International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:12, No:8, 2018

Optimization of Tolerance Grades of a Bearing and Shaft Assembly in a Washing Machine with Regard to Fatigue Life

M. Cangi, T. Dolar, C. Ersoy, Y. E. Aydogdu, A. I. Aydeniz, A. Mugan

Abstract**—**The drum is one of the critical parts in a washing machine in which the clothes are washed and spin by the rotational movement. It is activated by the drum shaft which is attached to an electric motor and subjected to dynamic loading. Being one of the critical components, failures of the drum require costly repairs of dynamic components. In this study, tolerance bands between the drum shaft and its two bearings were examined to develop a relationship between the fatigue life of the shaft and the interaction tolerances. Optimization of tolerance bands was completed in consideration of the fatigue life of the shaft as the cost function. The following methodology is followed: multibody dynamic model of a washing machine was constructed and used to calculate dynamic loading on the components. Then, these forces were used in finite element analyses to calculate the stress field in critical components which was used for fatigue life predictions. The factors affecting the fatigue life were examined to find optimum tolerance grade for a given test condition. Numerical results were verified by experimental observations.

Keywords**—**Fatigue life, finite element analysis, tolerance analysis, optimization.

I. INTRODUCTION

FATIGUE failure is the principal failure mechanism in mechanical components in engineering structures. Motivated by this fact, numerous studies are undertaken to predict the fatigue failure and estimate the fatigue life of mechanical components, e.g. see [1] for an overview. The methods to predict the fatigue failure is separated into two groups such as frequency domain [2] and time domain [3]-[12] methods. It is noteworthy that time domain methods and commercial codes nCode and ABAQUS were employed in this study. In time domain analyses, the fatigue life is estimated by using stress history of a component where a counting method (e.g., range-mean stress, rain flow counting and range-pair methods [3]-[5]) and a damage accumulation rule (e.g., Miner-Palmgren rule and Haibach rule [6]-[8]) are employed. The counting methods essentially convert an irregular stress-time history data into equivalent stress cycles having a constant range and mean stress. Following, the damage accumulation rules are implemented to calculate the total damage by summing the contributions of each stress cycle found by the counting method. In literature, the rainflow counting method and Miner-Palmgren rule are widely used for fatigue damage predictions due to their accuracy [9]-[12].

In this study, the drum shaft of a washing machine is examined in consideration of fatigue life. The drum shaft activates the drum of the washing machine in which the clothes are washed and spin. Being one of the critical components in a washing machine, it is attached to an electric motor and subjected to dynamic loads whose failure results in the replacement of the whole dynamic system of the washing machine, and such a warranty service brings about notably high costs to the washing machine producers. It is observed in tests that the tolerance grades of the drum shaft and its bearings have an important effect on fatigue life of the drum shaft. To this end, fit tolerances between the drum shaft and its two bearings were studied to develop a relationship between the fatigue life of the shaft and tolerance grades. By selecting the fatigue life of the shaft as the cost function, tolerance grades of the assembly of shaft and its bearings were optimized. First, a dynamic model of the washing machine was prepared, and dynamic analyses were completed to obtain the dynamic loads acting on the drum shaft during a quelle. Then, radial pressures on the shaft applied by the interaction between the shaft and its bearings were analytically calculated. After obtaining all dynamic and static loads, by the use of ABAQUS software, a finite element analysis was completed to find the stress distribution and the critical areas on the drum shaft. Following, by the use of nCode software, fatigue life estimation of the drum shaft was completed for different tolerance grades for the assembly of shaft and its bearings. Results of the analyses revealed that, only one of the bearings had a significant impact on the drum shaft fatigue life, whereas the other one had very slight effect. It is observed that fatigue life estimation of the drum shaft calculated by the software nCode agrees well with the test results. Accordingly, an optimum fit pressure is determined. Any fit pressure above or below that optimum value causes a decrease in the fatigue life of the shaft. Optimum tolerance grades for the shaft and bearings which ensure the optimum pressure on the shaft are determined for the shaft fatigue life optimization.

II. MODELING AND DESIGN CONSTRAINTS

A washing machine fulfills the design task of washing the

M. Cangi Arçelik A.Ş. Washing Machine Plant, Çayırova Campus, 34950 Tuzla and Istanbul Technical University, Faculty of Mechanical Engineering, 34437 Gumussuyu, Beyoglu, Istanbul, Turkey (corresponding author, e-mail: cangi@itu.edu.tr, phone: (90) 212 293 1300).

