Optimization of Distributed Processors for Power System: Kalman Filters using Petri Net

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***Abstract*—The growth and interconnection of power networks in many regions has invited complicated techniques for energy management services (EMS). State estimation techniques become a powerful tool in power system control centers, and that more information is required to achieve the objective of EMS. For the online state estimator, assuming the continuous time is equidistantly sampled with period Δt, processing events must be finished within this period. Advantage of Kalman Filtering (KF) algorithm in using system information to improve the estimation precision is utilized. Computational power is a major issue responsible for the achievement of the objective, i.e. estimators' solution at a small sampled period. This paper presents the optimum utilization of processors in a state estimator based on KF. The model used is presented using Petri net (PN) theory**.

***Keywords*— Kalman filters, model, Petri Net, power system, sequential State estimator**.

I. INTRODUCTION

TODAY, power system networks are more complicated and interconnected; therefore, the demand of modern Energy Management Systems (EMS) has also increased. State estimation (SE) is among tasks carried out at Energy Control Center (ECC). In order to estimate the real-time system state, a powerful processor and reliable communication must be in place [1]. Telemetry system gathers measurement quantities in the system and transmits them to ECC [2], [3], [4], [5], [6], [8], [9] and [12]. Using these measurements, SE obtains the best estimate of system state variable, i.e., complex bus voltages (voltage magnitude, $\mathbb{V}$ and voltage phase angle, θ) [4] and [5]. If the data from the system is sufficient for running state estimation, the network is said to be observable [6].

In power system, a reliable estimate of the system must be determined before any security assessment or control actions taken [4] and [5]. Reliable estimation depends on the state estimator algorithm and the information available.

Recently, there has been an increasing interest in various types of state estimation algorithms [4] and [19]. Currently, Weighted Least Squares (WLS) and Kalman Filtering are the most widely used. In this paper, allocating processing event to the distributed processors for KF estimator is discussed. The KF objective function is to minimize estimation error covariance. Kalman Filtering advantage over the WLS is based on using system information to improve state estimation precision [1], [20], [21] and [23]. The use of Petri Net model is to optimize the processing power in case of central state estimation (CSE) KF algorithm by allocating operations (processing events) to the distributed processors assuming no limit on communication channel.

Discussion of KF estimator in this paper is based on its advantages mentioned above. When well designed, fewer processors are able to perform the intended function while maintaining the speed of the system. It should be noted that, the KF estimator has the capability of tracking system state status versus time.

Many algorithms have been published regarding SE algorithms, such as the use of binary genetic algorithm explained in [7]. In [8] and [9] tabu search algorithms are explained in these publications, an iterative search that starts from some initial feasible solution and attempt to determine a better solution in the manner of hill-climbing algorithm, etc. Several numerical methods are explained in [10],[11] and [12]. Particle swarm optimization (PSO) is one of the global optimization method expressed in [13] where the basic assumption behind the algorithm is the birds finding food by flocking and not individually. In [14] the exploitation of the optimization using genetic algorithm (GA) is explained.

II. POWER SYSTEM STATE ESTIMATION

The state estimator calculates an estimate state of the bus voltage magnitudes and angles, based on the nonlinear equations relating the measurements vector z and the state vector x. The state estimation problem is usually formulated mathematically as a weighted least square (WLS) problem and solved by an iterative scheme [10] or as a Kalman Filtering problems are the most kind of algorithms used. Power system state estimation is based on the following stochastic difference models [20]

$$x_k = Ax_{k-1} + w_{k-1} \qquad (1)$$

$$z_k = Hx_k + v_k \qquad (2)$$

where $x \in \Re^n$ and $z \in \Re^m$ are state and measurement vectors, respectively. While the mutual independent random variables w and v represents process noise and measurement noise respectively. The mutual random probabilities are as presented in (3) and (4).

$$p(w) \approx N(0, Q) \qquad (3)$$

$$p(w) \approx N(0, R) \qquad (4)$$

where Q and R are process and measurement noise covariance, respectively.

III. KALMAN FILTERING

A. Optimization Problem

As explained in section I above, Kalman Filtering algorithm has an advantage of utilizing available information so that it can improve the estimation processing. KFs have the ability to fuse multiple sensor readings together, taking advantages of their individual strength, while gives readings with a balance of noise cancelation and adaptability. It is a tool for estimation and performance analysis of estimators [1].

Suppose that a measurement has been made at time t_k and that the information it provides is to be applied in updating the estimate of the state x of a stochastic system at time, t_k then from (2), then the system is said to be observable if the matrix H which related measurements to state at time k is of rank n if the state variables vector $x \in \Re^n$ [10]. The observability depends, not only the network topology but also the measurements units (sensors) locations within the power system network [20].

