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Optimization Using Simulation of the Vehicle Routing Problem

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***Abstract*—**A key element of many distribution systems is the routing and scheduling of vehicles servicing a set of customers. A wide variety of exact and approximate algorithms have been proposed for solving the vehicle routing problems (VRP). Exact algorithms can only solve relatively small problems of VRP, which is classified as NP-Hard. Several approximate algorithms have proven successful in finding a feasible solution not necessarily optimum. Although different parts of the problem are stochastic in nature; yet, limited work relevant to the application of discrete event system simulation has addressed the problem. Presented here is optimization using simulation of VRP; where, a simplified problem has been developed in the ExtendSim^{TM} simulation environment; where, ExtendSim^{TM} evolutionary optimizer is used to minimize the total transportation cost of the problem. Results obtained from the model are very satisfactory. Further complexities of the problem are proposed for consideration in the future.

***Keywords*—**Discrete event system simulation, optimization using simulation, vehicle routing problem.

I. Introduction

THE vehicle routing problem (VRP) is one of the most intensively studied problems in operations research, and this is due to its structural charm as well as practical relevance. Many papers have been devoted to the development of optimization[1-3]and approximation algorithms for vehicle routing and scheduling problems[4, 5]. This interest is due to the practical importance of effective and efficient methods for handling physical distribution situations as well as to the intriguing nature of the underlying combinatorial optimization models.The standard Vehicle Routing Problem (VRP)is an extension of the Travelling Salesman Problem (TSP), introducing demand at the customers and a fleet of vehicles, each having the same fixed capacity [6, 7].

Numerous methods have been proposed to solve the TSP. Finding the optimal route for a particular problem has not been practical for such problems when they contain many points or require a solution to be found quickly. Computational time on the fastest computers for optimization methods has been too long for many practical problems. Cognitive, heuristic, or combination heuristic-optimization solution procedures have been good alternatives [8].

The aim of this work is threefold; to present a new mathematical formulation of the VRP problem that uses fewer decision variables, to show how to model the TSP problem as a discrete event simulation model, and to employ the developed simulation model in finding the optimum/near optimum solution of the problem.

This paper is organized as follows: in Section II, the basic concepts of VRP and the solution techniques found in literature will be briefly discussed. In Section III, proposed problem formulations will be presented followed by the simulation model development and optimization using simulation in sections IV and V. Finally, in section VI, the conclusions drawn from this work are presented.

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II. Literature Review

A. Problem Types

The most addressed problem types in literature related to this work are:

1. The Travelling Salesman Problem

The TSP is one of the simplest, but still NP-hard, routing problems. In this problem, a set of cities to be visited and a way to measure the distances between any 2 cities is given. The tour is not complete until the vehicle returns back to its starting point (depot). The objective is to find the shortest tour that visits all cities exactly once [8, 9].

2. The m-Travelling Salesman Problem

The *m*-TSP is a generalization of the TSP that introduces more than one salesman. In the *m*-TSP;*n* citiesare given, *m* salesmen, and one depot. All cities should be visited exactly once by one of the *m*salesmen. Each tour must start and end at the depot and salesmen are not allowed to be unassigned to cities [9].

3. The Vehicle Routing Problem

The VRP calls for the determination of a set of minimum cost routes to be performed by a fleet of vehicles to serve a given set of customers with known demands; where, each route originates and terminates at a single depot. Each customer must be assigned to only one vehicle and the total demand of all customers assigned to a vehicle does not exceed

its capacity [10].

4. The Vehicle Routing Problem with Time Windows

This VRPTWis an extension of the basic VRP in which vehicle capacity constraints are imposed and each customer *i* is associated with a time interval [a_i , b_i], called a time window, during which service must begin. In any vehicle route, the vehicle may not arrive at customer *i* after $b_i$to begin service. If a vehicle arrives before a_i, it waits[10].

In these problems, the special aspect of routing is blended with the temporal aspect of scheduling which must be performed to ensure the satisfaction of the time window constraints[11].

B. Solution Approaches

Vehicle Routing Problems have been studied extensively in the Operational Research literature. A good overview of exact and heuristic methods, together with descriptions of some application areas is to be found in *The vehicle routing problem* book by Toth and Vigo [12].