T. Dolar, C. Ersoy, A. I. Aydeniz and A. Mugan are with the Istanbul Technical University, Faculty of Mechanical Engineering, 34437 Gumussuyu, Beyoglu, Istanbul, Turkey (phone: (90) 212 293 1300).

Y. E. Aydogdu is with the Arçelik A.Ş. Washing Machine Plant, Çayırova Campus, 34950 Tuzla, İstanbul, Turkey (e-mail: yunusemre.aydogdu@arcelik.com).

clothes inside the drum. The dynamic components of a drum assembly are shown in Fig. 1.

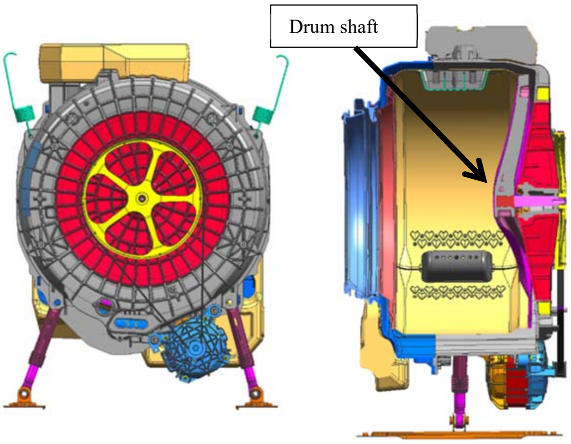

Fig. 1 Dynamic components in a washing machine

The drum is activated by the drum shaft which is connected to an electric motor through a pulley, e.g. see Fig. 2. The drum shaft is supported by two ball bearings.

In this study, a washing machine having the maximum rotational velocity of 1000 rotation per minute (RPM) in a quelle cycle, maximum clothe capacity of 63 liters and maximum laundry mass of 9 kg is considered. A quelle cycle is formed of washing and spinning phases of the washing operation. The loadings acting on the drum shaft in a quelle cycle is time varying. The drum shaft is supported by two ball bearings having different sizes. Tolerances of these ball bearings should be optimized separately that is pursued in this study. Typical tolerances employed between the inner ring of a ball bearing and shaft are presented in Fig. 3 [13].

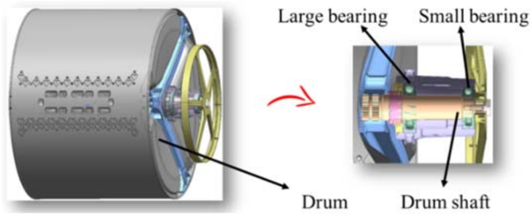

Fig. 2 The drum and drum shaft

Nominal design of the drum shaft has the roller bearing inner ring tolerance of P6 and shaft tolerance of g6. In analyses, the worst possible scenarios for the assembly of shaft and bearings were considered corresponding to the largest possible shaft diameter and smallest possible bearing inner ring. Associated tolerance bands in microns are as follows: P6/g6 where $P6^{-18}_{-31}$and $g6^{-7}_{-20}$.

Bearing Inner Race Inner Diameter Nominal Size (mm)																				
from	3	6	10	18	30	40	50	65	80	100	120	140	160	180	200	225	250	280	315	355
to	6	10	18	30	40	50	65	80	100	120	140	160	180	200	225	250	280	315	355	400
micrometer (um)																				
F7	22 10	28 13	34 16	41 20	50 25		60 30		71 36		83 43			96 50			108 56		119 62	
H7	12 0	15 0	18 0	21 0	25 0		30 0		35 0		40 0			46 0			52 0		57 0	
M7	0 -12	0 -15	0 -18	0 -21	0 -25		0 -30		0 -35		0 -40			0 -46			0 -52		0 -57	
N7	-4 -16	-4 -19	-5 -23	-7 -28	-8 -33		-9 -39		-10 -45		-12 -52			-14 -60			-14 -66		-16 -73	
P6	-9 -17	-12 -21	-15 -26	-18 -31	-21 -37		-26 -45		-30 -52		-36 -61			-41 -70			-47 -79		-51 -87	
P7	-8 -20	-9 -24	-11 -29	-14 -35	-17 -42		-21 -51		-24 -59		-28 -68			-33 -79			-36 -88		-41 -98	
Shaft Diameter Nominal Size (mm)																				
from	3	6	10	18	30	40	50	65	80	100	120	140	160	180	200	225	250	280	315	355
to	6	10	18	30	40	50	65	80	100	120	140	160	180	200	225	250	280	315	355	400
micrometer (um)																				
g6	-4 -12	-5 -14	-6 -17	-7 -20	-9 -25		-10 -29		-12 -34		-14 -39			-15 -44			-17 -49		-18 -54	
j6	6 -2	7 -2	8 -3	9 -4	11 -5		12 -7		13 -9		14 -11			16 -13			16 -16		18 -18	
k6	9 1	10 1	12 1	15 2	18 2		21 2		25 3		28 3			33 4			36 4		40 4	
m6	12 4	15 6	18 7	21 8	25 9		30 11		35 13		40 15			46 17			52 20		57 21	
n6	16 8	19 10	23 12	28 15	33 17		39 20		45 23		52 27			60 31			66 34		73 37	
p6	20 12	24 15	29 18	35 22	42 26		51 32		59 37		68 43			79 50			88 56		98 62	
r6	23 15	28 19	34 23	41 28	50 34		60 41	62 43	73 51	76 54	88 63	90 65	93 68	106 77	109 80	113 84	126 94	130 98	144 108	150 114

