International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:3, No:2, 2009

Optimization of Structure of Section-Based Automated Lines

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Abstract - Automated production lines with so called 'hard structures' are widely used in manufacturing. Designers segmented these lines into sections by placing a buffer between the series of machine tools to increase productivity. In real production condition the capacity of a buffer system is limited and real production line can compensate only some part of the productivity losses of an automated line. The productivity of such production lines cannot be readily determined. This paper presents mathematical approach to solving the structure of section-based automated production lines by criterion of maximum productivity.

Keywords - optimization production line, productivity, sections

I. INTRODUCTION

Researchers on automated lines, in area of metal cutting industry, developed many models to simulate line balancing, layout analysis, production timing, etc. **[1; 2]**. Productivity and reliability of automated production lines can be improved by segmentation of lines into some sections and placing buffers between them that enables reduction in line idle time and in turn results in increased productivity **[3; 4]**. In general, an automated production line is segmented on sections with equal reliability and equal indices of productivity losses and sections may have different number of stations. Practically it is possible to design such lines when the level of reliability of stations is known. Small variations of reliability stations will not have big influence on the result of output of an automated line. Under this arrangement, if one machine tool stops, the entire line will not stop except for the section with the stopped machine tool. This is because the other sections will continue operation by filling and exhausting the buffer located before the stopped section with the broken machine tool. The capacity of buffers should provide enough supply to the line until the stopped machine tool is fixed. Such approach enables decrease in idle time and increase in line productivity **[5-7]**. In the case when reliability of sections is different, the problem of output of automated line should be decided by probabilistic approach of work time of sections. In our case of optimization of structure for the section-based automated line is discussed situation mentioned above.

The scheme of automatic production line that is segmented into *n* sections with embedded buffers *B* is presented on Fig.1 **[8]**.

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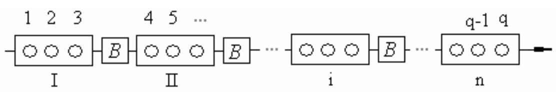

Fig. 1. Scheme of a section-based automated production line with *q* stations, segmented on *n* sections with *B* buffers

In real production condition the capacity of a buffer system is limited by many factors such as its cost, space and some technical parameters. Real buffer system can compensate only some part of the productivity losses of automated line and each section will have not only its own productivity losses but also so called uncompensated productivity losses due to the limited capacity of the buffer system located between sections.

The productivity of a section-based automated production line with limited capacity of buffers is expressed as **[8]**:

$$Q = \frac{1}{\frac{t_m}{q} + t_a + \frac{q\sum t_e}{n}[1+\Delta(n-1)]} = \frac{1}{\frac{t_m}{q} + t_a + \frac{q\sum t_e}{n}W} \quad (1)$$

where Q is the productivity of automated production line (part/min); t_m is the machining time (min/part); t_a is the auxiliary time (min/part); t_e is the index of productivity loss of single machine tool located in line due to idle time (min/part); q is the quantity of machine tools in an automated line; n is the number of sections in a line; $W = 1 + \Delta(n-1) > 1$ is coefficient of growth of non-cyclic losses due to uncompensated productivity losses by reason of the limited capacity of the buffer system; Δ is average coefficient of intersectional imposition of productivity losses

The coefficient of growth of productivity losses, *W,* of *i*-section of automated line is defined by coefficient Δ_i, that shows which part of productivity losses of *i*-section is transferred to the next section of a production line ($1 \geq \Delta_i \geq 0$). The expression $\Delta_i = 1$ means that there is no compensation of productivity losses because there is no buffer and all productivity losses are transferred to the next section, and $\Delta_i = 0$ means that there is buffer with endless capacity and productivity losses do not transfer to the next section of an automated line. For example $\Delta_i = 0.2$ means that 20% of productivity losses of section is transferred to the next section due to the limited capacity of buffer located between sections. The more buffer capacity the more compensation of productivity losses and the less transfer of productivity losses to the neighboring sections **[4-6; 8].** It was proven that the analytical expression of the coefficient of growth of productivity losses, W, of *i*-section by Eq. (1) is incorrect and cannot give true result **[9]**. The new analytical approach of productivity for section-based automated line is here developed.

II. ANALYTICAL APPROACH

Any section of production line can have additional productivity losses due to the following reasons:
1) any section can stop due to the overfilling by work pieces in the buffer located before a stopped section according to the technological route of machining parts;
2) any section can stop due to lack of work pieces in the buffer located after a stopped section.

The new corrected formula for the productivity of section-based automated production lines with limited capacity of buffer system is shown below **[9]**:

$$Q = \frac{1}{\frac{t_m}{q} + t_a + \frac{q\sum t_e}{n}\left[1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}\right]} \qquad (2)$$

This equation consists new formula of the coefficient of growth of productivity losses of section that has the following expression: $W = 1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}$. This expression is different if compare with one of Eq. (1). Numerically, new coefficient W (Eq. 2) gives higher result for small number of sections (n = 3 – 5) and less when the number of sections are large, if compare with same coefficient W of Eq. (1). The numerically results of two coefficients can have difference 10% – 15% that depends from coefficient Δ of the buffer.