1. Exact

Vehicle routing problems are classified as NP-hard optimization problems; where, solving this class ofproblemsto optimality has proven to be difficult to achieve. Only moderately sized problems can be solvedto optimality consistently[9].

Exact methods guarantee that the optimal solution is found if the methodis given sufficiently time and space. These algorithms have been used for solving the vehicle routing problem undercapacity constraints and the vehicle routing problem with time windows. These were addressedseveral times in literature with their mathematicalprogramming formulationas in [1, 2].

2. Approximate

A heuristic method is a procedure that is likely to discover a very good feasible solution, but not necessarily an optimal solution, for the specific problem being considered. No guarantee can be given about the quality of the solution obtained, but a well-designed heuristic method usually can provide a solution that is at least nearly optimal or conclude that no such solution exist. Heuristic methods are based on relatively simple common-sense ideas for how to search for a good solution. Heuristics tends to be ad hoc in nature. That is, each method usually is designed to fit a specific problem type rather than a variety of applications [13].

Several families of heuristics have been proposed for the VRP. These can be broadly classified into two main classes: *classical heuristics* and *metaheuristics*.

- Classical Heuristics

Classical heuristics can be classified into three categories: constructive heuristics, two phase heuristics and improvement methods [14].

Some of the well-known VRP heuristics found in literature are: the insertion algorithm [12, 15], the sweep algorithm [16], the Clarke and Wright algorithm, the Time-Oriented Nearest-Neighbor Heuristic, and the Time-Oriented Sweep Heuristic [17].

- Metaheuristics

A metaheuristic is a general solution method that provides both a general structure and strategy guidelines for developing a specific heuristic method to fit a particular kind of problem.The role of metaheuristics is to deal with problems that are too large and complicated to be solved by exact algorithms[13].

Metaheuristics are general solution procedures that explore the solution space to identify good solutions and often embed some of the standard route construction and improvement heuristics.In a major departure from classical approaches, metaheuristics allow deteriorating and even infeasible intermediate solutions in the course of the search process[5].

The most discussed metaheuristics approaches in literature for the VRP are: tabusearch [6, 10, 14, 18], genetic algorithms [5, 14, 19], and simulated annealing [5, 14].

3. Simulation

With the growing sizeof problems and the increasing uncertainties in the distribution environment, simple focusing on algorithm research is difficult to obtain satisfactory solution of VRP. Simulation can effectively handle the problem of complex systems[20]. However, as compared with other Operations Research fields, where simulation techniques are widely used, it is still in an early stage of implementation in the vehicle routing problem [21].

III. Mathematical Model Development

The traveling salesman problem (TSP) and the vehicle routing problem (VRP) are among the most widely studied combinatorial optimization problems. Both problems, as well as their numerous extensions, deal with optimally visiting customers from a central depot [14, 22].

In order to simplify the problem, if it isassumed that the fleet consists of *m*vehiclesof sufficiently large capacity, then the problem reduces to the *m*-Travelling Salesman Problem (*m*-TSP). Inits simplest version, if it is further assumed that there is only one vehicle of very large capacity, then theproblem reduces to the well-known Travelling Salesman Problem (TSP). It is for this reason that theformulation for TSP is taken as a core model for the development of the mathematical formulations for more complicated cases. Therefore, most of the mathematical formulations of VRP are variants and/orextensions of the well-known TSP [23].

TSP is a VRP involving only one uncapacitated vehicle, while the *m*-TSP involves *m*uncapacitated vehicles [11].

A. Formulation of the Travelling Salesman Problem

As mentioned earlier, in the TSP, a set of *n* cites and a way of measuring the distance between each city is given. The objective is to find shortest tour that visits all cities exactly once and returns back to the starting city (depot) [9].

In the model below, the starting city is considered node 1 (depot), where *i* represents the current visited node and *j*

represents the next node to be visited. A distance d_{X_i,Y_j} is associated with each arc and represents the distance travelled from node X_i to nodeY_j; as shown in Fig. 1.