Fig. 3 Tolerances between the inner ring of a ball bearing and shaft [13]

Scenario No	Small Bearing				Large Bearing			
	Shaft	Bearing	Fit [μm]	Pressure [MPa]	Shaft	Bearing	Fit [μm]	Pressure [MPa]
1	g6 m6 r6	N7 H7 F7	21	22,52	g6 m6 r6	N7 H7 F7	21	17,85
2	g6 m6 r6	N7 H7 F7	21	22,52	g6	P6	24	22,55
3	g6 m6 r6	N7 H7 F7	21	22,52	g6 n6	P7 H7	28	28,82
4	g6 m6 r6	N7 H7 F7	21	22,52	k6 p6	H7 F7	15	8,46
5	g6	P6	24	28,44	g6 m6 r6	N7 H7 F7	21	17,85
6	g6	P6	24	28,44	g6	P6	24	22,55
7	g6	P6	24	28,44	g6 n6	P7 H7	28	28,82
8	g6	P6	24	28,44	k6 p6	H7 F7	15	8,46
9	g6 n6	P7 H7	28	36,35	g6 m6 r6	N7 H7 F7	21	17,85
10	g6 n6	P7 H7	28	36,35	g6	P6	24	22,55
11	g6 n6	P7 H7	28	36,35	g6 n6	P7 H7	28	28,82
12	g6 n6	P7 H7	28	36,35	k6 p6	H7 F7	15	8,46
13	k6 p6	H7 F7	15	10,67	g6 m6 r6	N7 H7 F7	21	17,85
14	k6 p6	H7 F7	15	10,67	g6	P6	24	22,55
15	k6 p6	H7 F7	15	10,67	g6 n6	P7 H7	28	28,82
16	k6 p6	H7 F7	15	10,67	k6 p6	H7 F7	15	8,46

Fig. 4 Tolerance grade scenarios for interactions between the shaft and large/small bearings

In sum, the smallest possible bearing inner ring diameter is equal to 25–0.031=24.969 mm, and the largest possible shaft diameter is equal to 25–0.007=24.993 mm. Following, the maximum interaction is equal to 24.993–24.969=0.024 mm. Then, the maximum interactions were calculated for the possible matches of different tolerance grades. The cases providing an interaction larger or smaller than the nominal design were determined, that were analyzed by the finite element models, e.g. see Fig. 4.

In the following sections, the shaft and bearing tolerances will be determined for the worst possible interaction matches listed in Fig. 4 based on finite element analyses (FEA) results such that the fatigue life of drum shaft is maximized.

III. Tolerances of the Assembly of Shaft and Bearings

According to ISO standards, there are three different fits between the drum shaft and its bearings [13] which are determined by the tolerance grades such as no interaction (i.e., loose fit), moderate overlap of tolerance grades and strict overlap of tolerance grades (press fit) as shown in Fig. 5.

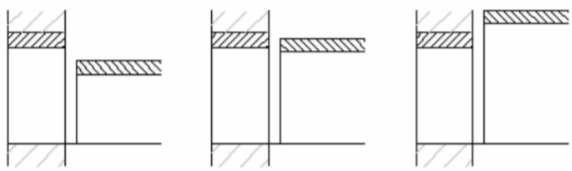

Fig. 5 Three different fits between the drum shaft and bearings [13]

There are 28 classes of tolerance grades in ISO standards for both holes and shafts. The hole and shaft tolerances are respectively designated by capital and small letters, e.g. see Fig. 6.

