$	1.2
$C_2$	0.8
Inertia $w$ factor	0.5
Number of particle	50
Searching iterations	200
Fitness	SSE

TABLE III  
MF PARAMETERS BEFORE AND AFTER THE PSO

MF output	Before PSO		After PSO	
	Mean(c)	STD( $\sigma$ )	Mean(c)	STD( $\sigma$ )
NB	-1	0.14	-1.03	0.14
NM	-0.66	0.14	-0.58	0.1
NS	-0.33	0.14	-0.33	0.18
ZE	0	0.14	-0	0.06
PS	0.33	0.14	0.33	0.17
PM	0.66	0.14	0.59	0.09
PB	1	0.14	0.94	0.14

TABLE IV  
SSE FOR THE FLC AND PSO- FLC

Control Structure	SSE
conventional FLC	27.23
tuned FLC(PSO-FLC)	22.68

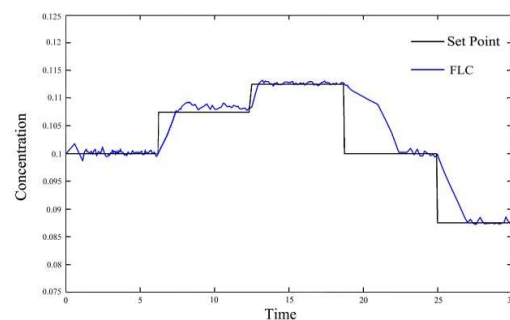


Fig. 13 Step response for FLC

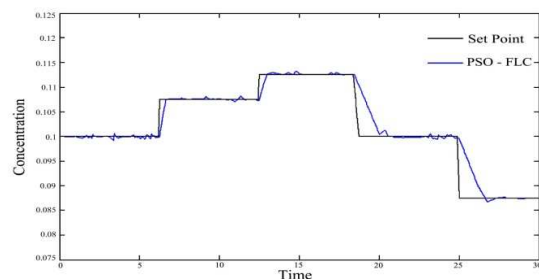


Fig. 14 Step response for PSO-FLC

Comparison between the control results obtained from FLC and PSO-FLC (in Fig. 13 and 14 respectively) clearly shows that PSO-FLC has more accurate and acceptable results rather than conventional FLC in control of the concentration of a CSTR in presence of additive random noise. Therefore, it is clear that the PSO-FLC control can achieve the desired output better than conventional FLC. The superior of PSO-FLC over

than FLC also can be seen in Table V where the sum of square error (SSE) of PSO-FLC is less than conventional FLC.

## VII. CONCLUSION

The results show clearly, that the optimized FLC has better performance in compare with a conventional controller in presence of additive random noise. The concentration of a CSTR is controlled by means of two different fuzzy controllers. According to the results of the computer simulation, the FLC with PSO algorithm is better than the conventional FLC without PSO algorithm. The major disadvantage of the fuzzy controller is lacking analytical technique design (the selection of the rules, the membership functions and the scaling factors). Therefore the PSO-FLC controller gives robustness improvement and very good results in compare with the conventional FLC controller.

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