B. Optimization Problem

The Kalman estimator objective is to minimizes the expected value of the square of the *posteriori* state estimation error [1], [20], and [21]

$$\min \quad E[x_k - \hat{x}_k^-] \qquad (5)$$

where $\hat{x}_k^- \in \Re^n$ is the priori at step k and $\hat{x}_k^+ \in \Re^n$ to be the posteriori state estimate at step k given measurement z_k.

The KF has two dependent equations, the *predict* equation as well as the *update* equation. The *predict* equation forward the current state and error covariance estimates to obtain the *a priori* estimates for the next step [1],[20 and [21] given that

$$\hat{x}_k^- = A\hat{x}_{k-1} \qquad (6)$$

$$P_k^- = E[(x_k - \hat{x}_k^-)(x_k - \hat{x}_k^-)^T] \qquad (7)$$

$$P_k^+ = E[(x_k - \hat{x}_k^+)(x_k - \hat{x}_k^+)^T] \qquad (8)$$

The KF update equations are related by feedback constant $K_k \in \Re^{n \times m}$ known as *Kalman gain* or *blending factor* to incorporate measurement z_k into the a *priori* estimates P_k^- to obtain an improved (i.e. minimized error covariance) a *posteriori* estimate P_k^+ expressed as

$$K_k = P_k^- H_k^T [H_k P_k^- H_k^T + R_k]^{-1} \qquad (9)$$

Therefore, an updated estimated $\hat{x}_k^+$ based on the observation z_k that is a function of the *a priori* estimate and the measurement z is given as

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(z_k - H_k\hat{x}_k^-) \qquad (10)$$

For the discrete-time each time t_k, measurements are received from remote measuring units and KF estimator obtains the state estimation. In order to obtain fast KF estimator, processing stages must be optimized. The following stages present the KF hardware model.

IV. KALMAN FILTERING MODEL

A. Petri Net

Fig. 1 depicts the Petri Net (PN) representation of general KF power system state estimator PN algorithm. The kth state variables vector x_k and measurement sensitivity matrix are used to obtain the measurements vector shown in (1).

An updated estimated $\hat{x}_k^+$ based on the observation z_k that is a function of the *a priori* estimate and the measurement z. Petri nets models are logic based-model interpreted by input places, transitions, and output places. Input places are defined by: preconditions, input data (or signals), buffers, etc. Transitions according to PNs are events, computational steps (or processors), task or jobs, etc. Output places represents post-conditions, output data (or signals), resources released, buffers, etc [22] and [23].

Petri net is a bipartite directed graph whose nodes belong to two different classes (places and transitions) and the edges (arcs) are allowed to connect only nodes of different classes. a Petri net as a 5-tuple defined by [22] and [23].

$$PN = (P; T; F; W; M_0) \qquad (11)$$

where $P = \{p_1, p_2, \ldots, p_n\}$ is the set of n places;
$T = \{t_1, t_2, \ldots, t_m\}$ is the set of m transitions;
$F \subseteq (P \times T) \cup (T \times P)$ is the set of arcs;
$W: F \to \{1, 2, \ldots, \}$ is the weight function; and
$M_0: P \to \{0, 1, 2, \ldots\}$ is the initial marking.
Petri net structures without any initial marking are denoted by N, such as; $N = (P; T; F; W)$.

Transition enabling and firing are sequential steps operated at the transition. Before firing, transition t must be enabled by the presence of *tokens* (activeness) in all of its input places.
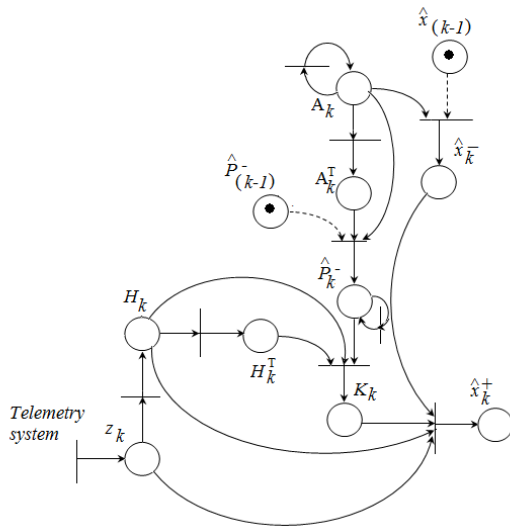
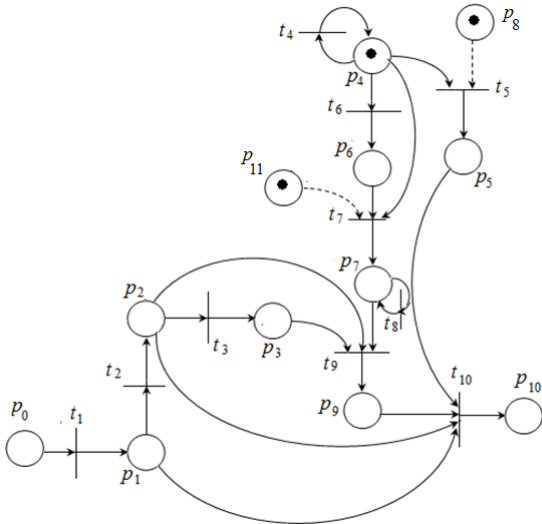Fig. 1 Kalman Filter estimator Petri Net Algorithm