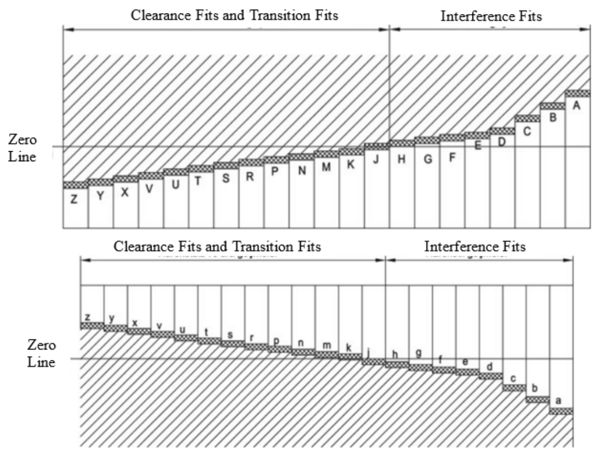

Fig. 6 Classification of ISO tolerances designated for the shafts and holes [13]

IV. Analysis of the Assembly of Drum Shaft and Bearings

Beforehand, the pressures applied to the shaft by the interactions of bearings were calculated by

$$P = \frac{\Delta d_{max,real}}{\frac{D_{ri}}{E_r}\left(\frac{D_{ro}^2+D_{ri}^2}{D_{ro}^2-D_{ri}^2}+\nu_r\right)+\frac{D_{mo}}{E_m}\left(\frac{D_{mo}^2+D_{mi}^2}{D_{mo}^2-D_{mi}^2}-\nu_m\right)} \quad (1)$$

where P is the pressure on the shaft applied due to the interaction, $\Delta d_{max,real}$ is the amount of interaction between the bearing inner ring hole and shaft diameter, D_{ri} and D_{ro} are respectively the inner and outer diameters of bearing inner ring, D_{mi} and D_{mo} are respectively the inner and outer diameters of the shaft, E_r and E_m are respectively the elasticity modulus of bearing inner ring and shaft materials, and ν_r and ν_m are respectively the Poisson's ratios of bearing inner ring and shaft materials. For the nominal tolerance of 25 P6/g6 having the tolerance bands of $P6_{-31}^{-18}$ and $g6_{-20}^{-7}$ for the small bearing, we find that $\Delta d_{max,real}$=(-7)-(-31)=24 μm. The loss in interaction due to smashing of surface roughness is equal to

$$z = 1.2\left(R_{t,shaft} + R_{t,bearing}\right) = 1.2(4+4) = 9.6\ \mu m \quad (2)$$

where R_t=4 μm is the maximum surface roughness value. Note that the shaft material is hot rolled EN 10025-2:2014.

Following, we find $\Delta d_{max,real}$=24-9.6=14.4 μm that yields fit pressure of P=28.44 MPa for the small bearing. For FEA by using the software ABAQUS, the drum shaft is meshed by hexahedral elements as shown in Fig. 7. Number of elements of the FEA model was determined by refining the elements and carrying out a mesh convergence test. Then, static loads shown in Fig. 8 were applied to the drum shaft. The loads were distributed to the nodes of the shaft by the use of RBE3 elements in ABAQUS software. The boundary conditions were that only the rotations around z axis were allowed in torsion, while support points were bearing locations for the other loads.