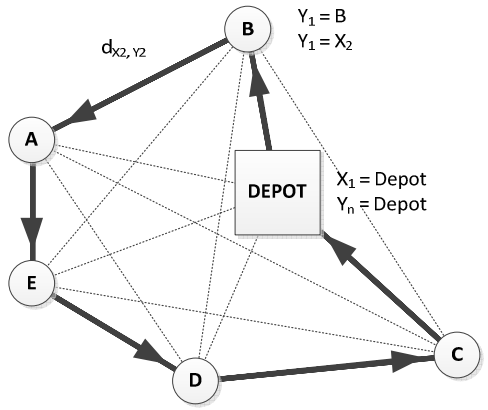

Fig. 1 Illustration of the TSP problem

Given a symmetric network, the decision variable isY_j; where,Y_j determines the value of the next customer to be visited by the vehicle. X_i variable represents the value of the start node of the arc; while,Y_j represents the next destination node, which is then considered as the start node of the following arc. Generally the use of loop segments is not allowed (leaving a node then arriving to same node,$X_i \neq Y_j$), as all nodes must be visited exactly once.S_{X_i,Y_j} is a binary variable to represent the passing of the vehicle on arc (X_i,Y_j). S_{X_i,Y_j} is given a value of 1 if arc (X_i,Y_j) belongs to the optimum route, 0 otherwise.The problem can be formulated as follows:

$$Minimize Z = \sum_{j=1}^{n}\sum_{i=1}^{n} S_{X_i,Y_j} * d_{X_i,Y_j} \quad (1)$$

Subject to:

$$X_1 = 1 \quad (2)$$

$$Y_n = 1 \quad (3)$$

$$X_{i+1} = Y_j \quad (4)$$

$$X_i \leq n \quad (5)$$

$$Y_j \leq n \quad (6)$$

$$\sum_{j=1}^{n} S_{X_i,Y_j} = 1 \quad \forall\, i = 1, \dots, n \quad (7)$$

$$\sum_{i=1}^{n} S_{X_i,Y_j} = 1 \quad \forall\, j = 1, \dots, n \quad (8)$$

$$X_i, Y_j > 0 \text{ and integer} \quad (9)$$

The objective function (1) minimizes the total travel distance. Constraints (2) and (3) ensure that the route starts and ends at the depot. Constraint (4) ensures that routes are not segmented, that is, if a vehicle arrives at a city, it eventually leaves the city again; where,*i* and *j* are equal for the same arc. Constraints (5) and (6) state the range of values given (the number of nodes, *n*). Constraints (7) and (8) ensure that every city is visited exactly once. Finally, constraint (9) is the non-negativity constraint and guarantees that the variables can assume integer values only.

B. Formulation of the m-Travelling Salesman Problem

As mentioned earlier, the *m*-TSP is a generalization of the TSP that introduces morethan one salesman (*m*); hence, *m*number of tours can be done; each starting and ending at thedepot. None of the salesmen is allowed to remain unassigned to any city.For formulating the *m*-TSP, the starting city is considered node 1 (depot); where,*i* represents the current visited node and *j* represents the next node to be visited. Now, *m* routes are introduced to the model; where, distance d_{X_i,Y_j} is associated with each arc and represents the distance travelled from node X_i^k to nodeY_j^kon route *k*, as shown in Fig. 2.

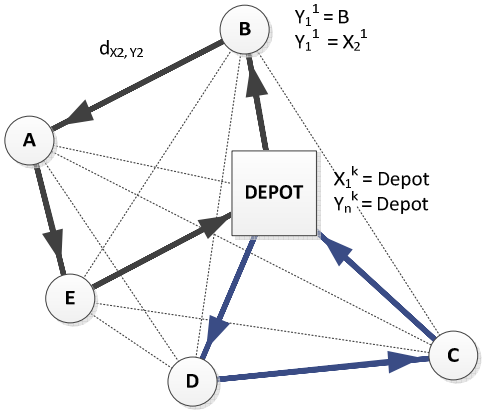

Fig. 2 Illustration of the *m*-TSP problem

The decision variables are Y_j^k; where,Y_j^k determines the value of the next customer to be visited on route *k*. The X_i^k variable represents the value of the start node of the arc on route *k*. the binary variable $S_{X_i,Y_j}^k \in$to a set of all possible arcs connecting any two nodes on route *k*. S_{X_i,Y_j}^k is given a value of 1 if arc (X_i^k, Y_j^k) belongs to route *k*; 0 otherwise. The problem can be formulated as follows:

$$Minimize\ Z = \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} S_{X_i,Y_j}^{k} * d_{X_i,Y_j} \quad (10)$$