Fig. 2 Kalman Filter estimator Petri Net Model

V.   METHOD USED EXPLAINED

The mechanism is made in such a way that fast processing technique is obtained. At k unit of time upon the receipt of the measurement readings in the control center (token made available in place (p_0), thus, t_1 is enabled the measurement Jacobian (sensitivity) matrix is formed. This matrix is sent to the buffer which then used to computer P_k (3). At this moment t_3 is enabled assumed that the previous P_{k-1} is in buffer (has token). Since the information in the buffer will be needed later when state variables are estimated, then t_1 will have to fire when t_3 is made to fire so as to keep the token at buffer, p_1 place. The information in place p_3 is used to find the Kalman gain K. Therefore, t_4 fires to place token at p_4. Finally, the

token at p_4 enables t_5 and state estimation variables vector is estimated.

A.   Parallelism

The parallelism in PN model defines the related firing (processing) steps in the system. In this paper, the advantage of *parallelism* behavior of PN model is considered so as to obtain the number of firing (i.e. processing events) in each parallel sequence. For simplicity, each processing sequence takes t_p unit of time. In practice, process time varies in each processing sequence. In order to obtain the minimum time for each parallel sequence, find the sum of processing time in each processor in the parallel sequence. The parallel sequence with the maximum processing time defines the minimum time of the system processing time T_s.

B.   Processing matrix

Processing matrix $\mathcal{B}$ presents the hierarchical processing time allocation for each processor. The starting processor in the parallel sequence is of the highest hierarchy level. The level of hierarchy decreases from starting processor by one towards the next processor, and so on. This can be achieved by the use of firing sequence and state equation.
Knowledge of the events (transition) hierarchy level helps in determination of the sequence of the events. Same events in a hierarchical level cannot be processed by the same processors in an optimal operation of the distributed processors. Therefore, each even will be allotted in a separate processor. The next level will start after the previous level has been finished. The released processors will be available for the transition events in the next level.

C.   State Equation

The marking vector $M_k \in \Re^m$, and incident matrix $A \in \Re^{m \times n}$ are used to obtain the state equation given by

$$M_k = M_{k-1} + A^T u_k \qquad k = 1, 2, \dots \qquad (12)$$

where u is a firing sequence $\{u_1, u_2, \dots\}$.
For example, consider the simple PN model shown in Fig. 3, with token in p_1 and p_2 as a precondition. For simplicity, assume the equal distribution of a unit processing time t_p. Note that t_4 does not active any other place but itself. It can be synchronized with t_1 and t_2 so that anytime t_1 or t_2 fires, the token is still maintained in p_2. The token in this place will be needed by t_3 so as to fire. Since it is a synchronized transition, and does not change the state of the system when fired alone, it is termed as a *dummy* transition. However, dummy transitions are very important when representing buffers in a PN model. Dummy transitions are not included in the parallel sequences.

The parallel sequences are as follows:

Parallel processing sequences

$\sigma_1 = \{t_1, t_3\}$
$\sigma_2 = \{t_3\ \}$
$\sigma_3 = \{t_2, t_3\}$

The σ_μ $\mu = \{1,2,\dots\}$ is the μth parallel processing sequence.

Hierarchical level is as follows: Level 1consists of t_1 and t_2 since they can fire at the starting time. Level 2 is t_3 since it can only fire after the firing of t_1 and t_2. Then $M_0 = [1,1,0]^T$; $M_1 = [0,1,1]^T$.The weight function can be extended towards the buffer level, such that it can only contain one token so that although the token in p_2 enables t_2 but it does not fire. This is because, its output buffer, place p_4 is full. Hence, $M_1 = [0,0,1]^T$ and processing matrix is obtained as