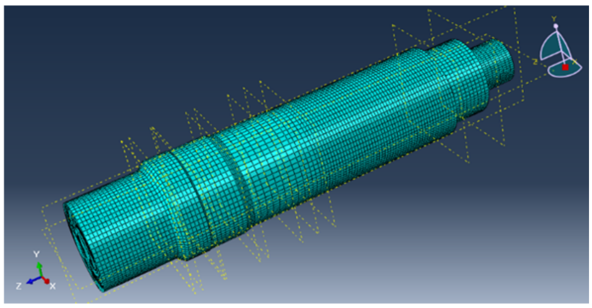

Fig. 7 Mesh of the drum shaft

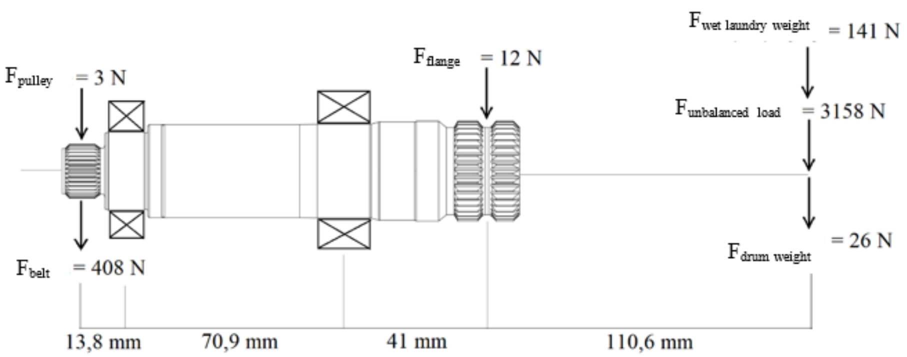

Fig. 8 The loads acting on the drum shaft

The FEA was completed in two stages. In the first stage, the calculate pressure values were applied, then external dynamic forces were applied. Following, all stress values were superposed. The stress distribution due to the interaction between the shaft and small bearing was calculated by the ABAQUS software that is shown in Fig. 9. Nonetheless, it is observed that the interaction between the shaft and big bearing is more critical. Convergence of numerical solutions was checked by increasing the number of elements. As a result, the stress distribution shown in Fig. 10 was obtained by using 971,388 elements and had the maximum Von Mises stress of 68.56 MPA on the edge of contact surface of the big bearing.

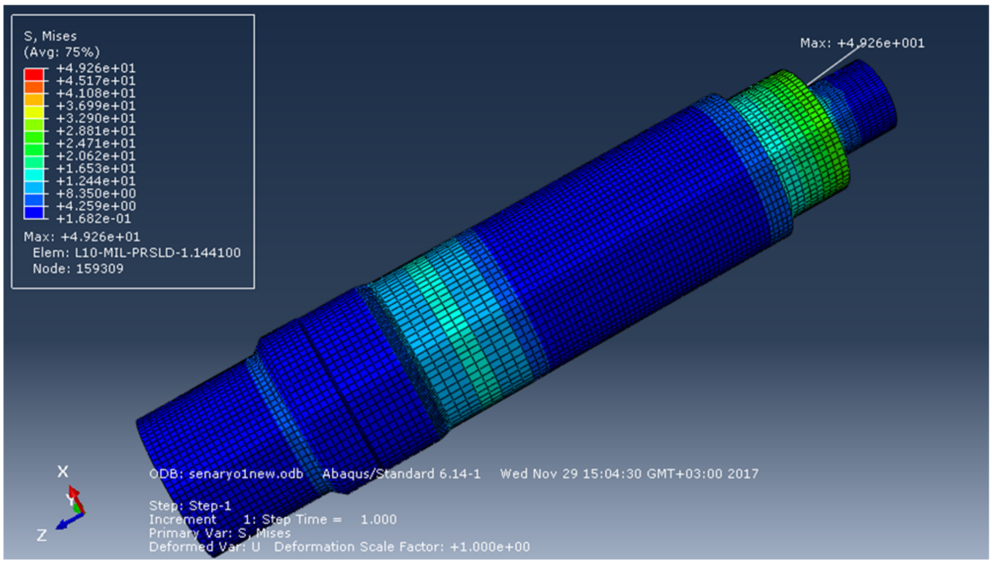

Fig. 9 Von Mises stress distribution due to interaction between the shaft and small bearing

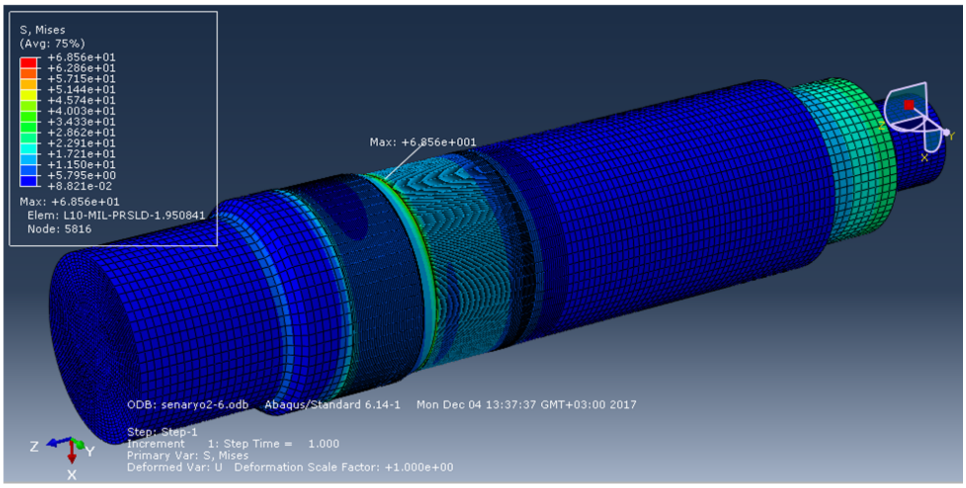

Fig. 10 Von Mises stress distribution due to interaction between the shaft and big bearing