Subject to:

$$X_1^k = 1 \quad (11)$$

$$Y_n^k = 1 \quad (12)$$

$$X_{i+1}^k = Y_j^k \quad \forall k = 1,\dots,v \quad (13)$$

$$X_i^k \le n \quad \forall k = 1,\dots,v \quad (14)$$

$$Y_j^k \le n \quad \forall k = 1,\dots,v \quad (15)$$

$$\sum_{j=1}^{n} S_{1,Y_j}^{k} = 1 \quad \forall k = 1,\dots,m \quad (16)$$

$$\sum_{i=1}^{n} S_{X_i,1}^{k} = 1 \quad \forall k = 1,\dots,m \quad (17)$$

$$\sum_{k=1}^{v} S_{X_i,Y_j}^{k} = 1 \quad \forall j = 1,\dots,n \quad (18)$$

$$\sum_{k=1}^{v} S_{X_i,Y_j}^{k} = 1 \quad \forall j = 1,\dots,n \quad (19)$$

$$X_i^k, Y_j^k > 0\ and\ integer \quad (20)$$

The objective function (10) minimizes the total travel distance on all *k* routes; where, *m* is the number of routes. Constraints (11) and (12) ensure that each route starts and ends at the depot. Constraint (13) ensures that routes are not segmented, that is, if a vehicle arrives at a city, it eventually leaves the city again, where *i* and *j* are equal for the same arc. Constraints (14) and (15) state the range of values given (the number of nodes, *n*). Constraints (16) and (17) ensure that all vehicles are being used; whereas, constraints (18) and (19) state that every city is visited exactly once. Finally, constraint (20) is the non-negativity constraint and guarantees that the variables can assume integer values only.

C. Formulation of the Vehicle Routing Problem

Further modifications are introduced to the *m*-TSP formulation to account for additional complexities of the capacitated vehicle routing problem in its basic form. Knowing that at each city, customers' demand q_{Y_j} is present and that each vehicle has a limited capacityQ_k; constraint (21) ensures that the total demand of all customers assigned to a route*k* does not exceed the vehicle's capacity.

$$\sum_{j=1}^{n}\sum_{i=1}^{n} S_{X_i,Y_j}^{k} * q_{Y_j} = Q_k \quad \forall k = 1,\dots,m \quad (21)$$

Also, if there is a constraint on the total distance travelled by the vehicles on a route,D_k, it can be presented as shown (22).

$$\sum_{j=1}^{n}\sum_{i=1}^{n} S_{X_i,Y_j}^{k} * d_{X_i,Y_j} = D_k \quad \forall k = 1,\dots,m \quad (22)$$

D. Models Validation

In order to check the validity of the aforementioned models, a problem was taken from literature and was implemented in MS Excel. Then, using the Solver Add-in, the problem was solved to optimality.

The problem, which will be used in the simulation model as well, assumes that a truck is to be routed from its depot to five stops (points coordinates are given in

TABLE I). Stops and the origin point are identified with linear coordinate points. Euclidean (straight-line) distances are computed in terms of these coordinate points [8].

TABLE I
DEPOT AND STOPS COORDINATES

No.	Stop	X	Y
1	Depot	2	2
2	Stop 1	3	4
3	Stop 2	5	3
4	Stop 3	4	1
5	Stop 4	5	5
6	Stop 5	4	3

The optimal solution of the problem is as follows: Depot → Stop 1→ Stop 4→ Stop 2→ Stop 5→ Stop 3→ Depot; with a total route distance of 11.71.

The other formulations were tested for validation using the MS Excel solver add-in, and results showed satisfactory, all constraints were fulfilled.