$$B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

System processing stages will be $t_1 \cup t_2$ then followed by t_3. In order to optimize the uses of processors, while maintaining the minimum possible system processing time, two processors can be used. The first processor $P_{s1}$and the second processor P_{s2} processes t_1 and t_2, respectively at the same time. After a period of t_p, processes t_1 and t_2 will be finished. The next processes t_3 will be processed within the next t_p period. During this period, P_{s1} and P_{s2}will be idle and thus can be used to process t_3. Hence, only two (2) processors can be used for this system instead of three (3) processors as shown in Fig. 4. It can be seen easily when dealing with a simple system such as the one shown in Fig. 3. Many practical systems are complex and the use of PN model analysis is appreciated.
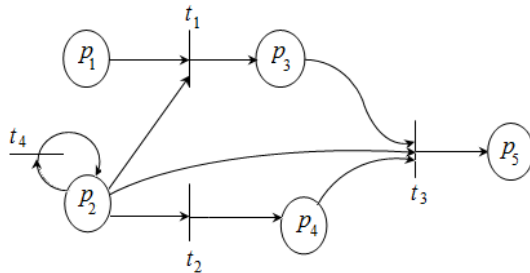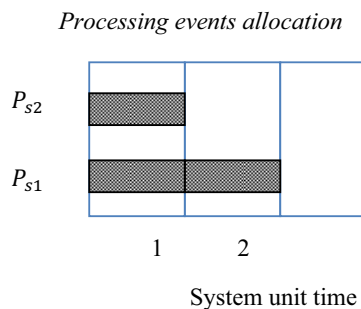

Fig. 3 Example of a simple Petri Net Model

Processing events allocation

System unit time

Fig. 4 Allocating processing events

VI. RESULTS

The KF state estimator shown in Fig. 2 is analyzed and results obtained are presented in this section.   In this model, there are several parallel firing sequences. Logically, the KF estimator solution at k will depend on availability of measurements readings from telemetry system. Obtaining z_k will place a token at place p_0, which will enable t_1. These are considered as precondition sets. The following are the parallel sequences in KF model shown in Fig. 2.

Parallel sequences
$\sigma_1 = \{t_1, t_{10}\}$
$\sigma_2 = \{t_1, t_2, t_{10}\}$
$\sigma_3 = \{t_1, t_2, t_3, t_9, t_{10}\}$
$\sigma_4 = \{t_1, t_2, t_9, t_{10}\}$
$\sigma_5 = \{t_{10}\}$
$\sigma_6 = \{t_6, t_7, t_9, t_{10}\}$
$\sigma_7 = \{t_5, t_{10}\}$

The σ_μ $\mu = \{1,2,\dots\}$ is μ^{th}parallel processing sequence.

Hierarchical level is as follows: Level 1consists t_1, t_5, and t_6 since they can fire at the starting time. Level 2 is set by firing of first level are t_2 and t_7. Level 3 consist of t_3. Level 4 consist of t_9 and level 5 consist of t_{10}

According to preconditions set by k^{th}-time processing cycle, and z_k received measurements, initial state of KF power estimator shown in Fig. 2 including dummy, becomes $M_0 = [1,0,0,1,1,1,0,0,0]^T$. Excluding dummy processors t_4 and t_8, the reduced initial system state is given as $M_0 = [1,0,0,1,1,0,0,0]^T$.

Three (3) transitions in level 1 require three processors which represent the maximum number of processors required for the system. Each processor is allotted by the transition event in level 1 such as shown in Fig. 5. The next level starts after the period of t_p. Since this level has only two events, only two idle processors will be allotted with their transition events, say, The first processor $P_{s1}$and the second processor P_{s2}. The third processor P_{s3} will be idle during this period. The third level has only one transition event which can be allotted to the third processor P_{s3} for equal processing distribution, although all processors are idle for this task. Finally the last level, level 4 also has only one transition event and can be allotted to any processor since all processors are idle.

The processing matrix for the KF model is given as

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
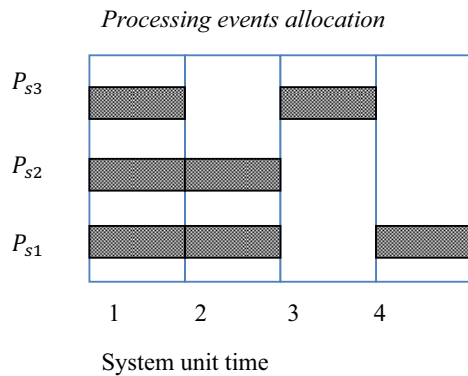
Processing events allocation

Fig. 5 Allocating KF estimator processing events

VII. CONCLUSION

This research work concludes that, if all processors have the same speed, only logical allocation of a transition event will be based on the task distribution. That is, if three processors are idle and two tasks have to be allotted then the processor that has been idle for a longer period would be a suitable for the task. If processors speeds are not equal, then the period of t_p of that level (task with maximum transition period) can lead in the processor's choice among the idle processors, so as to maximize the system speed. During the allotting process, the speed and the idle period have to be balanced.

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