The actual level of engineering has some limitations due to technical and technological reasons and with increase number of serial stations *q*, automated lines cannot give productivity increase due to reliability reasons. The new Eq. 2 for productivity rate of the section-based automated line gives the extreme of this function. In such case, it is possible to decide mathematical task of optimization of the structure of the automated line that means to find optimal number of serial stations *q* and optimal number of sections *n* **[10-12]**. This task can be defined by the first derivative of the Eq. 2. Since there are two variables, *n* and *q*, it is necessary to find first partial derivative of the Eq. 2 with respect to parameter *q* when other parameters are constant **[13].** This approach can give decision of optimal number of serial stations *q* of the section-based automated line.

$$\frac{\partial Q}{\partial q} = \frac{\partial}{\partial q}\left(\frac{}{\frac{}{q} + \quad + \frac{q\sum t_e}{n}\left[1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}\right]}\right) = 0,$$

giving

$$-\left(-\frac{}{q^2} + \frac{\sum t_e}{n}\left[1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}\right]\right) = 0, \qquad (3)$$

$$q_{opt} = \sqrt{\frac{n}{\left[1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}\right]\sum t_e}}$$

But it is not the necessary and sufficient condition for the judgment of Q value. In order to judge the Q value for the maximum productivity , the second derivative of Q is needed to find the optimized q . The second derivative of Eq. 2 is next

$$\frac{\partial^2 Q}{\partial q^2} = \frac{\partial}{\partial q^2}\left\{\frac{\frac{}{q^2} - \frac{\sum t_e}{n}\left[1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}\right]}{\left(\frac{}{q} + t_a + \frac{q\sum t_e}{n}\left[1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}\right]\right)^2}\right\} \qquad (4)$$

giving

$$-\frac{2\left\{\frac{t_m}{q^3}\left(\frac{}{q} + t_a + \frac{q\sum t_e}{n}\left[1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}\right]\right) + \left(\frac{}{q^2} - \frac{\sum t_e}{n}\left[1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}\right]\right)^2\right\}}{\left(\frac{}{q} + t_a + \frac{q\sum t_e}{n}\left[1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}\right]\right)^4}$$

The second derivative is $\frac{\partial^2 Q}{\partial q^2} < 0$, then the graph of Eq. (2) is concave down, it means the function of productivity rate Q has maximum value.

The formula of the maximum productivity rate of the section-based automated line is defined after substituting the expression of the optimal number serial stations q_{opt} (Eq. 3) into the Eq. 2 and after transformation will have next expression.

$$Q_{\max} = \frac{1}{2\Big/\sqrt{n\Big/\left[1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}\right]\frac{t_m\sum t_e}{n}} + t_a} \qquad (5)$$

Analysis of Eq. 2 shows that it can be the solution for the expected optimal number of sections n by criterion of maximum productivity. In such case, it is necessary to find first derivative of the Eq. 2 with respect to variable parameter n when other parameters are constant.

$$\frac{\partial Q}{\partial n} = \left(\frac{}{\frac{}{q} + \quad + \frac{q\sum t_e}{n}\left[1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}\right]}\right)' = 0 \qquad (6)$$

$$-\left(-\frac{q\sum t_e}{n^2}\right)\left[1 + \frac{2\Delta(1-\Delta^{n-1})}{1-\Delta}\right] - \left(\frac{q\sum t_e}{n}\right)\left(\frac{2\Delta}{1-\Delta}\right)[-(n-1)\Delta^{n-2}] = 0$$

After transformation Eq. 6 will have next expression

$$\Delta^n = (1+\Delta)/2n \qquad (7)$$

Expression (7) is a transcendental equation where roots cannot be found analytically. Solutions of a transcendental equation can be found by using of graphical methods. The left hand side of the equation is expressed as exponential function, and the right hand side is a rational function.

Let us present $f(n) = \Delta^n$, and $g(n) = (1+ \Delta)/2n$. For a given $\Delta = 0.1$, the functions $f(n)$ and $g(n)$ are decreased with increasing of n. The intersection of $f(n)$ and $g(n)$ is impossible based on their natures. Fig. 2 gives graphical presentation where there is no intersection of the two equations $f(n)$ and $g(n)$ for any number of sections n. It proofs there is no solution of optimal number of sections n for section-based automated line.