IV. SIMULATION MODEL DEVELOPMENT

A simulation model has been developed for the problem given in the previous section. The model is developed in the ExtendSim^{TM} environment and is divided into three main sections: router, customers, and total route distance calculator.

A. Router

The router (Fig. 3) is responsible for routing the vehicle and calculating travelling distance to next customer using a built-in database having the inputs of the problem.

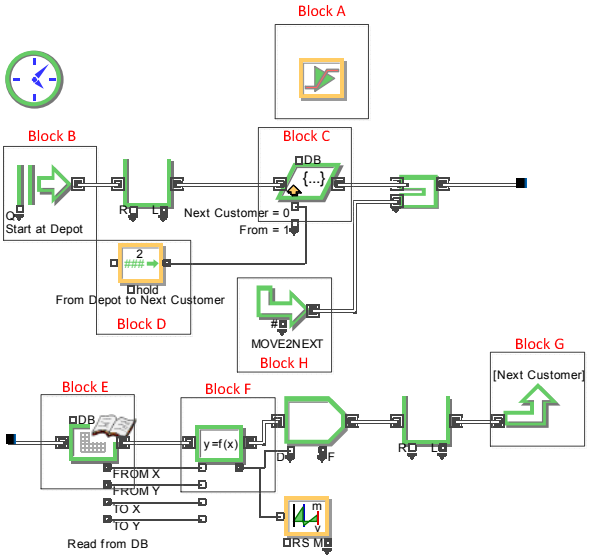

Fig. 3 Snapshot of the Router

In Fig. 3, Block B is the create block which introduces items into the model; items here represent vehicles requiring routing starting at the depot. The vehicle's next destination point is using the attribute Block C, which receives its attribute value from Block D. This attribute value is one of the decision variables and is defined in the optimizer Block A.

The Euclidean distances between any two connected nodes is calculated at Block F using the XY coordinates of these nodes. XY coordinates of the current node and the destination node are retrieved from the built-in database using Block E. The database table shown in Fig. 4 includes the co-ordinates of all stops. After defining the attribute value of the next customer to visit, Block G routes the vehicle to that customer.

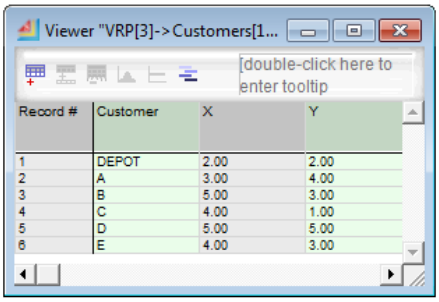

Fig. 4 Input Data

B. Customers

The customers section (Fig. 5) represents stop points and basically determines the number of times this customer has been visited. If the number of visits is zero, a penalty is incurred; if it is greater than one (multiple visits), a penalty that is in multiples of the number of visits is incurred. Finally, if it is visited only once, no penalties occur. These penalties act as soft constraints that are required by the optimization block to ensure that each customer is visited exactly once. Also, that section of the model determines the customer to be visited next.

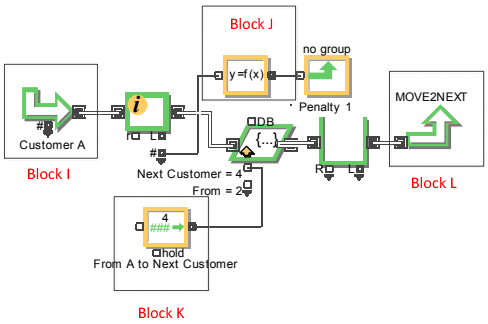

Fig. 5 Snapshot showing a customer

Fig. 5 shows a part of the model representing one of the cities to be visited by the vehicle. This part is replicated according to the number of stops to be modeled. Block I receives the vehicle that is sent to the customer by Block G. Block J calculates the penalties based on the number of visits as described above. Block K is as same as Block D; where, at this block the following city to be visited is determined. Finally, the vehicle leaves the current city moving to Block H in Fig. 3, going through distance calculation and then to the next destination.