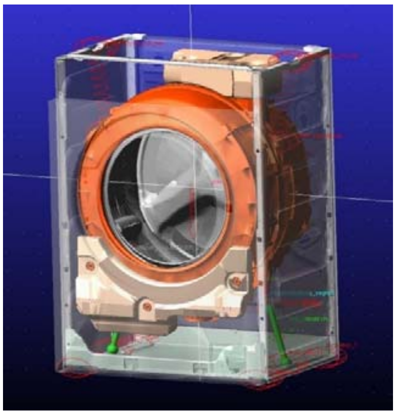

Fig. 11 Washing machine model run in the software ADAMS

To estimate the fatigue life of the drum shaft, the dynamic model of the washing machine shown in Fig. 11 was prepared in ADAMS software and run for a quelle cycle of 150 second where the unbalanced load was 1.150 kg and maximum drum rotational velocity was 1000 RPM. Note that the following loads obtained by ADAMS runs were used in fatigue life estimations: force components of F_x, F_y and F_z acting on the center of the mass of the shaft, pressures applied respectively by the interactions with the big and small bearings P_b and P_s, and torsional moment of M_t. While the variations of F_x, F_y, F_z, M_t and angular velocity of the shaft α_z (where $M_t = I\alpha_z$, $I=575416x10^{-6}$ kgm^2) were calculated by the ADAMS model, the pressures P_b and P_s were calculated by (1); they are all shown in Fig. 12 where the sampling frequency was 100 Hz. Then, the ABAQUS output file having the stress distributions of six load cases in response to unit loadings of P_b, P_s, F_x, F_y, F_z and M_t is imported by the nCode Design Life module. These load cases are scaled by the software nCode according to the load variations shown in Fig. 12 and the stress history of

International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:12, No:8, 2018

the drum shaft during a quelle cycle is obtained by superposition of these six load cases. Following, fatigue life estimations were calculated by the software nCode that were used to determine the optimum tolerance grades for the interactions of big and small bearings with the drum shaft.