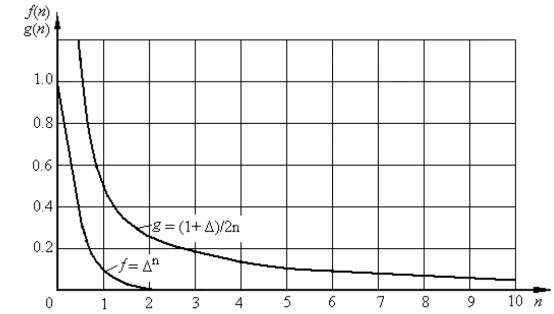

Fig. 2. Graphical solution of optimal number of sections for the automated line

In an industrial area there are many types of automated line where buffers are embedded after each station, i.e $q = n$, number of stations are equal to number of buffers. In such case, it is necessary to substitute q in place of n. The equation of productivity rate for this type of automated lines is as follows:

$$Q = \frac{1}{\frac{t_m}{q} + t_a + \left[1 + \frac{2\Delta(1-\Delta^{q-1})}{1-\Delta}\right]\sum t_e} \qquad (8)$$

Optimal number of q stations that gives maximum productivity is defined by the first derivatives of Eq. 8.

$$\frac{\partial Q}{\partial q} = \frac{\partial}{\partial q}\left(\frac{}{\frac{}{q} + \quad + \frac{[1+2\Delta(1-\Delta^{q-1})]}{(1-\Delta)}\sum t_e}\right) = 0 \qquad \text{giving}$$

$$-\left[\left(-\frac{}{q^2}\right) - \frac{2\Delta(q-1)\Delta^{q-2}}{(1-\Delta)}\sum t_e\right] = 0, \qquad -\frac{}{q^2} - \frac{2(q-1)\Delta^{q}}{(1-\Delta)\Delta}\sum t_e = 0,$$

Analysis of this derivative shows no solution exists about of the number of stations q in automated line with buffers embedded after each station because solution has minus sign. In such case, Eq. 3 can be used to calculate the optimal number of stations q that gives maximum productivity of the section-based automated lines.

III. A CASE STUDY

Assume that $\Delta = 0.1$ is the average coefficient of the intersectional imposition of productivity losses, $n = 10$ is the number of sections in the automated production line, q = 20 is the number of machining stations in the line, $t_m = 1$ min/part is the machining time, $t_a = 0.3$ min/part is the auxiliary time, and $\Sigma t_e = 0.02$ min/part is the productivity losses due to the reliability of machine units of production line.

After substitution of these data into Eq. 2, the result of the automated line productivity as a function of the number of sections in a line is presented in Fig. 3. Fig. 3 represents diagrams of productivity change of automated lines divided on sections with increase of number of stations q. The optimal number of machining stations q_{opt} by different number of sections n can give maximum productivity of the line. Eq. 4 gives mathematical solutions of the optimal number of machining stations that cannot be an integer number. In such case, it is necessary to take the nearest integer number of stations in automated line. This decision will not give big error on solution of maximum productivity of section-based automated line because no graphs have sharp form at extreme area of functions.

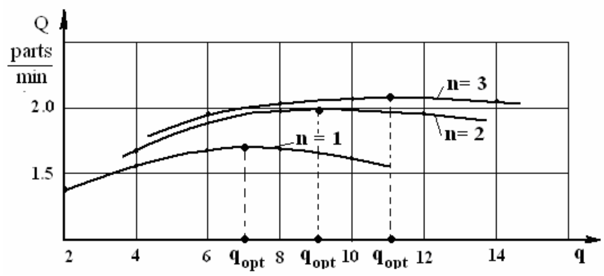

Fig. 3. Productivity increase of section-based automated line with n sections calculated from Eq. 2 versus the number of stations q.

In the case when number stations, q, is equal to the number of sections n, $(q = n)$, the change of the productivity of section-based automated line shows that there is an asymptotical limit to the productivity line with increasing number of sections n (Fig. 4), and there is no optimal solution. This result responds to the solution that was received by the Eq. 8.

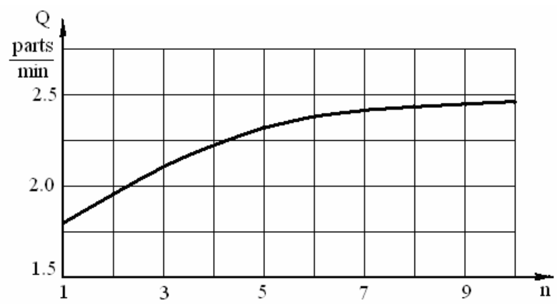

Fig. 4. Productivity increase of automated line with $q = 10$ stations calculated from Eq. 2 versus the number of sections n.

IV. RESULTS AND DISCUSSION

Eq. 2 enables calculation of the productivity of section-based production lines and predicts the real output. Eq. 3 enables calculation of the optimal number of stations in section-based automated line by criterion of maximum productivity rate. Eq. 4 enables calculation of the maximum productivity of section-based automated line and this equation enables evaluation on the variations of productivity of automated line with different structures.

V. CONCLUSION

The equation for the productivity of section-based automated production lines, the equation for optimal number of stations, and the equations of maximum productivity of production line have been obtained. The equations will be useful in modeling the output of automated production line, defining the structure, determining the number of sections and the number of stations according to the level of productivity and enable calculation of economical and optimal parameters of structure of an automated production line by criterion of productivity rate. Results of this paper can be used in project stage of automated line design.

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