C. Total Route Distance Calculation

The third and final section of the model is the total route distance calculator (Fig. 6), which is responsible for calculating the total travel distance of the vehicle and ensuring that the vehicle returns back to its starting point (depot).

Fig. 6 shows that after all the routing decisions are made, the vehicle returns back to the depot (Block M). The total distance travelled is calculated and reported in Block O. All penalties incurred for violations of the number of visits per customer are summed up at Block N and added to the total distance travelled. The vehicle then leaves the model through the exit Block P.

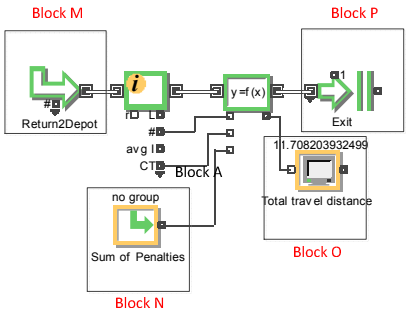

Fig. 6 Snapshot of the total route distance calculation

V. Optimization Using Simulation

After developing the model in ExtendSim™, optimization using simulation to find the best route that minimizes the total

travelling distance and satisfies all constraints. ExtendSim™ performs optimization using the Optimizer Block (Block A) shown in Fig. 3.

This optimizeruses an evolutionary algorithm to reduce the number of times the model has to run before a solution is found. The "problem" is stated as an objective function or cost equation that ExtendSim™ tries to minimize or maximize to save time going through the process of manually trying different values with each model run [24]; as shown in Fig. 7.

Fig. 7 shows the optimizer parameters window; where, the decision variables and objective function are defined.

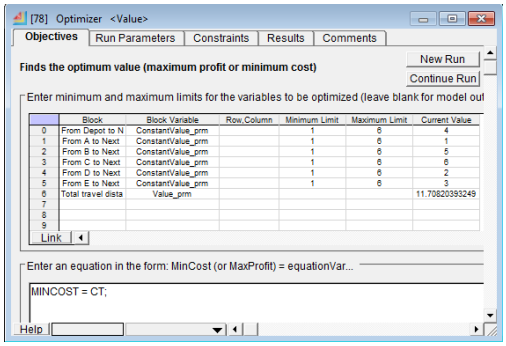

Fig. 7 Optimizer parameters window

The objective function is to minimize the total distance travelled that is reported by Block O. The different decision variables are the attribute values that define the next customer to visit; such as the values reported by Blocks D and K.

After executing the optimization of the model, the minimum total distance is 11.7082 as shown in Fig. 7, which is the same total distance obtained using MS Excel solver. Finally, the route is reported back to the built-in database as shown in Fig. 8.

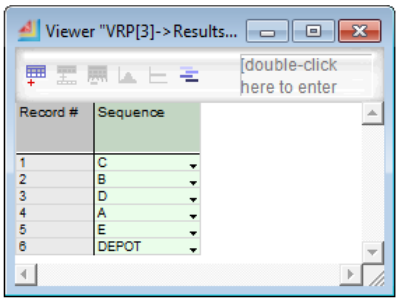

Fig. 8 Results table

VI. Conclusion

In literature, binary decision variables are used to formulate VRP, this paper introduces a new mathematical formulation where integer decision variables are used to determine which city/customer to be visited next instead of which routing segment to be taken. Decision variables in the proposed mathematical formulations are relatively small if compared to other formulations found in literature.

An optimization using simulation approach is introduced to the TSP and results showed satisfactory as the simulation output showed that the optimum solution can be obtained.

For future work, it is recommended that further research shall be undertaken to model the VRP using discrete event simulation, seeking for optimum/near optimum solutions considering time windows and vehicles capacity constraints and the stochastic nature of customers demand, customers' service time, and travel time.

Acknowledgment

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References

[1] R. Baldacci, P. Toth, and D. Vigo, "Exact algorithms for routing problems under vehicle capacity constraints," *Ann Oper Res,* vol. 175, pp. 213-245, 2010.

[2] N. Azi, M. Gendreau, and J.-Y. Potvin, "An exact algorithm for a single-vehicle routing problem with time windows and multiple routes," *European Journal of operational Research,* vol. 178, pp. 755-766, 2007.