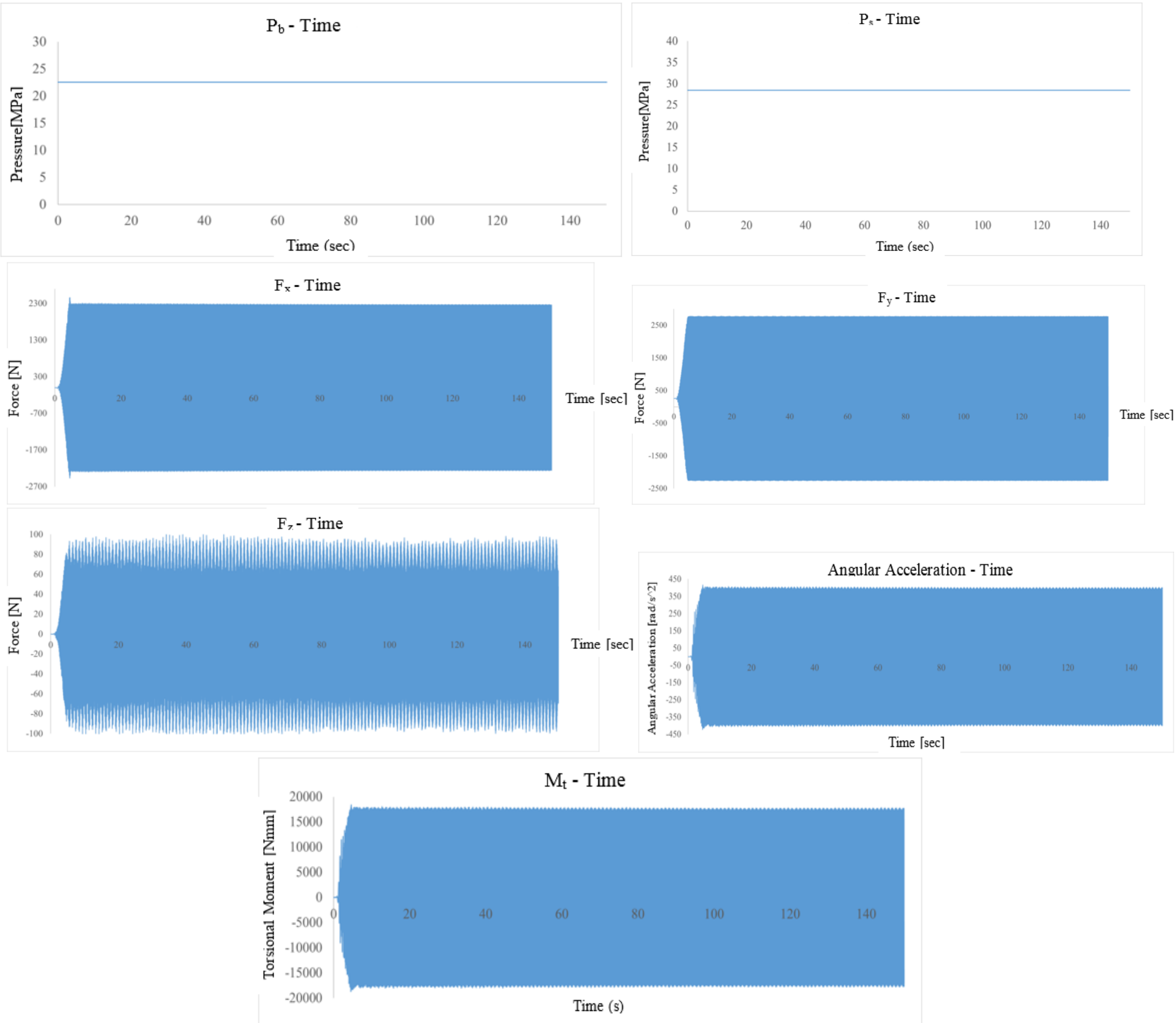

Fig. 12 Loads in a quelle cycle applied in fatigue life estimations

For the unit pressure/load/moment magnitudes of P_b, P_s, F_x, F_y, F_z and M_t, the FEA analyses in Abaqus were separately completed as six different load cases. To optimize the tolerance grades of the assembly of shaft and big/small bearings, the boundary conditions were implemented on the interaction surfaces to account for the interactions between the shaft and bearings given in the fourth and eighth columns of Fig. 4 that are respectively corresponding to interactions of small and big bearings with the shaft. In brief, the worst possible interaction cases were considered in analyses. The tolerance grades that yield longer fatigue life than that of the nominal design were given in Fig. 13.

In nCode fatigue life calculations, it is observed that the tolerance grades in the interaction between the shaft and small bearing had insignificant effect on the fatigue life estimation of the shaft. On the other hand, the tolerance grades in the interaction between the shaft and big bearing had considerably affected the fatigue life estimation of the shaft. It is observed in experiments on prototype washing machines that the nominal design of the drum shaft has the fatigue life of 1500 to 2000 quelle cycles. On the other hand, nCode simulations predicted that the fatigue life of drum shaft of nominal design is 1786 quelle cycles, that agrees well with the experiments. It is shown in Fig. 14 that the fatigue life of the drum shaft in terms of the number of quelle cycles changes with the pressure in the interaction between the shaft and big bearing that has an optimum value for approximately P=33 MPa.

Scenario No	Small Bearing		Large Bearing	
	Shaft	Bearing	Shaft	Bearing
1	g6 m6 r6	N7 H7 F7	g6 m6 r6	N7 H7 F7
4	g6 m6 r6	N7 H7 F7	k6 p6	H7 F7
5	g6	P6	g6 m6 r6	N7 H7 F7
8	g6	P6	k6 p6	H7 F7
12	g6 n6	P7 H7	k6 p6	H7 F7
13	k6 p6	H7 F7	g6 m6 r6	N7 H7 F7
14	k6 p6	H7 F7	g6	P6
15	k6 p6	H7 F7	g6 n6	P7 H7
16	k6 p6	H7 F7	k6 p6	H7 F7

Fig. 13 Tolerance grades that yield longer fatigue life than that of the nominal design

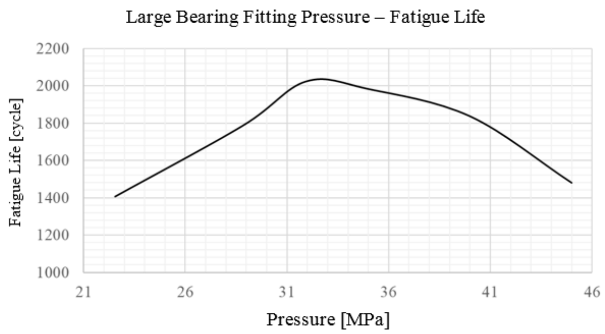

Fig. 14 Variation of fatigue life of the drum shaft with the pressure in the interaction between the shaft and big bearing

Table I summarizes the optimization trials where it is observed that the fatigue life of the drum shaft may be increased to 2025 cycles by selecting the tolerance 30 H7/n6 for the big bearing. Details of this study can be found in [14].

TABLE I
TOLERANCE GRADES OF NOMINAL DESIGN AND OPTIMUM SOLUTION FOR THE INTERACTION BETWEEN THE SHAFT AND BIG BEARING

	Nominal Design	Optimum Solution
Big Bearing Tolerance	30 P6/g6	30 H7/n6
Small Bearing Tolerance	25 P6/g6	25 P6/g6
Fatigue Life in terms of Quelle Cycles	1786	2025
Tolerance Bands for Big Bearing	Shaft$\varnothing 30^{-0.009}_{-0.017}$	Shaft$\varnothing 30^{+0.028}_{+0.015}$

V. CONCLUSIONS

Optimum tolerance grades of the assembly of the drum shaft and its bearings in a washing machine were determined such that the fatigue life of the drum shaft is maximized. While the pressures due to bearing-shaft interactions were calculated by analytical formulas, dynamic loads acting on the drum shaft were calculated by the dynamic model of the washing machine simulated by ADAMS software. Stress distributions in the shaft were calculated by ABAQUS models in response to unit loadings. Then, the software nCode was employed for fatigue life estimations in time domain by superposition of all load cases. Possible tolerance grades in the interactions between the drum shaft and bearings were determined. Following, by the virtue of nCode fatigue life estimations, optimum tolerance grades were determined that elongate the fatigue life of the shaft by 13.3%.

The methodology followed in this study is general and can be applied to determine the optimum tolerance grades in any other interaction between the shafts and hubs.

ACKNOWLEDGMENT

This project is supported in part by TUBITAK under the project number 3161220 titled “Development of a Scalable Boiler Assembly with a Passive Balancing System Produced by Insert Molding”.

REFERENCES

[1] J. Schijve. Fatigue of Structures and Materials. Springer, 2008.
[2] A. Halfpenny, A frequency domain approach for fatigue life estimation from finite element analysis. In M. D. Gilchrist, J. M. Dulieu Barton, and K. Worden, editors, DAMAS 99: Damage Assessment of Structures, volume 167-1 of Key Engineering Materials, pages 401-410, 1999.
[3] M. Matsuishi, T. Endo, Fatigue of metals subject to varying stress. Paper presented to Japan Society of Mechanical Engineers, Fukuoka, Japan, 1968.
[4] ASTM Designation E 1049-85 (1985). Standard practises for cycle counting in fatigue analysis.
[5] S. D. Downing, D. F. Socie (1982). “Simple rainflow counting algorithms.” Int. J Fatigue, January 1982, pp 31-40.
[6] A. Palmgren, Die lebensdauer von kugellagern. VDI-Zeitschrift, 68:339-341, 1924.
[7] M. A. Miner, Cumulative damage in fatigue. J. Appl. Mech., 12: A159-A164, 1945.
[8] E. Haibach, Betriebsfestigkeit-Verfaren und Daten zur Bauteliberechnung, Springer Berlin Heidelberg, 2006.
[9] I. Rychlik, Fatigue and stochastic loads, Scand. J. Stat., 23(4):387-404, 1996.
[10] W. Zhao, M. J. Baker, On the probability density function of rainow stress range for statonary gaussian processes. Int. J. Fatigue, 14(2):121-135, March 1992.
[11] R. Tovo, Cycle distribution and fatigue damage under broad-band random loading. Int. J. Fatigue, 24(11):1137-1147, 2002.
[12] G. Petrucci, B. Zuccarello, Fatigue life prediction under wide band random loading. Fatigue & Fract. Eng. Mater. & Struct., 27(12):1183-1195, December 2004.
[13] ISO 286-2. Geometrical product specifications (GPS) -- ISO code system for tolerances on linear sizes -- Part 2: Tables of standard tolerance classes and limit deviations for holes and shafts, International Organization for Standardization, 2010.
[14] C. Ersoy, M. Cangi, T. Dolar, Tolerance Analysis, Optimization, FEM Analysis and DOE of the Bearings and Drum Shaft of a Washing Machine. B.Sc. Graduation Cap Stone Design Project, Faculty of Mechanical Engineering, Istanbul Technical University, 2018.