[3] R. Baldacci, E. Hadjiconstantinou, and A. Mingozzi, "An Exact Algorithm for the Capacitated Vehicle Routing Problem Based on a Two-Commodity NetworkFlow Formulation," *Operations Research,* vol. 52, pp. 723–738, 2004.

[4] O. Bräysy and M. Gendreau, "Vehicle Routing Problem with Time Windows, Part I: Route Construction and Local Search Algorithms," *Transportation science,* vol. 39, pp. 104–118, 2005.

[5] O. Bräysy and M. Gendreau, "Vehicle Routing Problem with Time Windows, Part II: Metaheuristics," *Transportation science,* vol. 39, pp. 119-139, 2005.

[6] U. Derigs and K. Reuter, "A simple and efficient tabu search heuristic for solving the open vehicle routing problem," *Journal of the Operational Research Society,* vol. 60, pp. 1658-1669, 2009.

[7] M. Desrochers, J. K. Lenstra, and M. W. P. Savelsbergh, "A classification scheme for and scheduling problems vehicle routing," *European Journal of operational Research,* vol. 46, pp. 322-332, 1990.

[8] R. H. Ballou, *Business Logistics/ Supply Chain Management*, 5th ed. Pearson, 2004.

[9] S. Ropke, "Heuristics and exact algorithms for vehicle routing problems," Ph.D., Department of Computer Science University of Copenhagen, 2005.

[10] Z. Fu, R. Eglese, and L. Li, "A unified tabu search algorithm for vehicle routing problems with soft time windows," *Journal of the Operational Research Society,* vol. 59, pp. 663 --673, 2008.

[11] M. M. Solomon and J. Desrosiers, "Time window constrained routing and scheduling Problems," *Transportation science,* vol. 22, 1988.

[12] W. Maden, R. Eglese, and D. Black, "Vehicle routing and scheduling with time-varying data: A case study," *Journal of the Operational Research Society,* vol. 61, pp. 515-522, 2010.

[13] F. S. Hillier and G. J. Lieberman, *Introduction to Operations Research*, 8th ed.: McGraw-Hill, 2005.

[14] P. Toth and D. Vigo, *The Vehicle Routing Problem*. Philadelphia: SIAM, 2002.

[15] A. M. Campbell and M. Savelsbergh, "Efficient Insertion Heuristics for Vehicle Routing and Scheduling Problems," *Transportation science,* vol. 38, pp. 369-378, 2004.

[16] G. Laporte, "The Vehicle Routing Problem: An overview of exact and approximate algorithms," *European Journal of operational Research,* vol. 59, pp. 345-358, 1992.

[17] M. M. Solomon, "Algorithms for the Vehicle Routing and Scheduling Problems with Time Window Constraints," *Operations Research,* vol. 35, pp. 254-265, 1987.

[18] N. Wassan, "A reactive tabu search for the vehicle routing problem," *Journal of the Operational Research Society,* vol. 57, pp. 111–116, 2006.
[19] B. M. Baker and M. A. Ayechew, "A genetic algorithm for the vehicle routing problem," *Computers & Operations Research,* vol. 30, pp. 787–800, 2003.
[20] S. Zhongyue and G. Zhongliang, "Vehicle Routing Problem Based on Object-oriented Discrete Event Simulation," presented at the International Conference on Advanced Computer Control ICACC, 2010.
[21] J. Faulin, M. Gilibert, A. A. Juan, X. Vilajosana, and R. Ruiz, "SR-1: A Simulation-based Heuristic Algorithm for the Capacitated Vehicle Routing Problem," presented at the The 2008 Winter Simulation Conference, 2008.
[22] D. Feillet, P. Dejax, and M. Gendreau, "Traveling Salesman Problems with Profits," *Transportation science,* vol. 39, pp. 188-205, 2005.
[23] R. V. Kulkarni and P. R. Bhave, "Integer programming formulations of vehicle routing problems," *European Journal of operational Research,* vol. 20, pp. 58-67, 1985.
[24] "ExtendSim 8 User Guide," Imagine That Inc.2007.